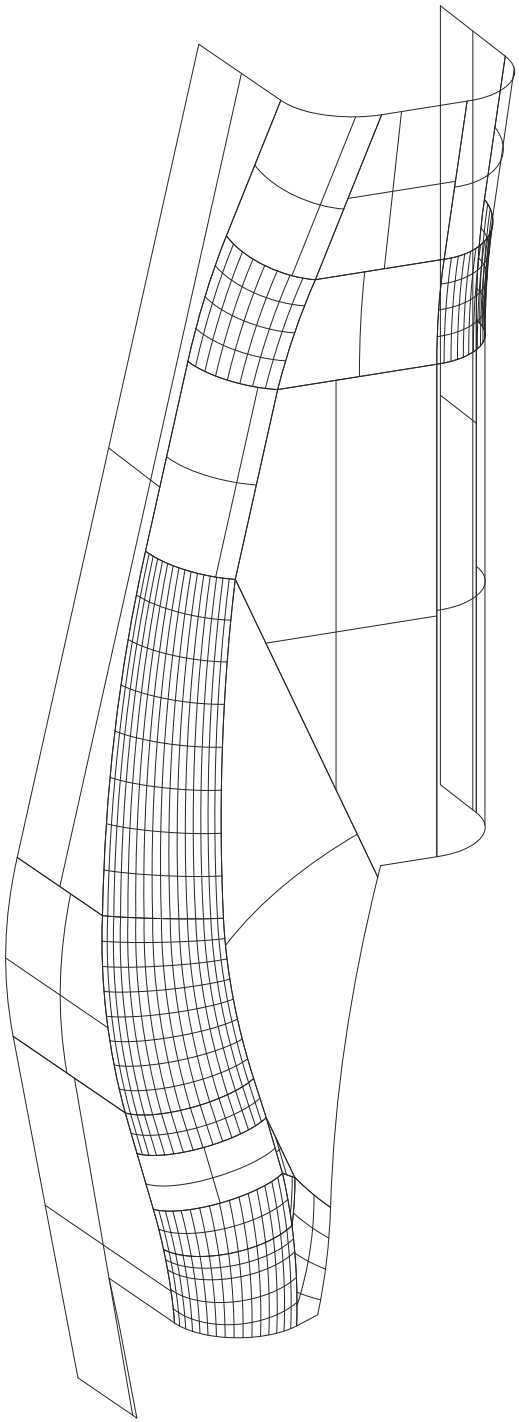
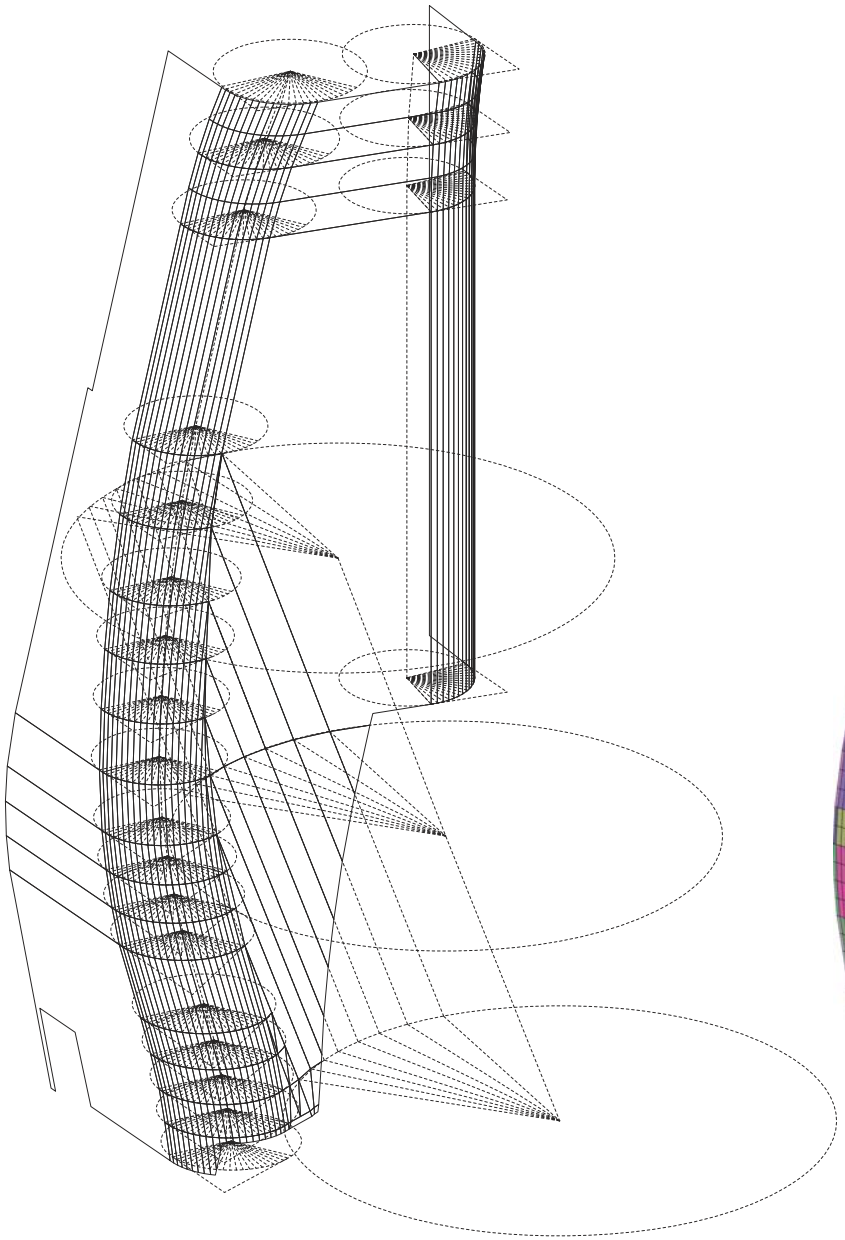
MORPH SEMINAR 2009

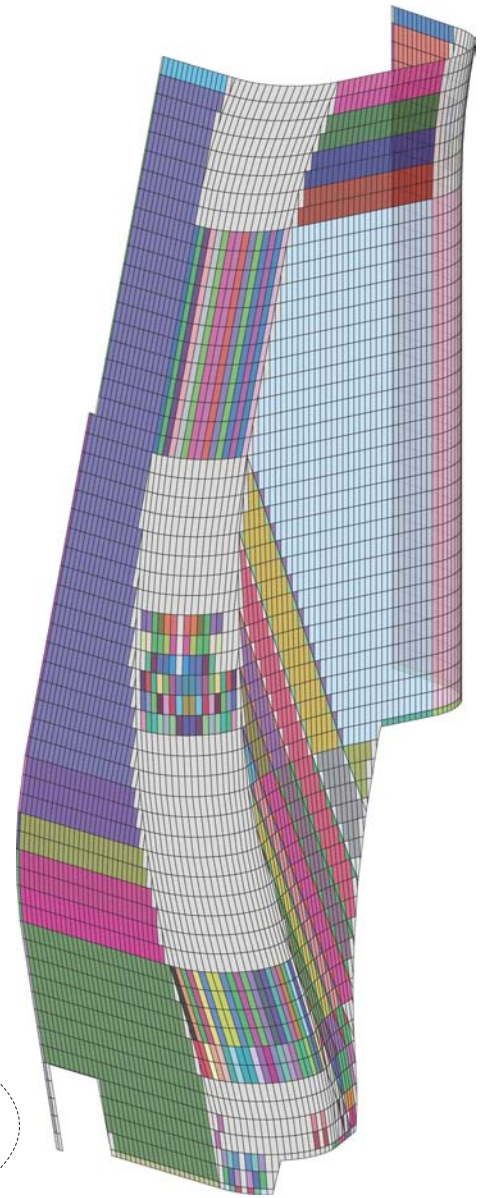
MATHEMATICS FOR SYSTEMATIC MODELING

WEEK1 POINT AND VECTOR

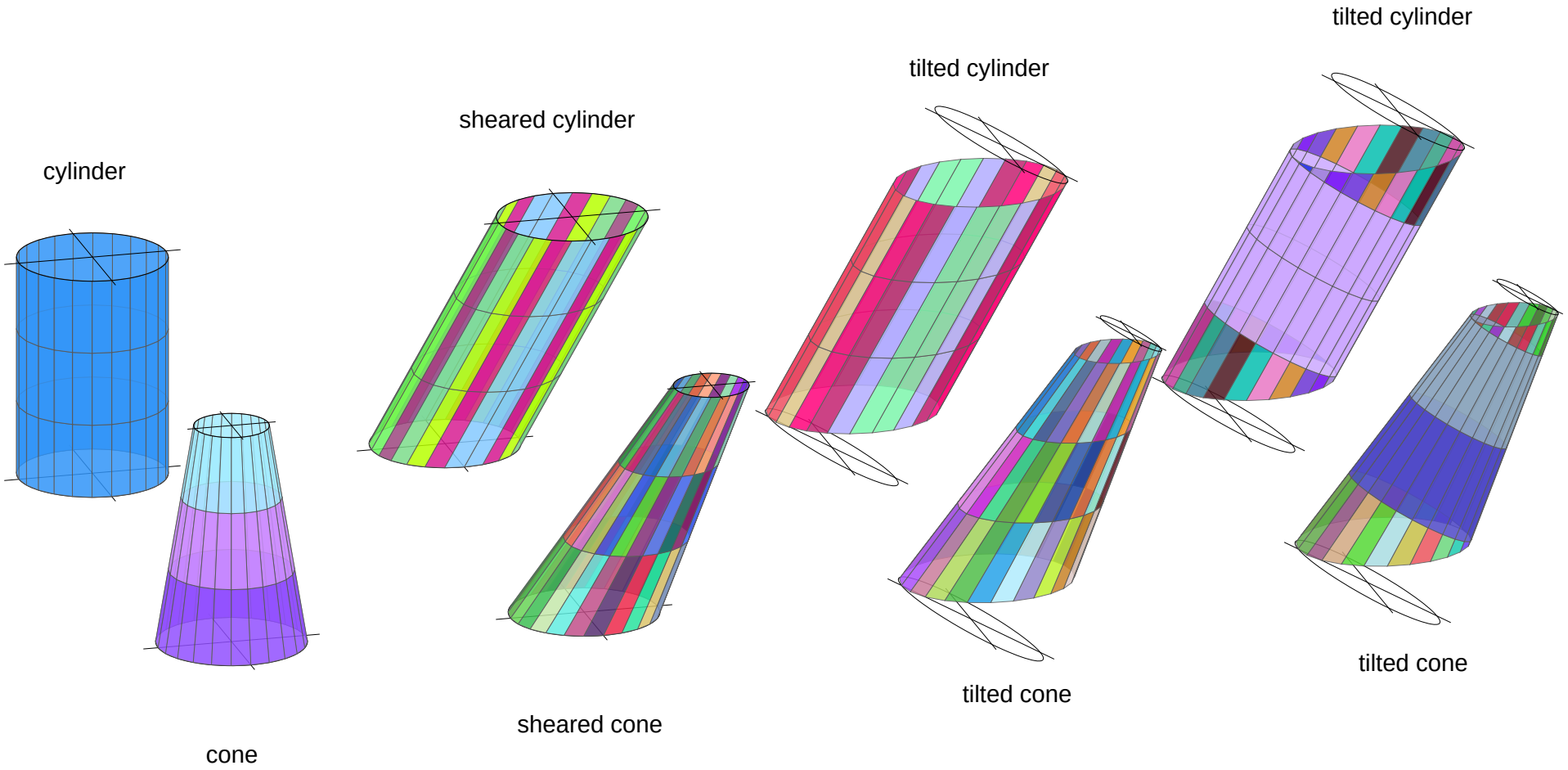
UNDERSTANDING OF GEOMETRY



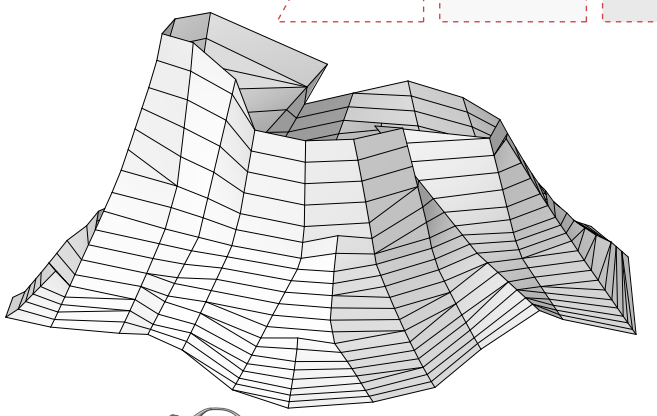


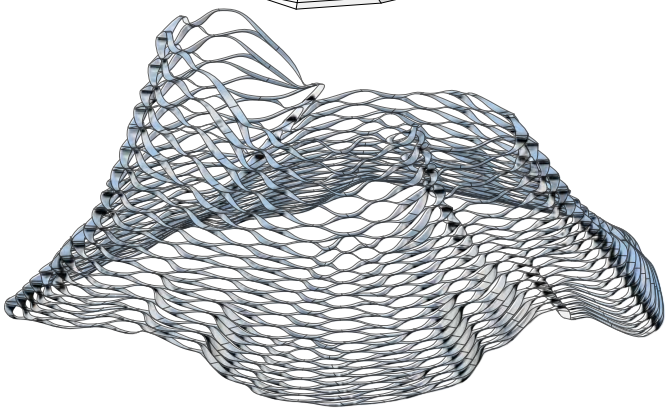


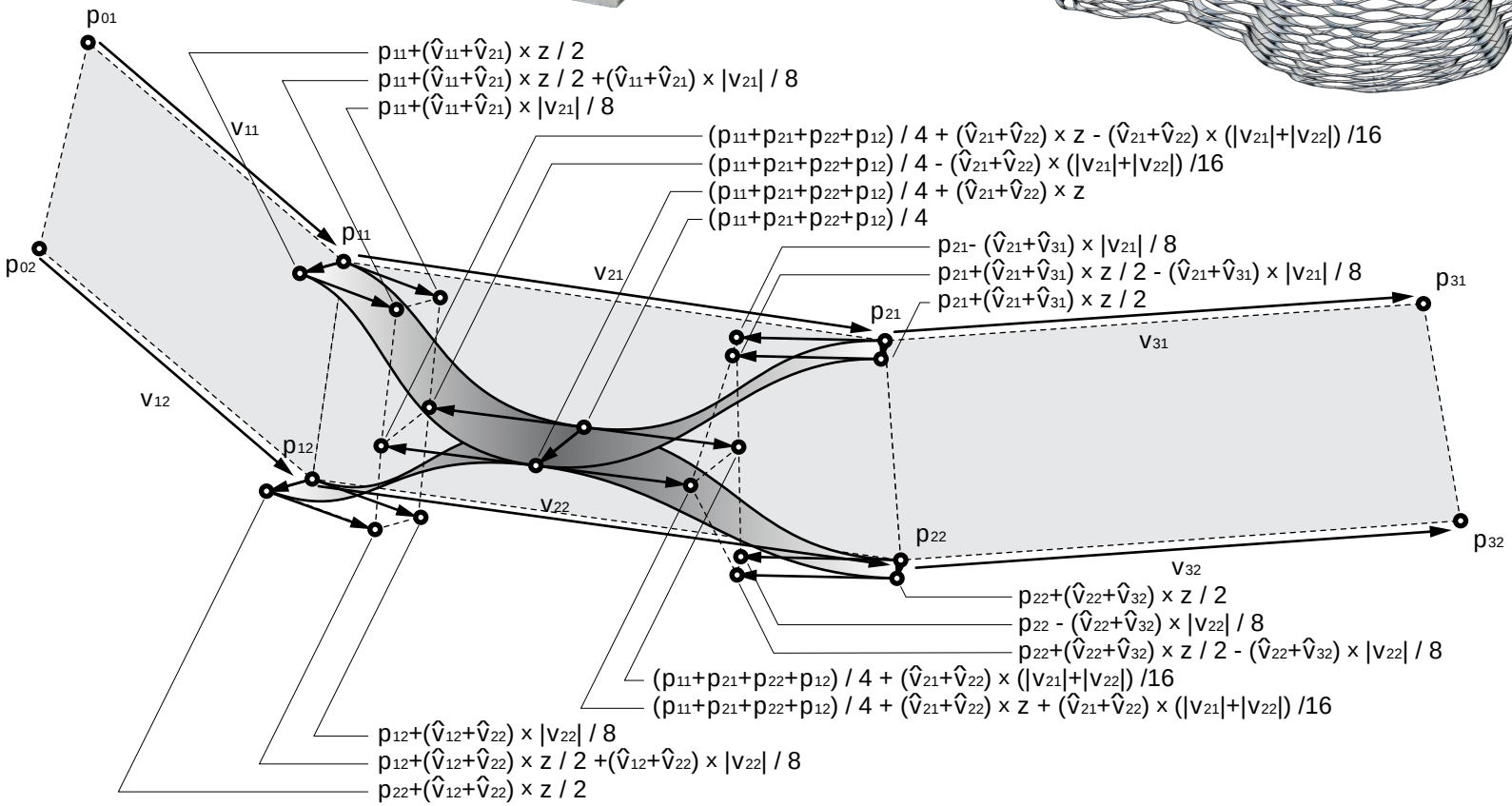
RATIONAL GEOMETRY AND PANELIZATION



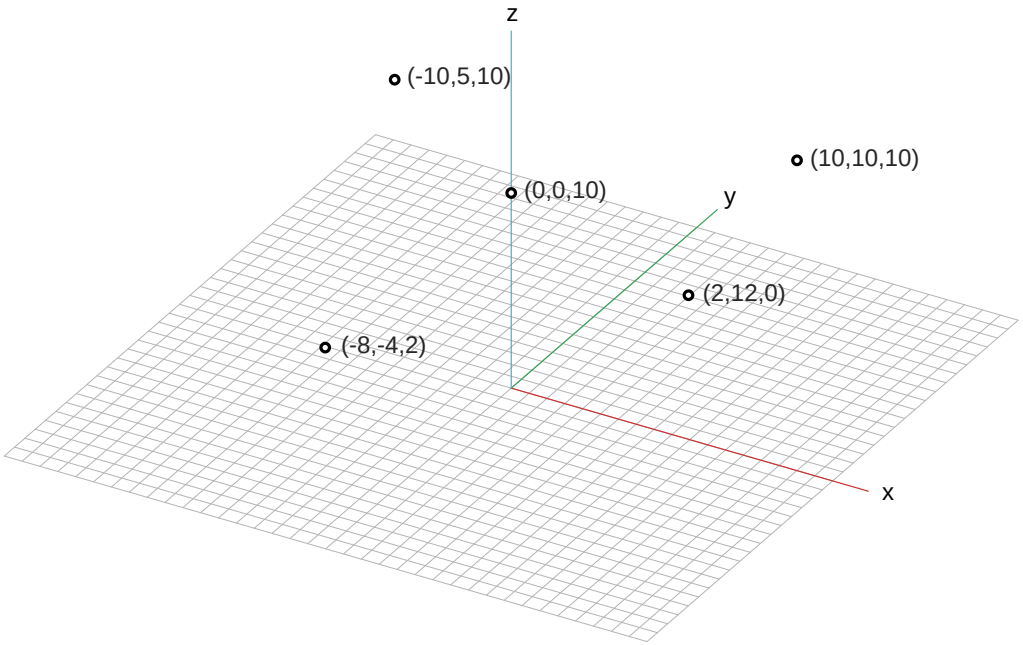
VECTOR MODELING EXAMPLE



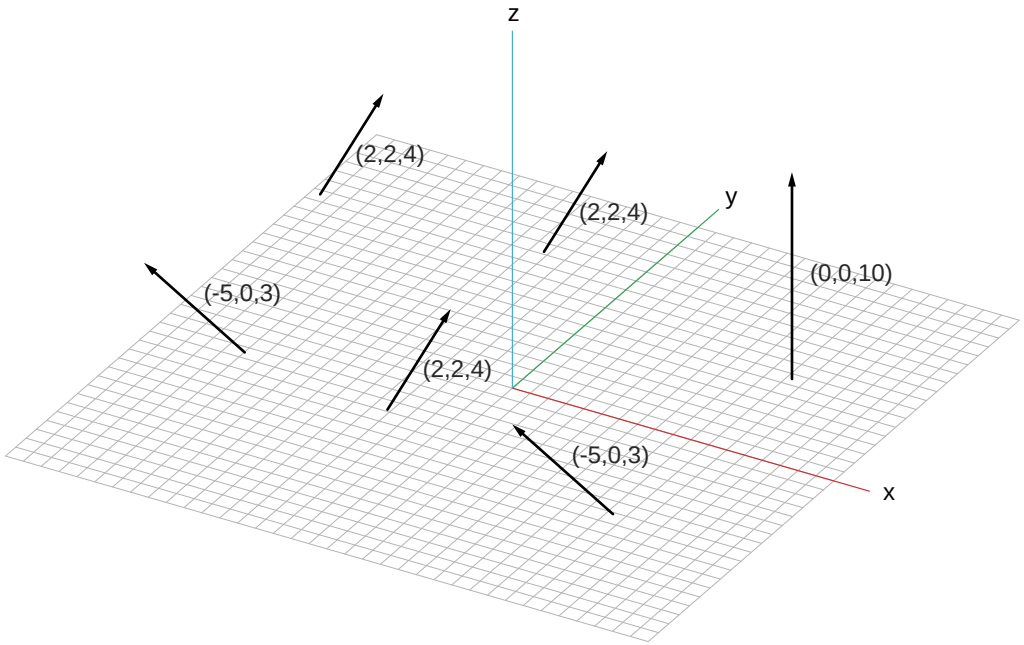




POINT AND VECTOR



POINT



VECTOR

p = (x, y, z)

p = (x, y, z)

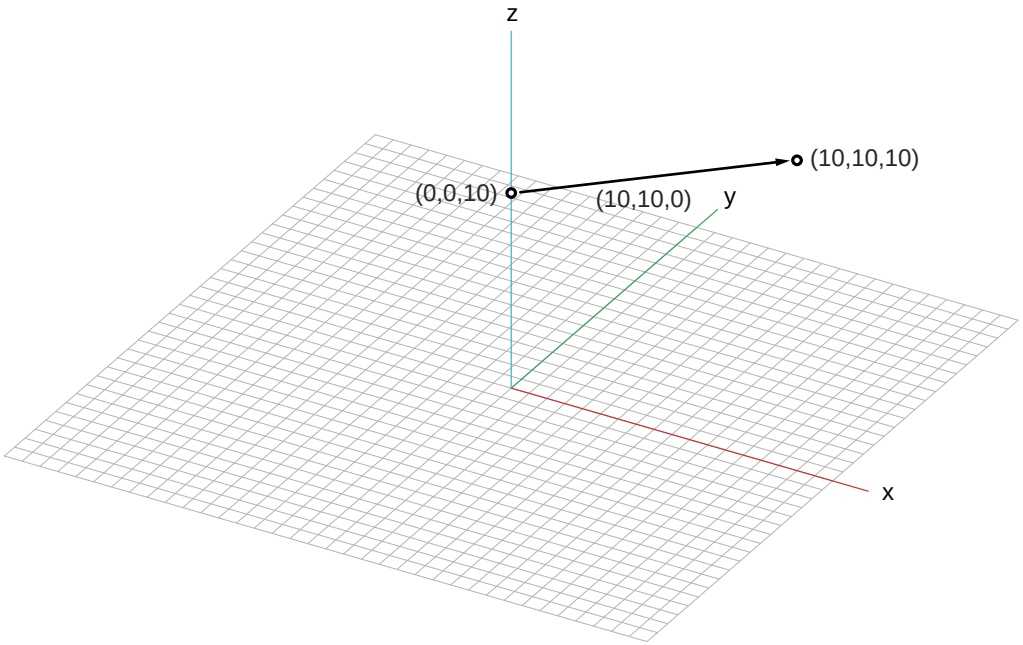
$\vec{p}$ = (x, y, z)

v = (x, y, z)

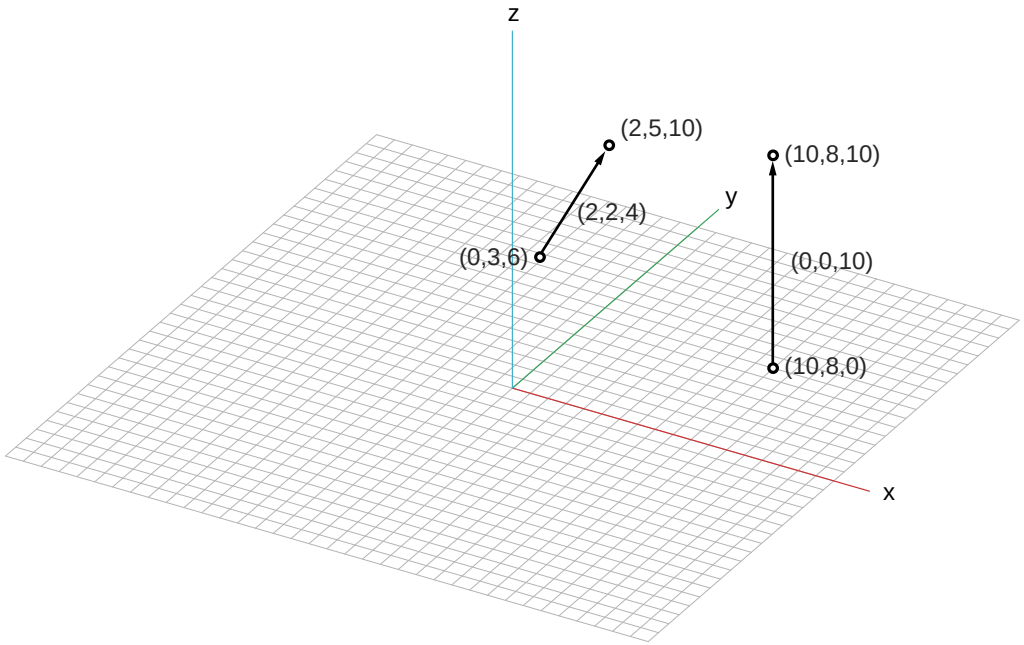
v = (x, y, z)

$\vec{v}$ = (x, y, z)

RELATIONSHIP BETWEEN POINT AND VECTOR

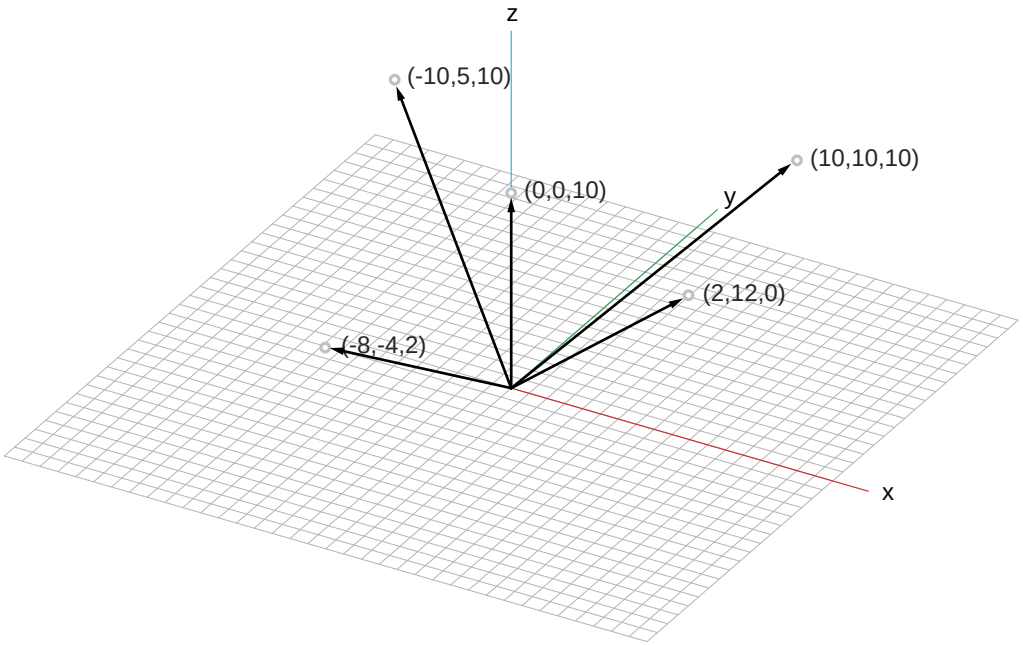


p = (x, y, z)



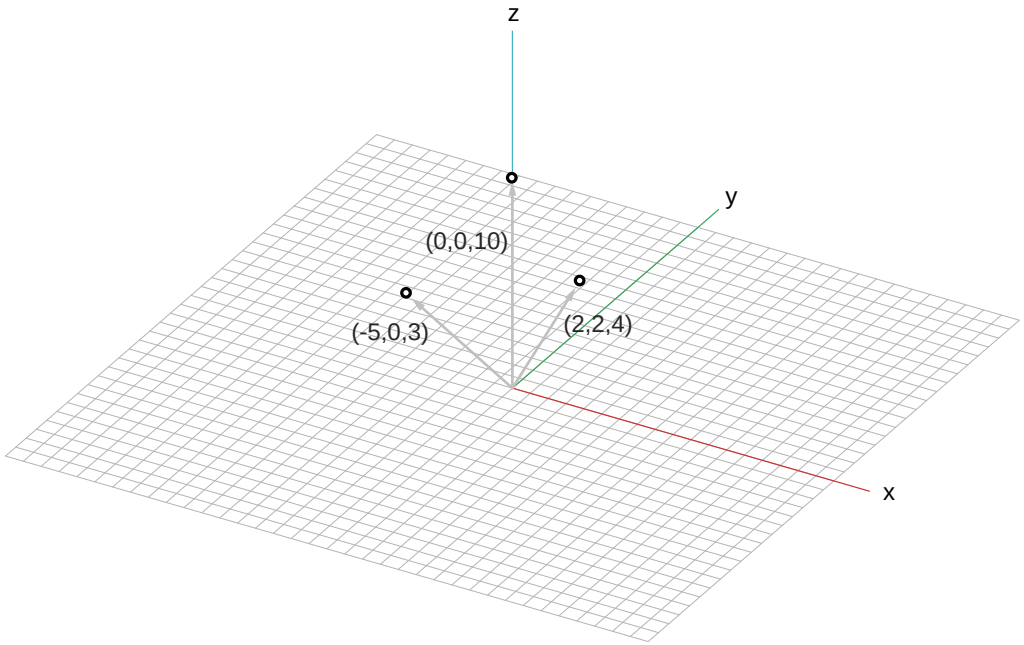
v = (x, y, z)

RELATIONSHIP BETWEEN POINT AND VECTOR



VECTOR

v = (x, y, z)

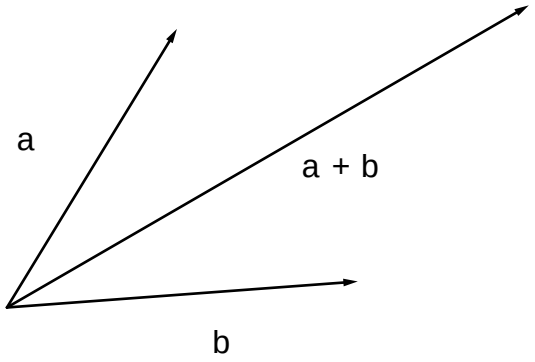


POINT

p = (x, y, z)

VECTOR OPERATION : ADDITION

Definition

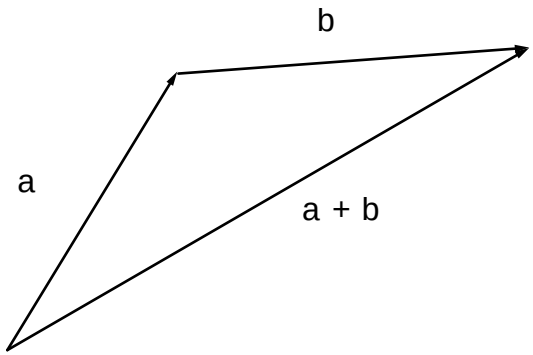


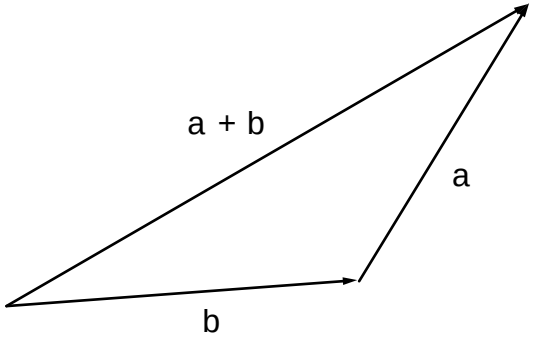
$a = (a_x, a_y, a_z)$

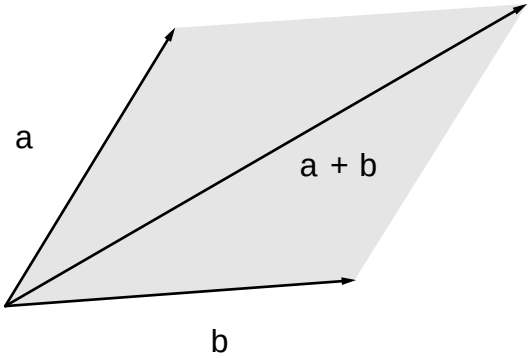
$b = (b_x, b_y, b_z)$

$a + b = (a_x+b_x, a_y+b_y, a_z+b_z)$

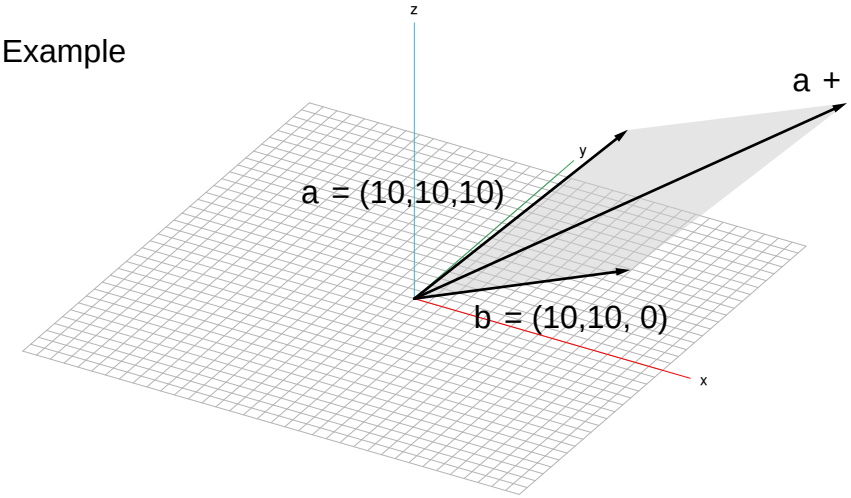
Geometrical Interpretation







Example



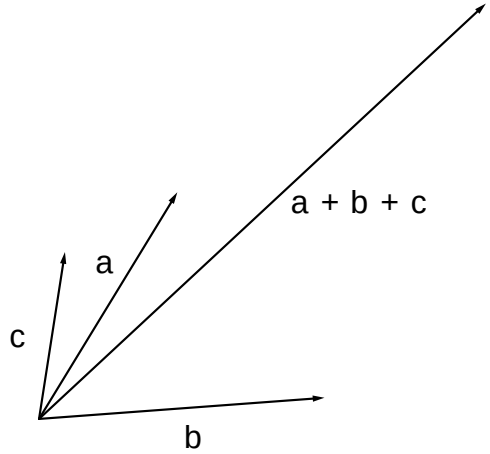
$a = (10, 10, 10)$

$b = (10, 10, 0)$

$a + b = (10+10, 10+10, 10+0)$

$= (20, 20, 10)$

VECTOR OPERATION : ADDITION

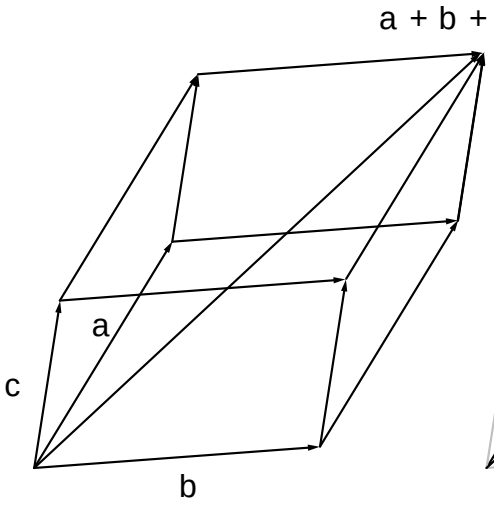


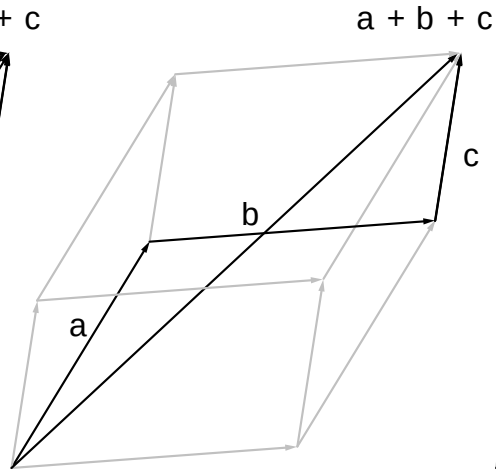
$a = (a_x, a_y, a_z)$

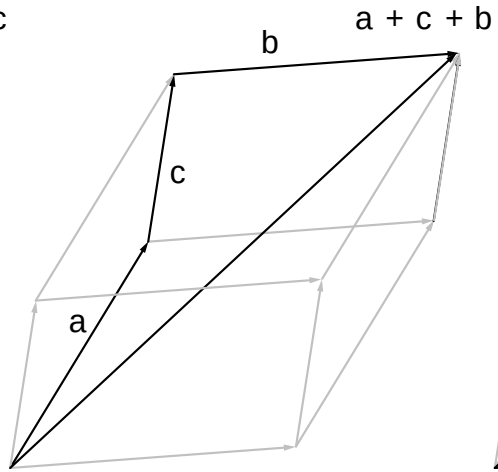
$b = (b_x, b_y, b_z)$

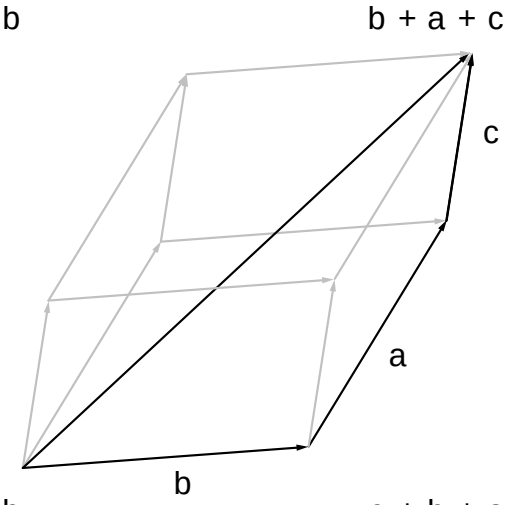
$c = (c_x, c_y, c_z)$

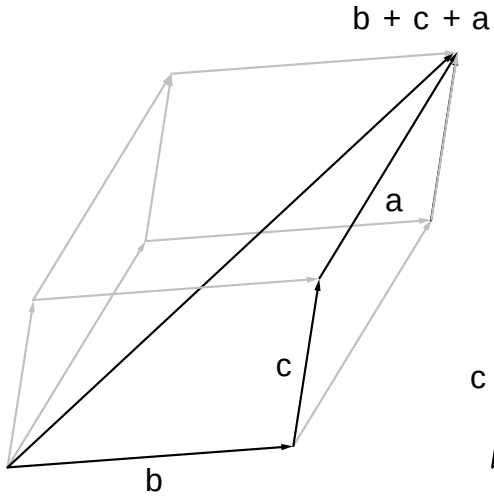
$a + b + c = (a_x+b_x+c_x, a_y+b_y+c_y, a_z+b_z+c_z)$

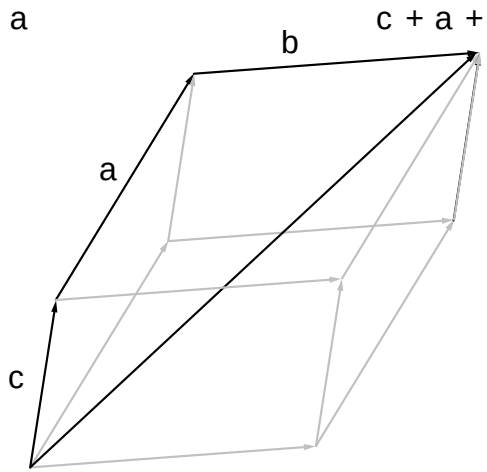


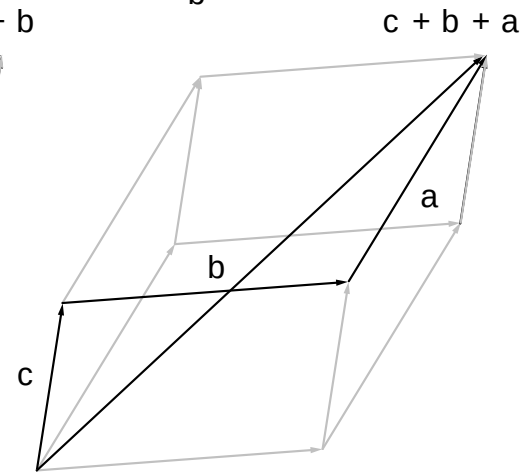




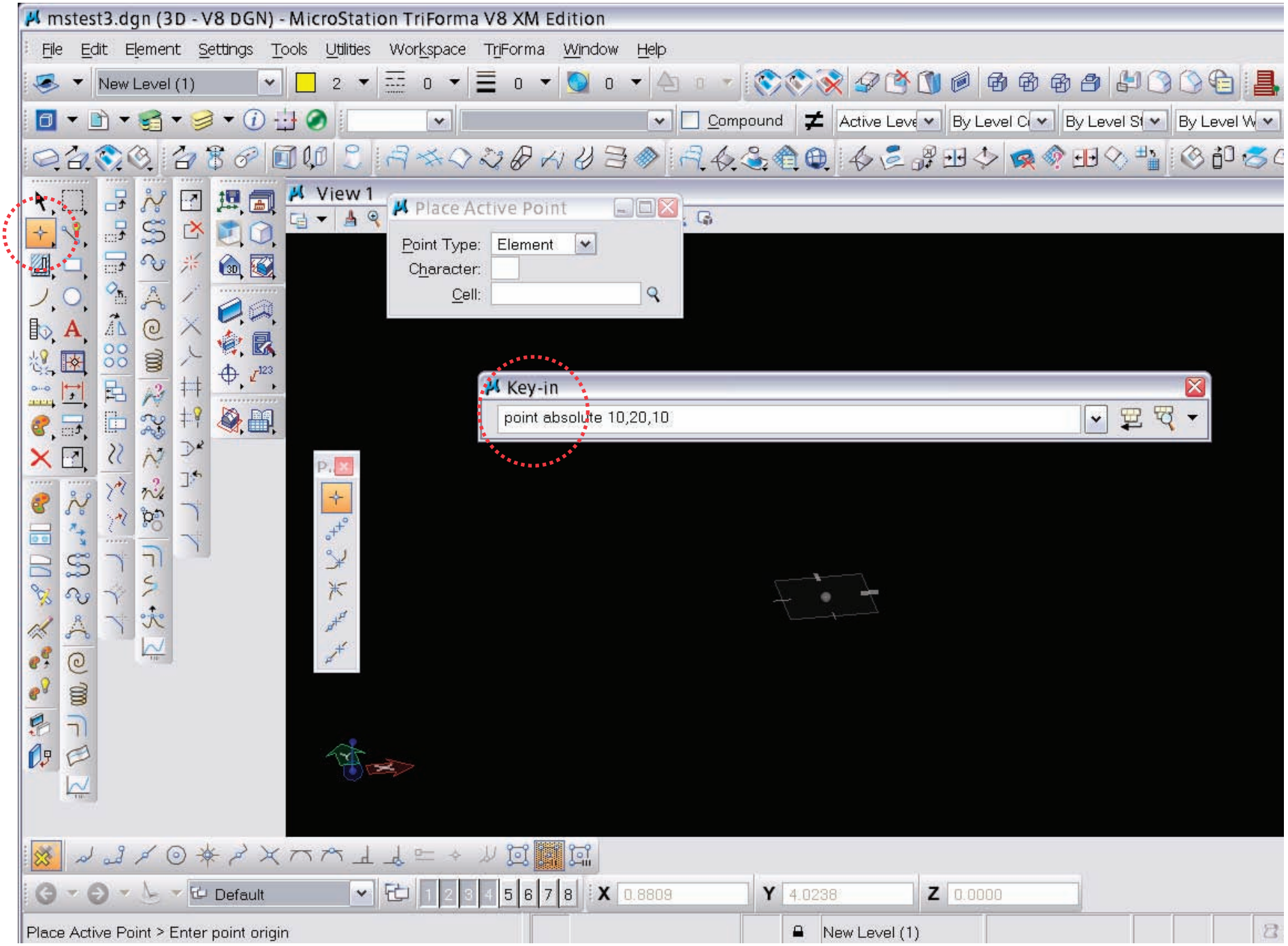






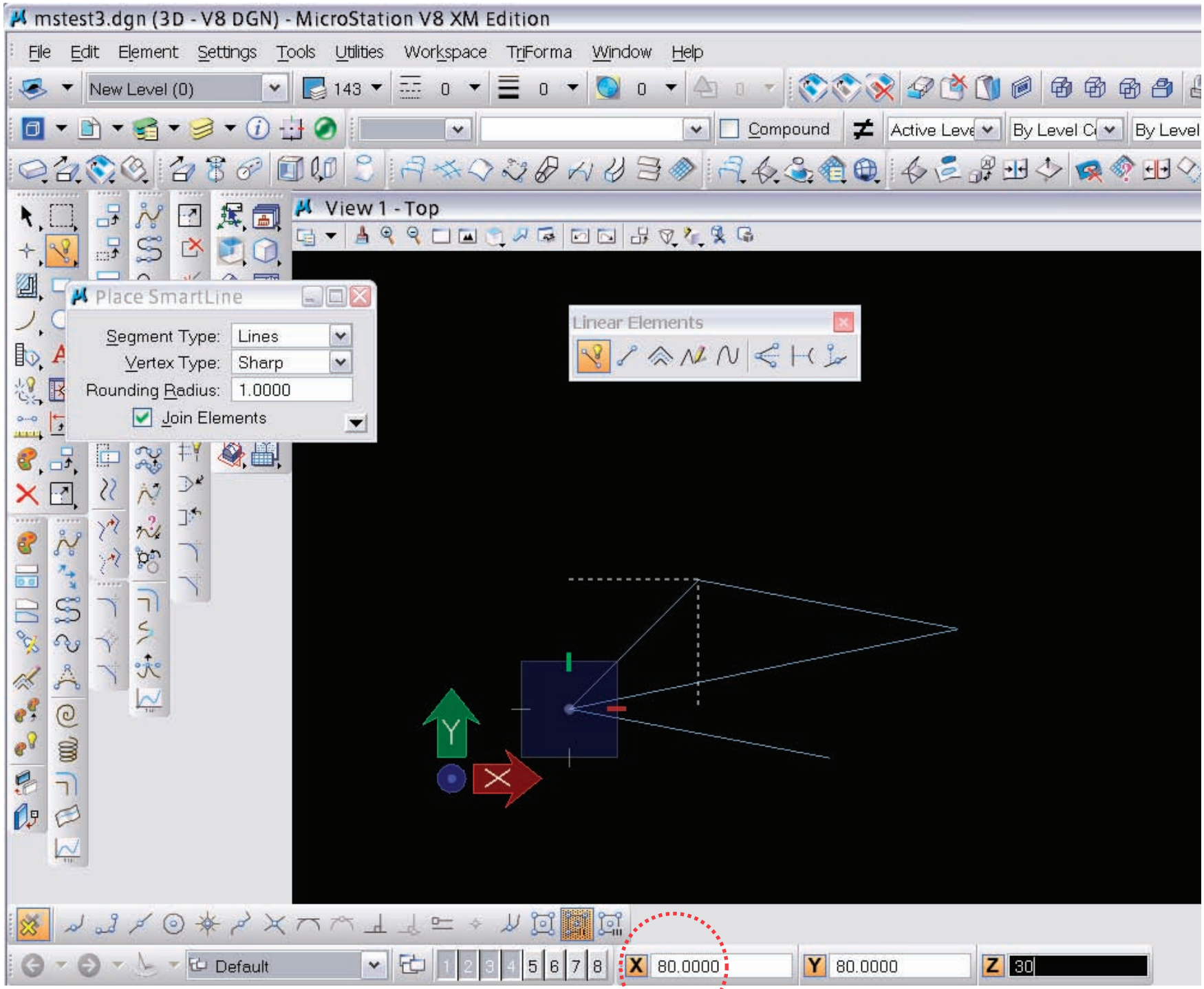


OPERATIONS IN MICROSTATION

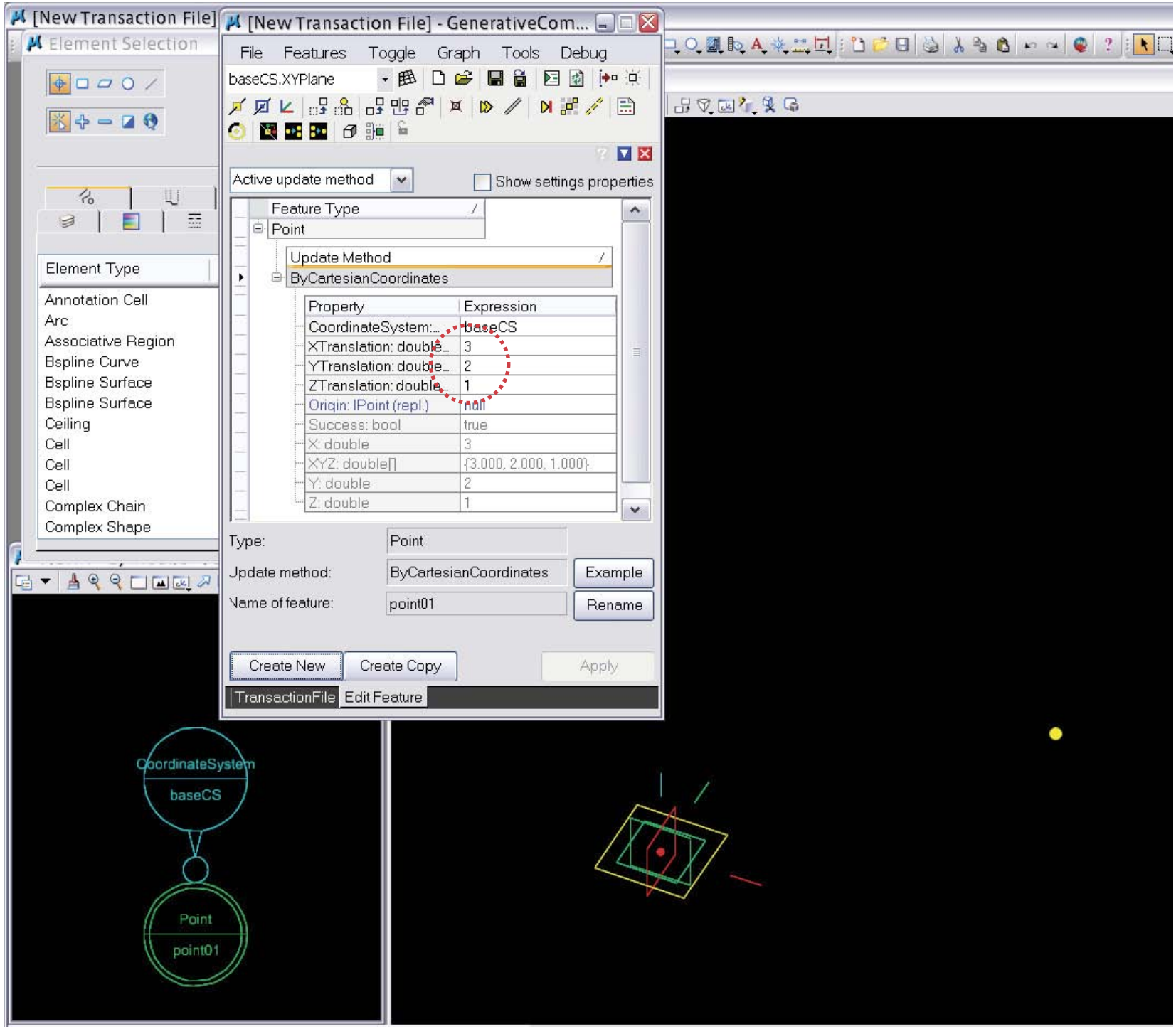


Point Creation by Coordinates

OPERATIONS IN MICROSTATION

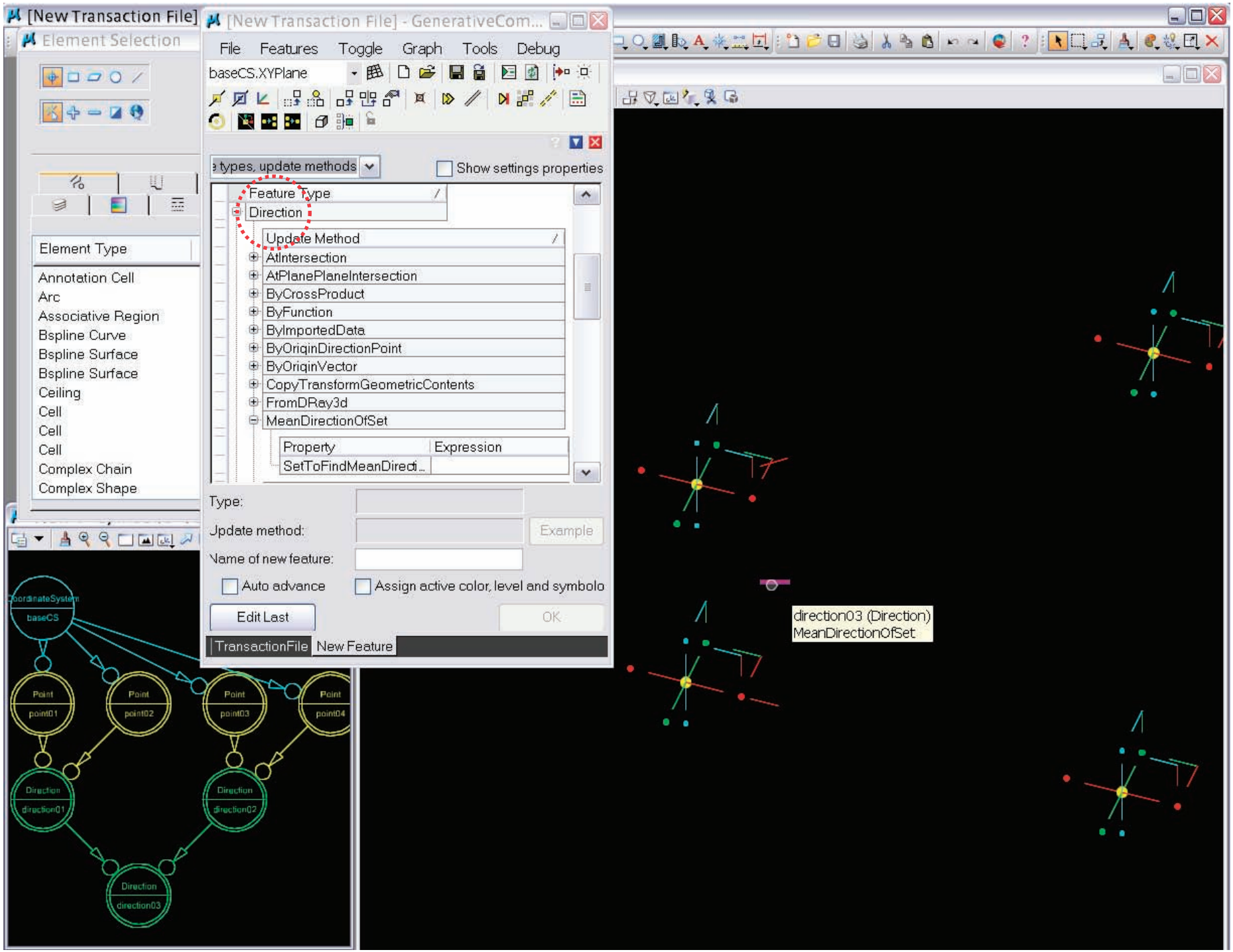


Vector Creation and Addition



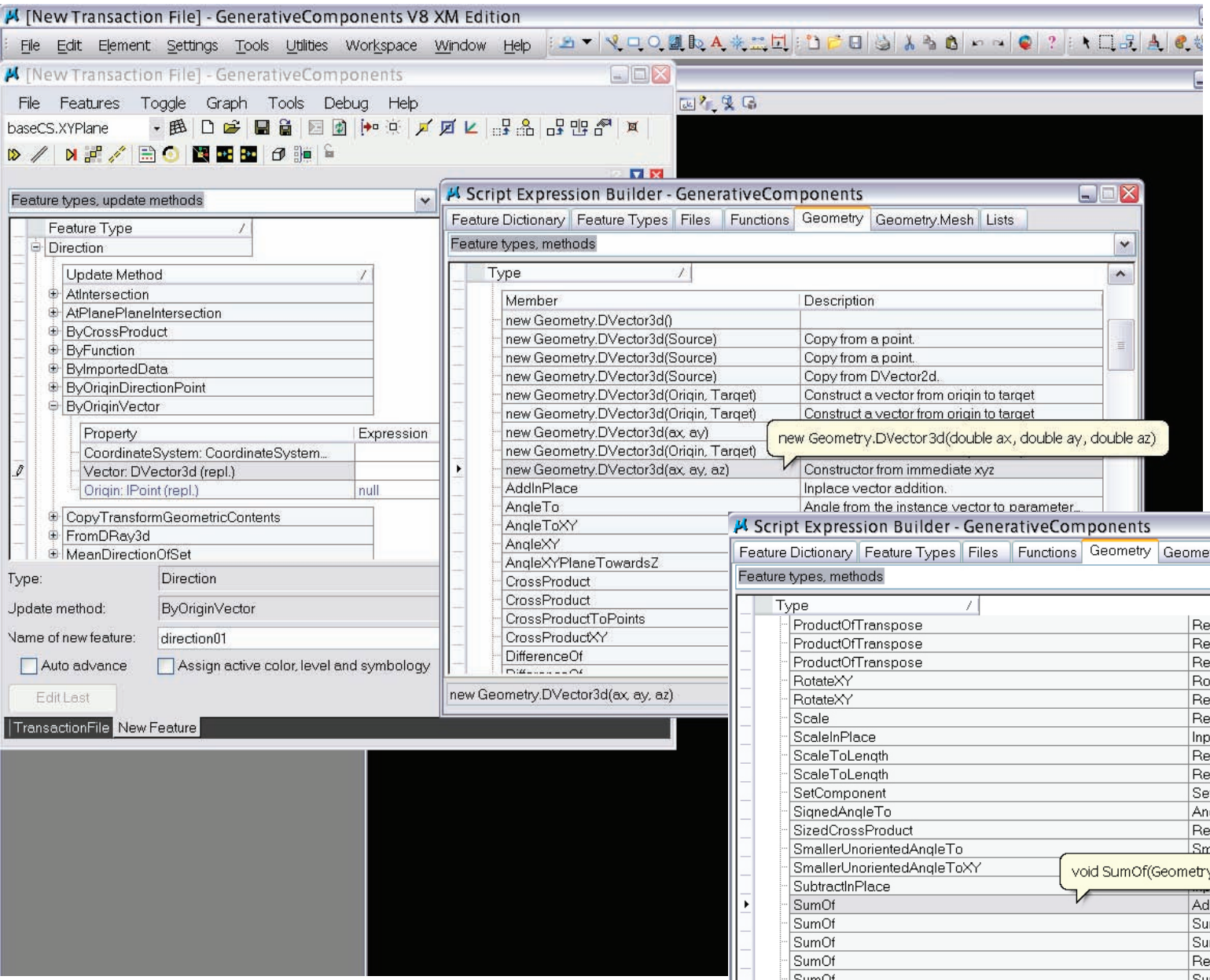
Point Creation by Coordinates

OPERATIONS IN GENERATIVE COMPONENTS



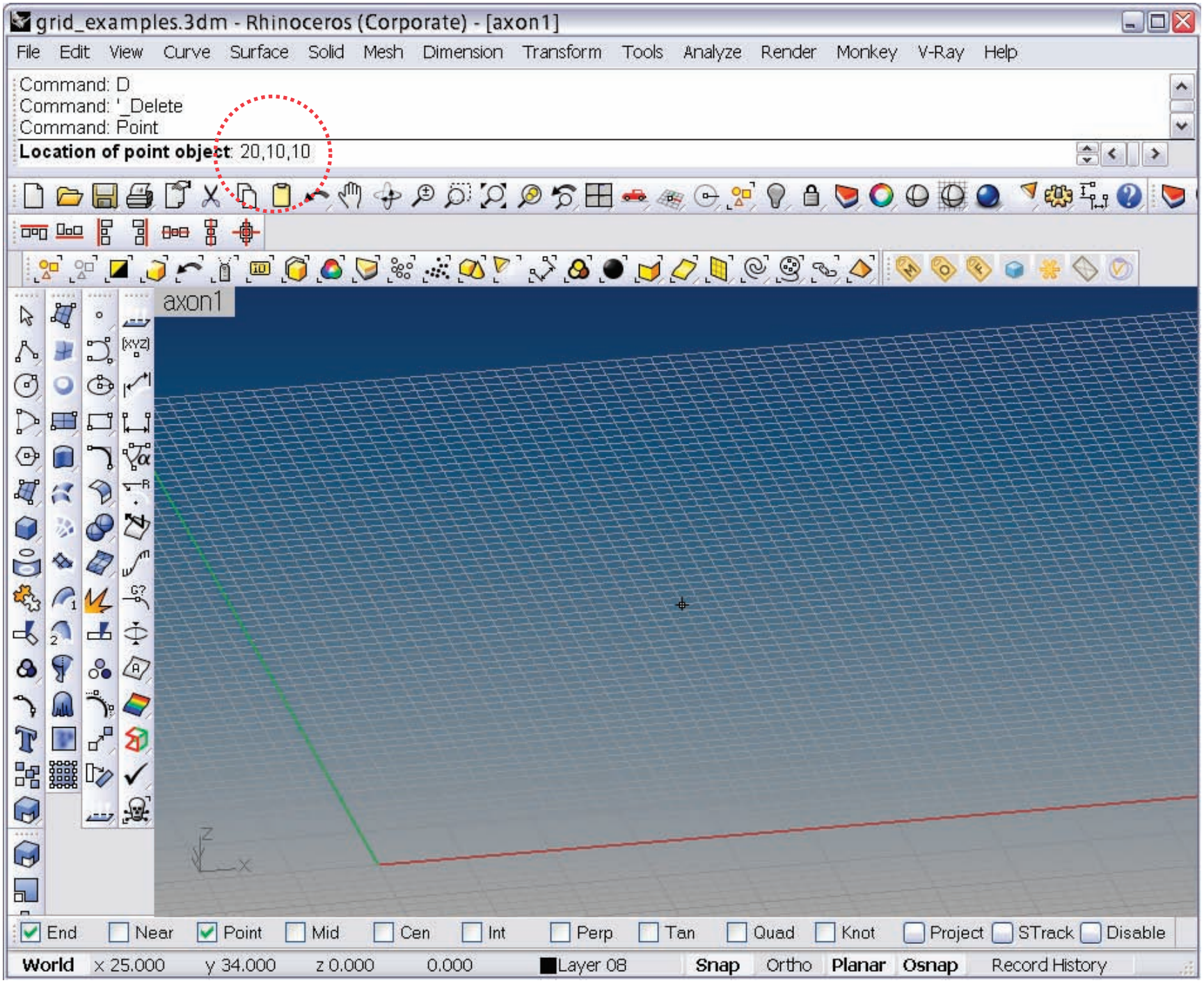
Vector Creation and Addition

OPERATIONS IN GENERATIVE COMPONENTS



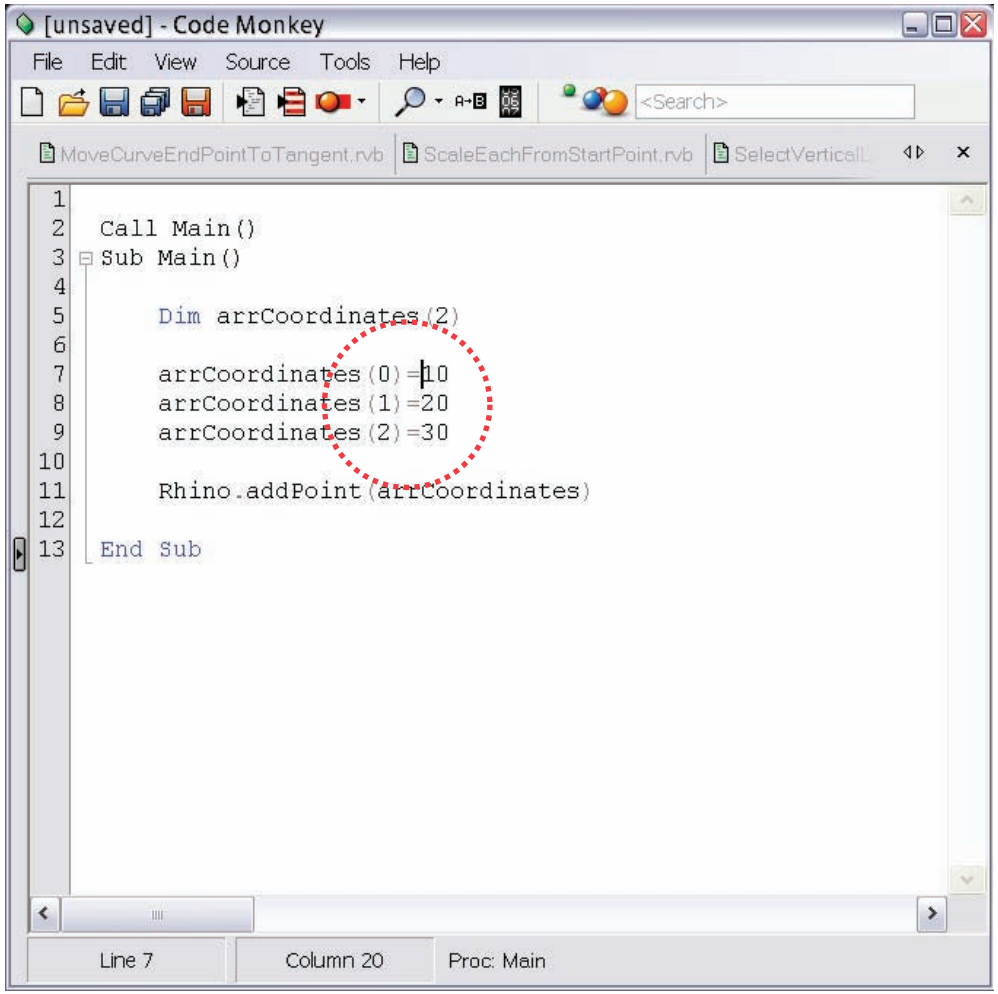
Vector Creation and Addition

OPERATIONS IN RHINOCEROS

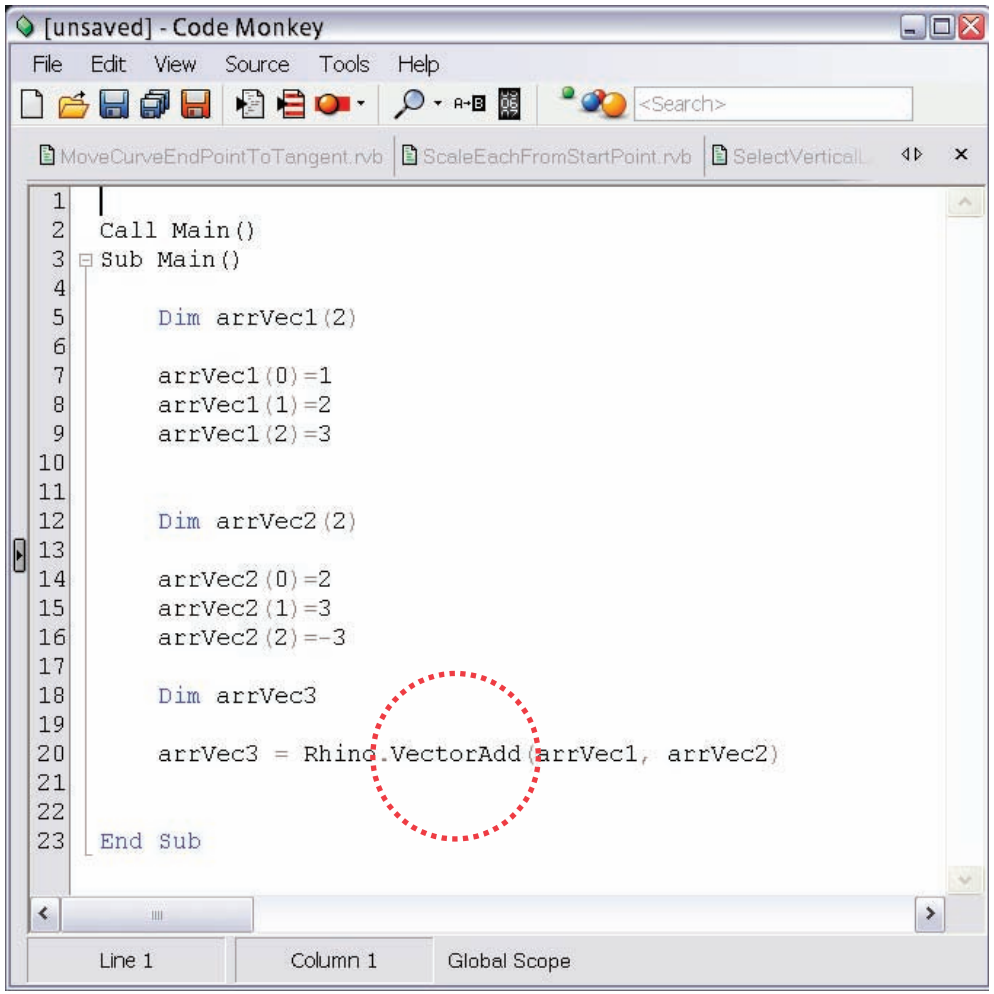


Point Creation by Coordinates

OPERATIONS IN RHINOSCRIPT

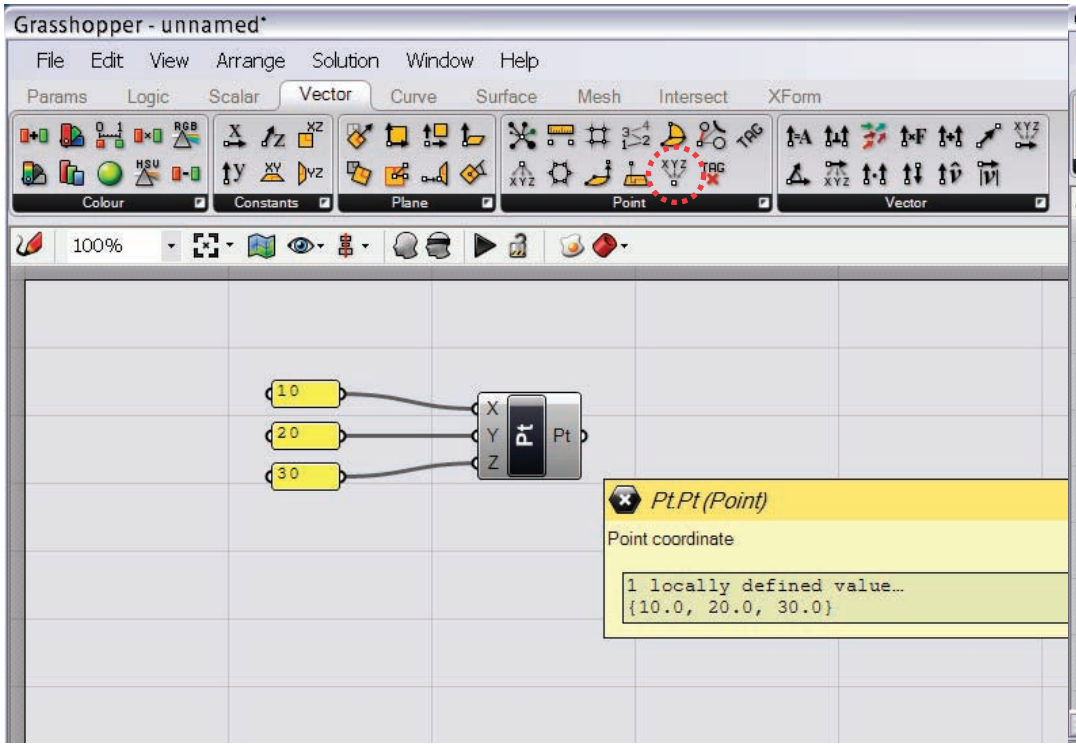


Point Creation by Coordinates

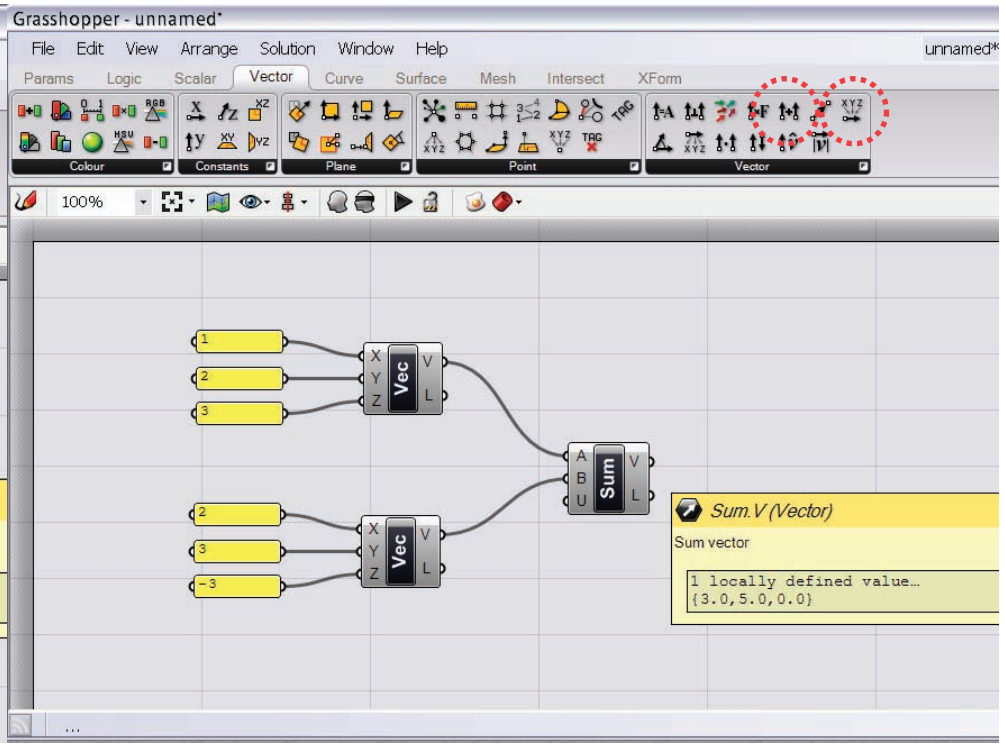


Vector Creation and Addition

OPERATIONS IN GRASSHOPPER



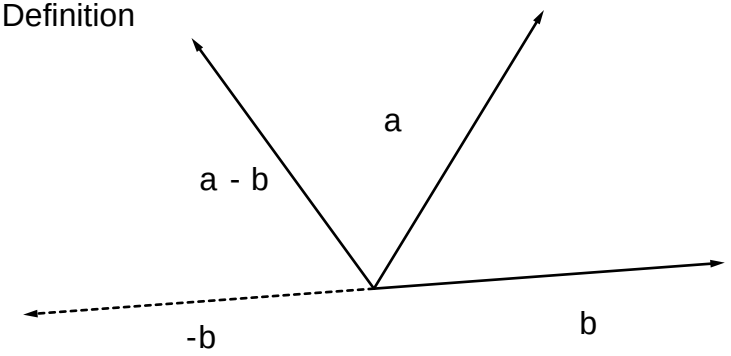
Point Creation by Coordinates



Vector Creation and Addition

VECTOR OPERATION : SUBTRACTION

Definition

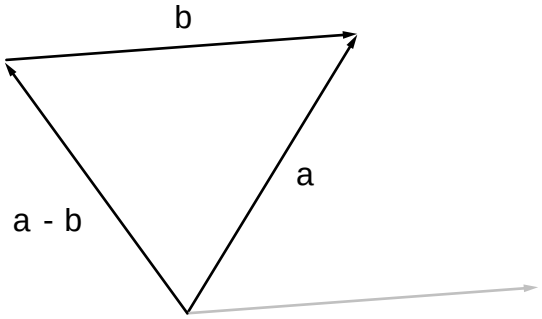


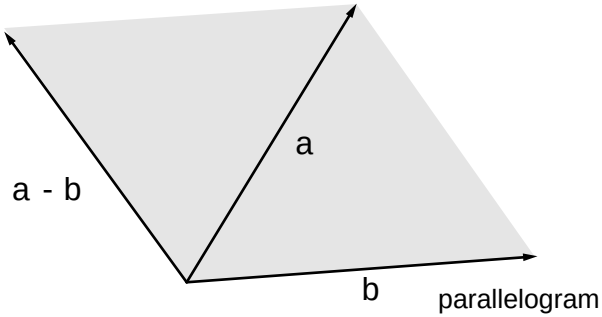
$a = (a_x, a_y, a_z)$

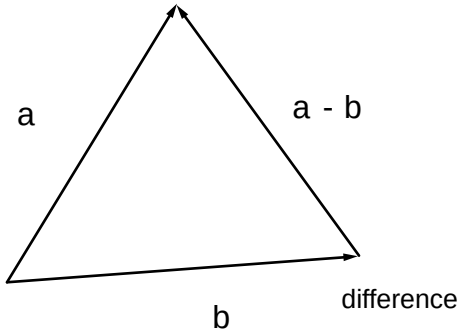
$b = (b_x, b_y, b_z)$

$a - b = (a_x - b_x, a_y - b_y, a_z - b_z)$

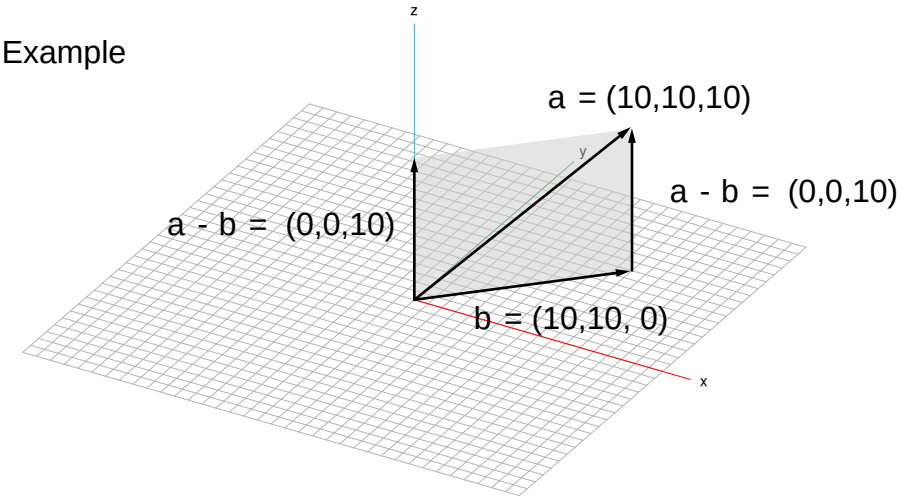
Geometrical Interpretation







Example



$a = (10, 10, 10)$

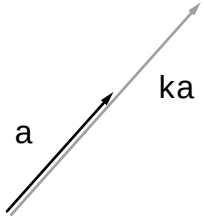
$b = (10, 10, 0)$

$a - b = (10-10, 10-10, 10-0)$

$= (0, 0, 10)$

VECTOR OPERATION : SCALING

Definition

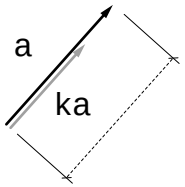


$a = (a_x, a_y, a_z)$

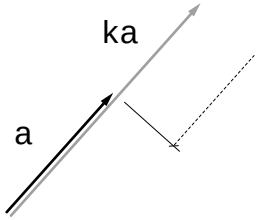
$ka = (ka_x, ka_y, ka_z)$

Geometrical Interpretation

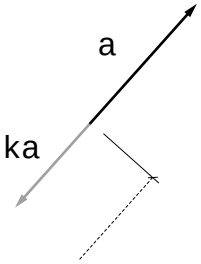
0<k<1



k>1



k<0

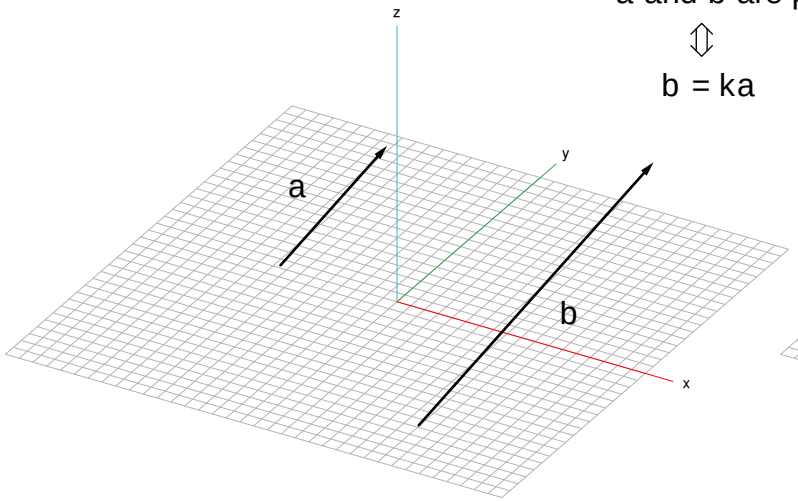


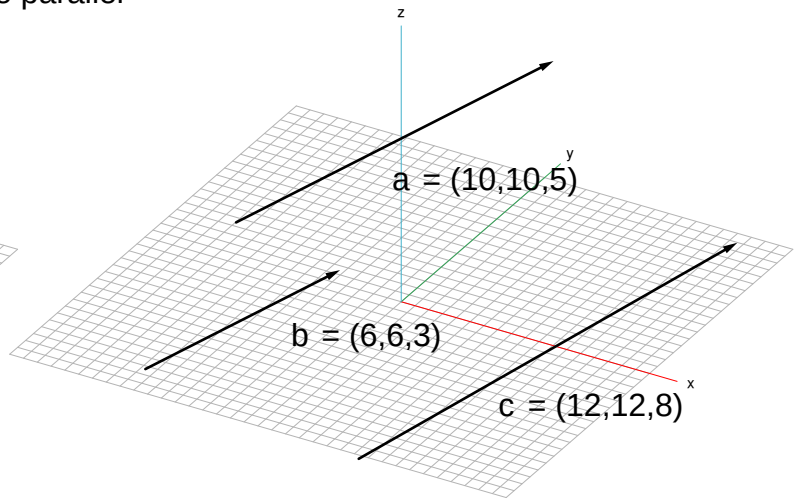
Parallel Vectors

a and b are parallel

⇕

b = ka

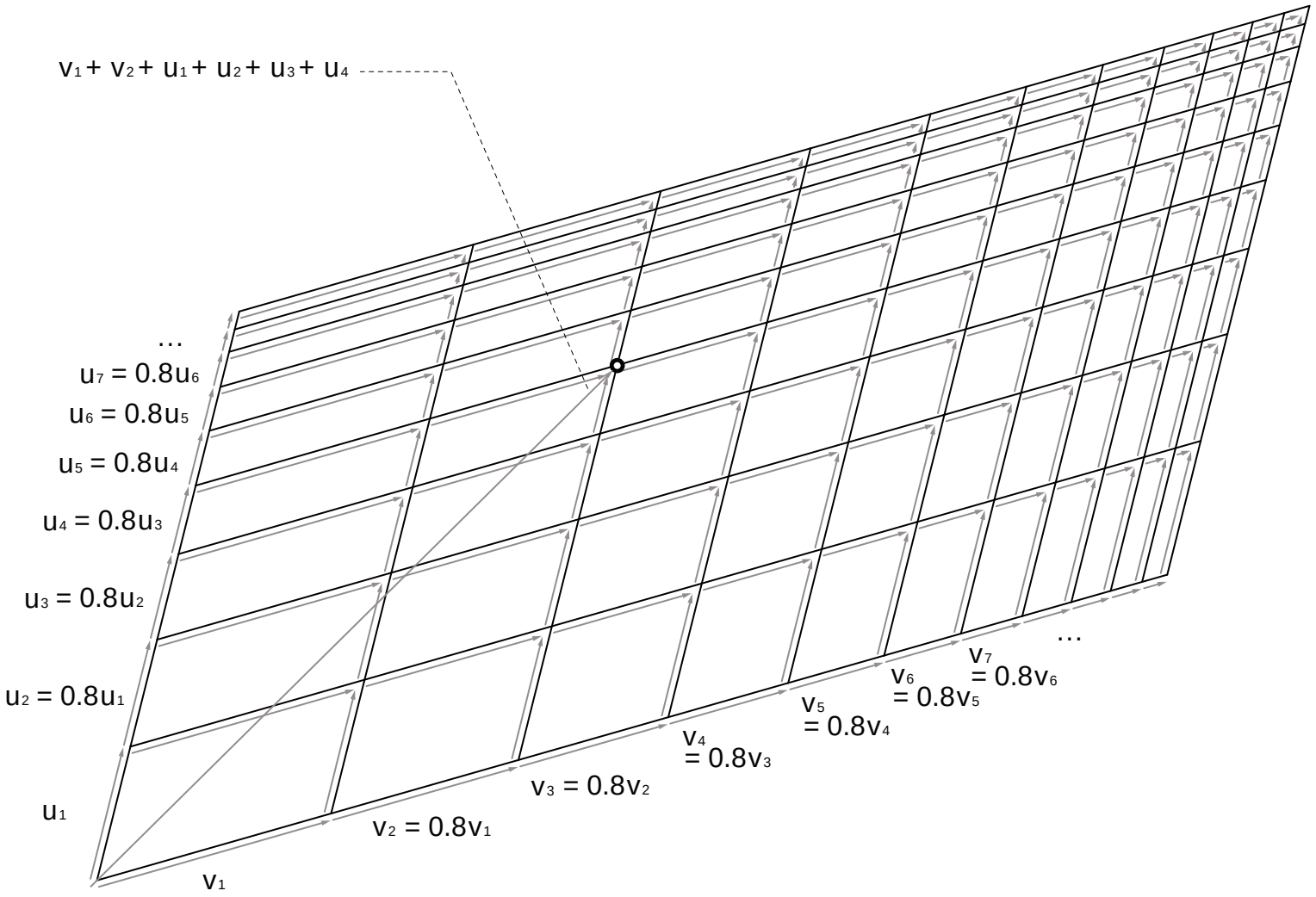




b = (6, 6, 3)
= (10x0.6 ,10x0.6, 5x0.6)
= 0.6 x (10,10, 5)
= 0.6 x a
⇒ a and b are parallel

c = (12, 12, 8)
= (10x1.2 ,10x1.2, 5x1.6)
≠ 1.2 x a
⇒ a and c are not parallel

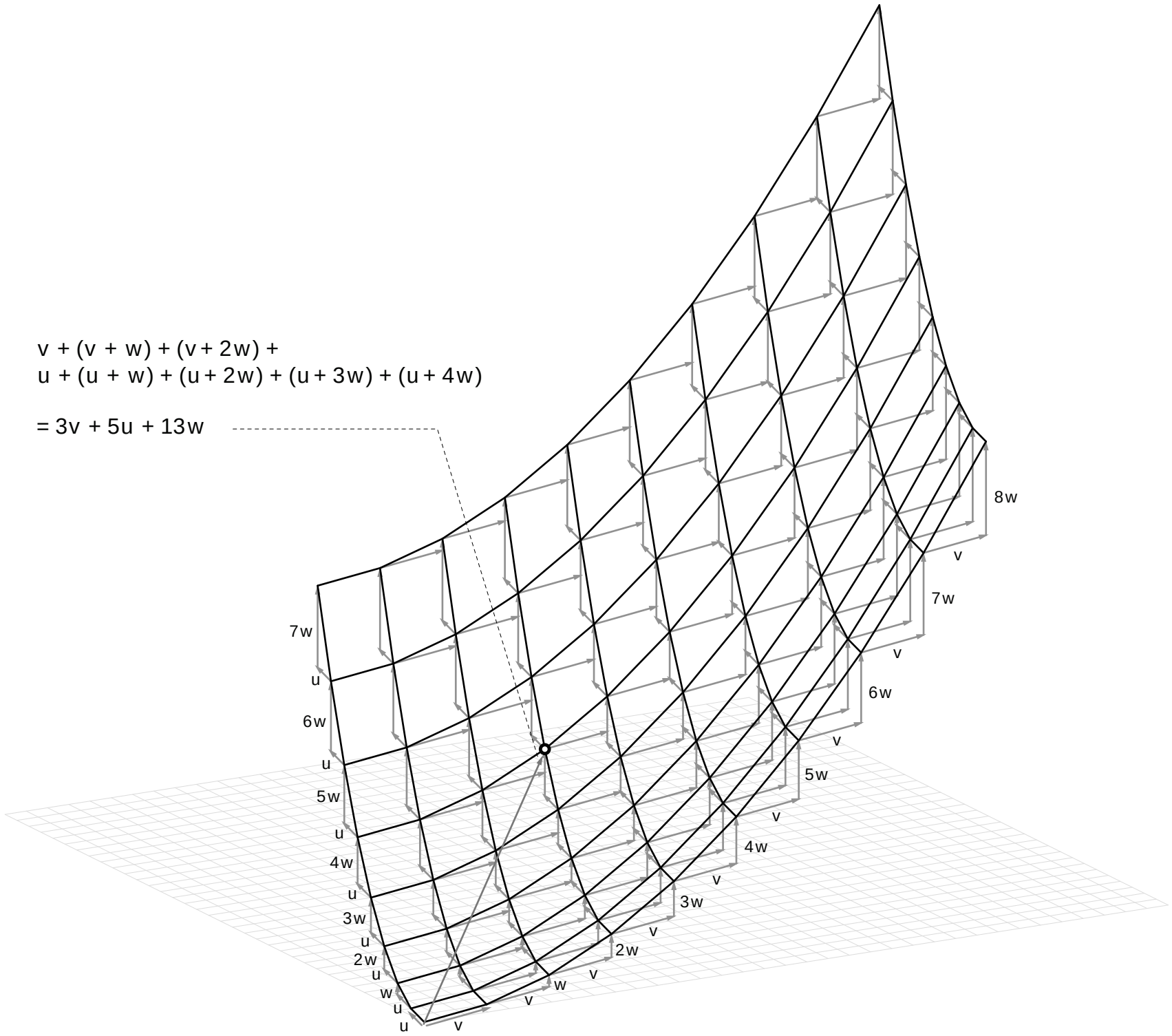
EXAMPLE OF VECTOR ADDITION AND SCALING



EXAMPLE OF VECTOR ADDITION AND SCALING

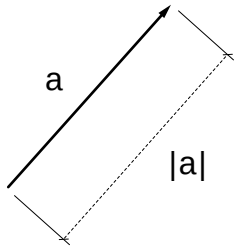
$v + (v + w) + (v + 2w) +$
$u + (u + w) + (u + 2w) + (u + 3w) + (u + 4w)$

$= 3v + 5u + 13w$



VECTOR OPERATION : LENGTH AND UNITIZATION

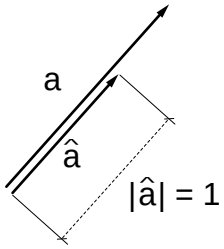
Length



$a = (a_x, a_y, a_z)$

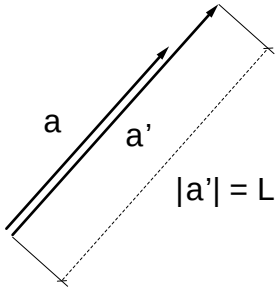
$|a| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

Unitization

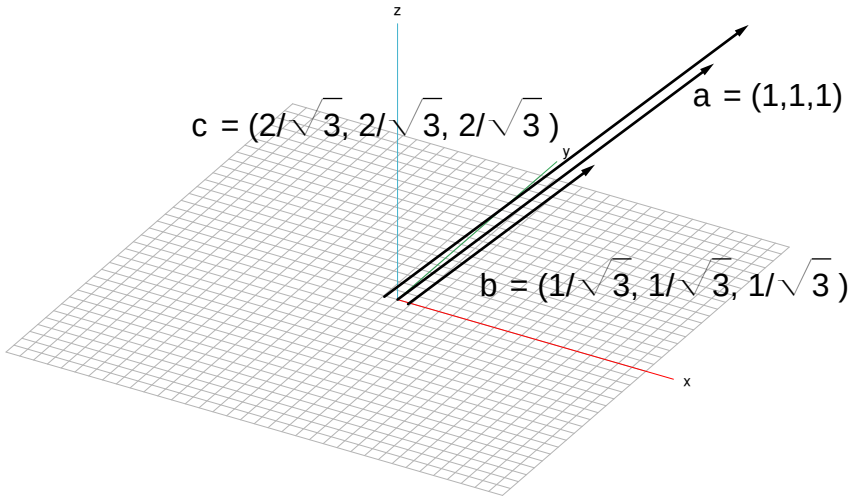


$\hat{a} = a/|a|$

Setting Length



$a' = \hat{a} \times L = a/|a| \times L$

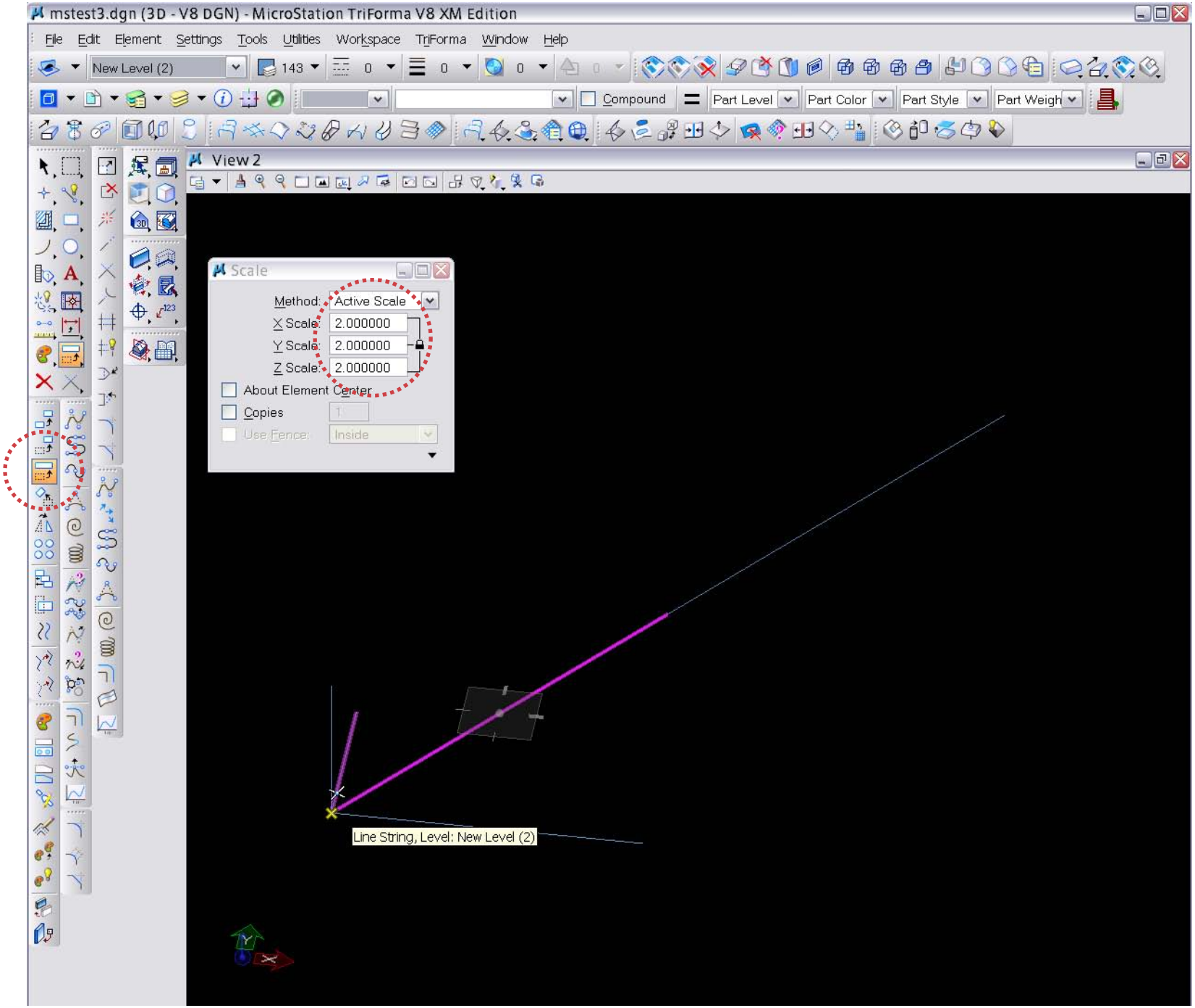


$|a| = \sqrt{1 + 1 + 1} = \sqrt{3}$

$|b| = 1$

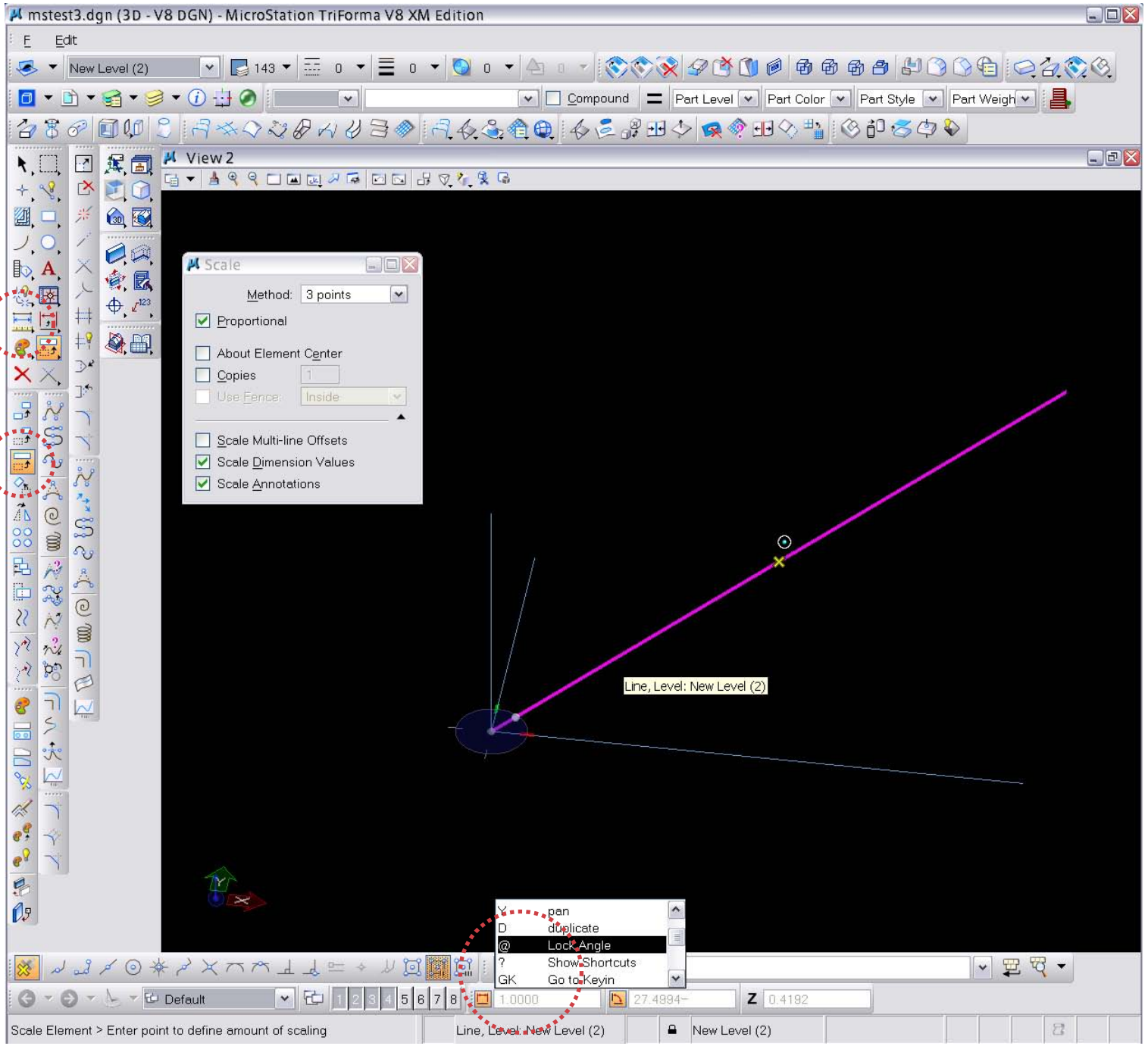
$|c| = 2$

OPERATIONS IN MICROSTATION



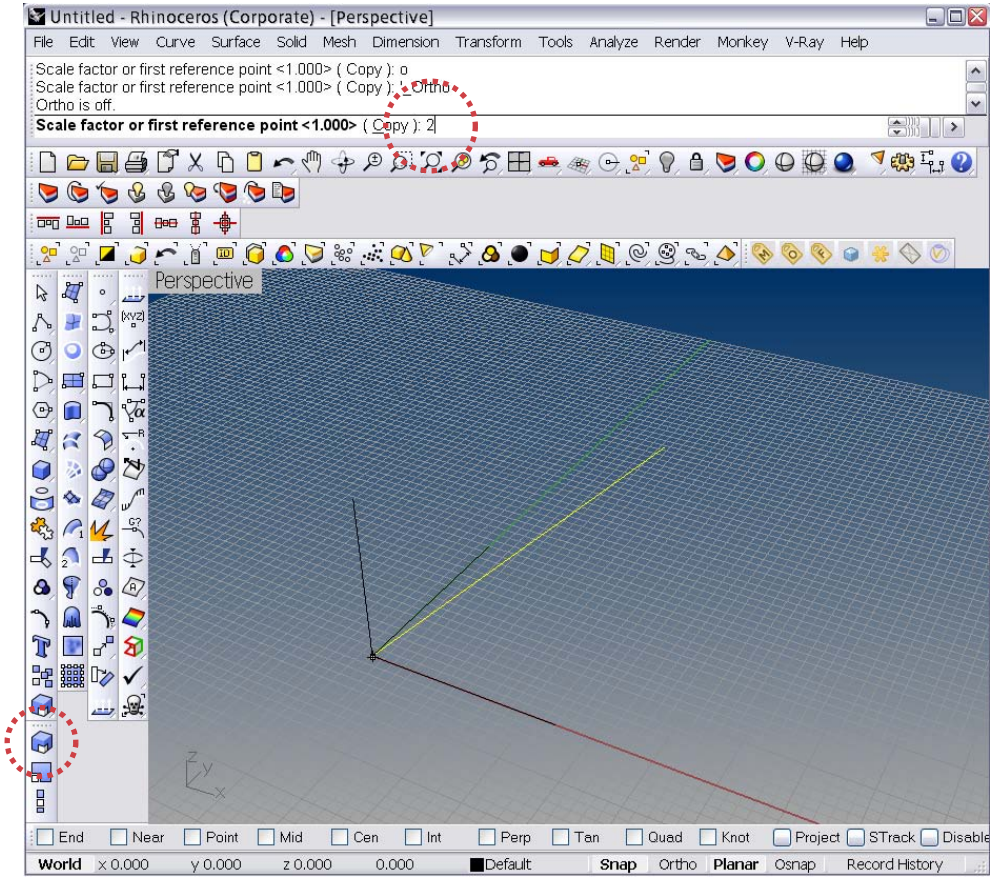
Scaling Vector (Line)

OPERATIONS IN MICROSTATION

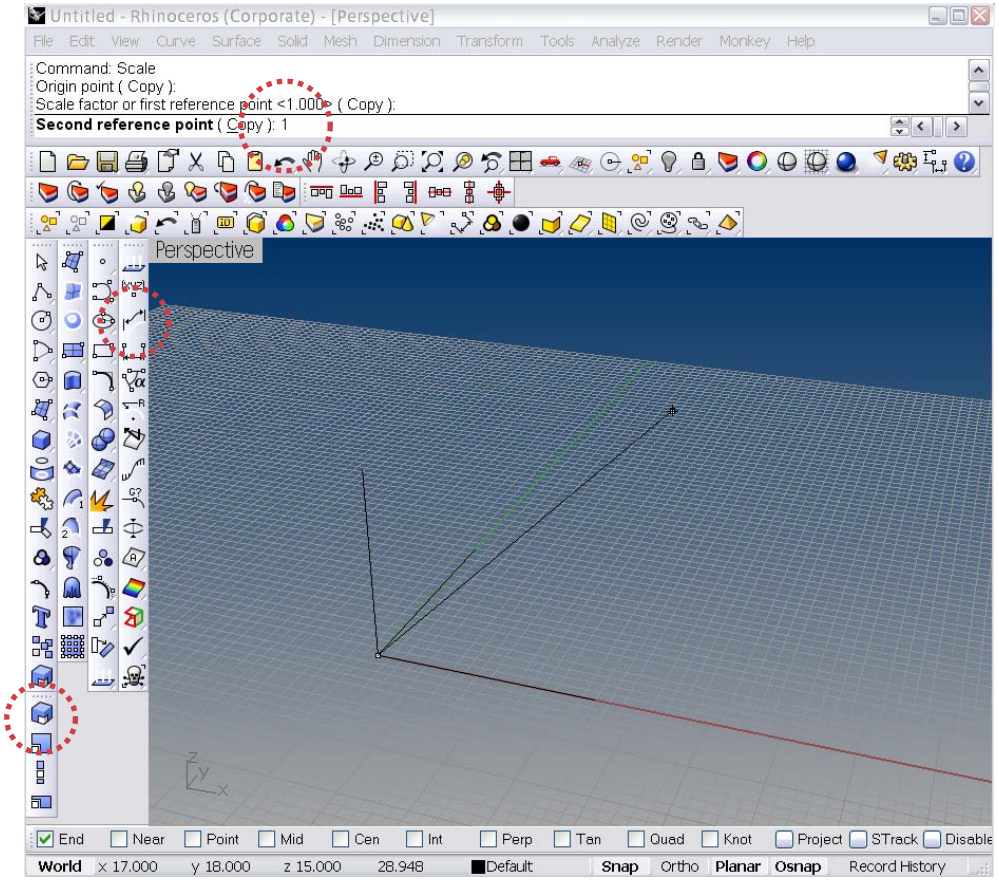


Unitizing Vector (Line)
and Vector Length

OPERATIONS IN RHINOCEROS

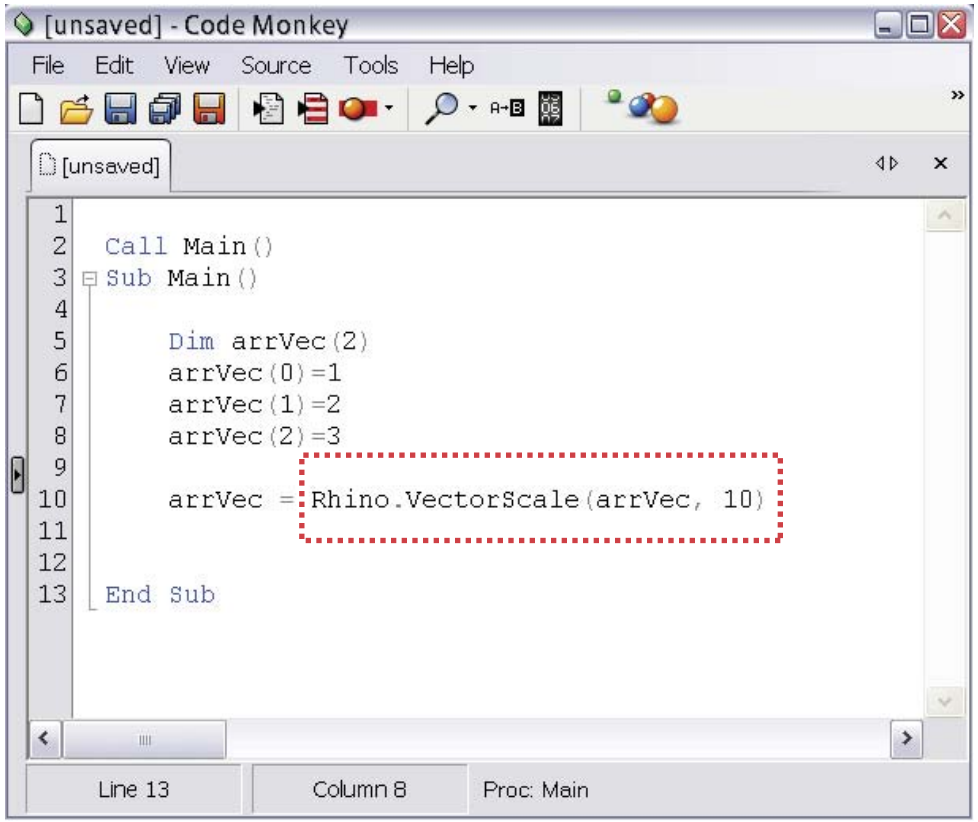


Scaling Vector (Line)

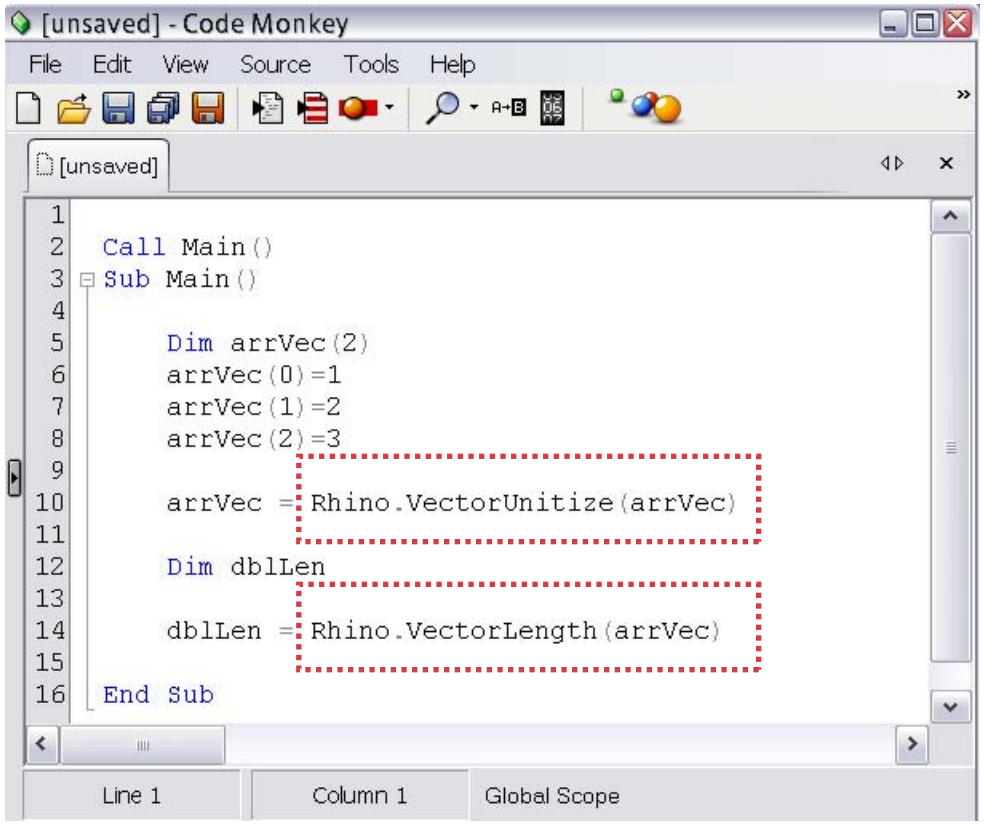


Unitizing Vector (Line)
and Vector Length

OPERATIONS IN RHINOSCRIPT

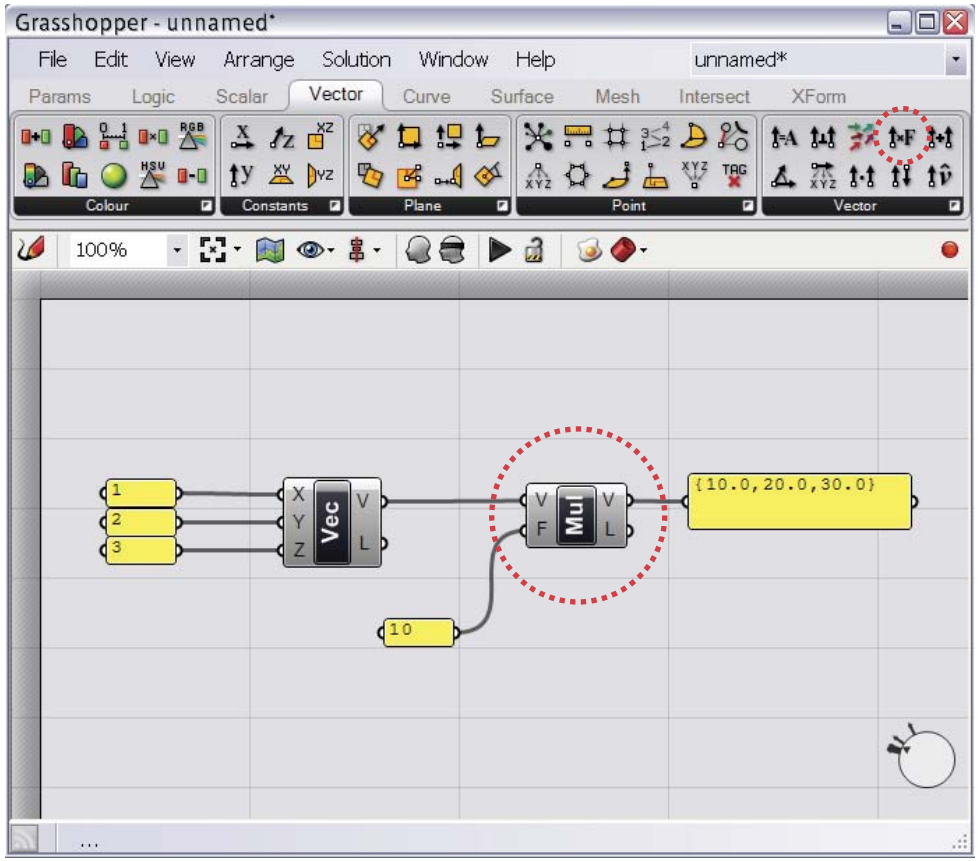


Scaling Vector

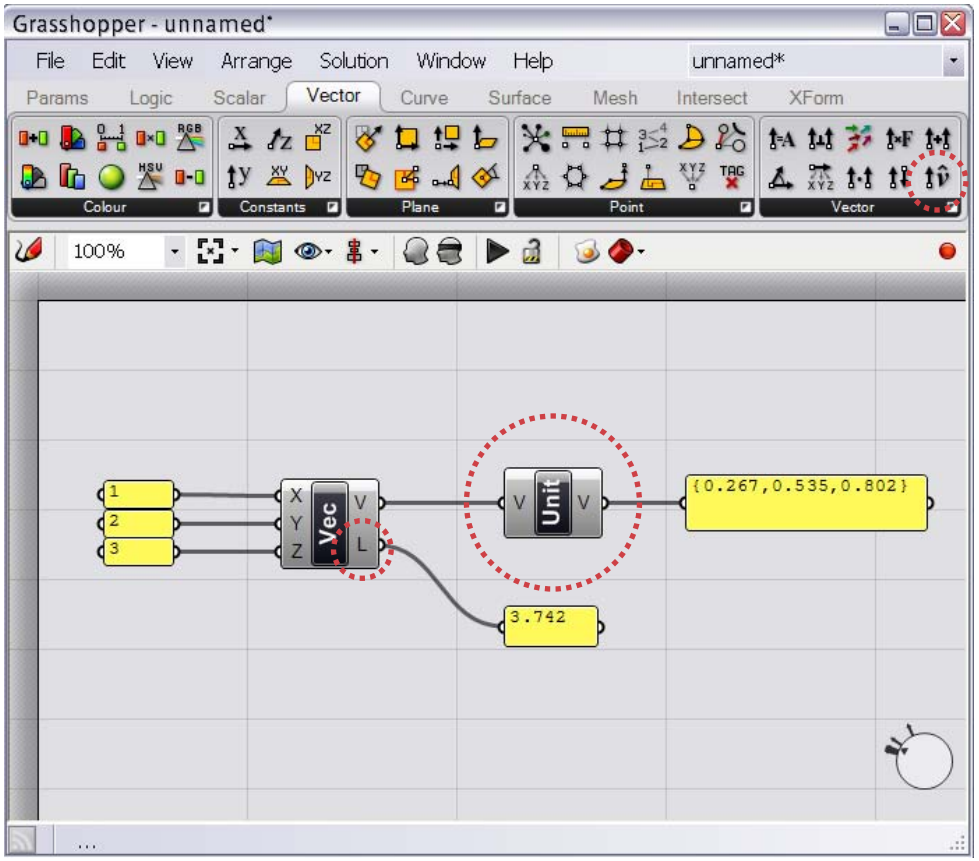


Unitizing Vector
and Vector Length

OPERATIONS IN GRASSHOPPER



Scaling Vector



Unitizing Vector

and Vector Length

VECTOR OPERATION : DOT PRODUCT

Definition

$a = (a_x, a_y, a_z)$

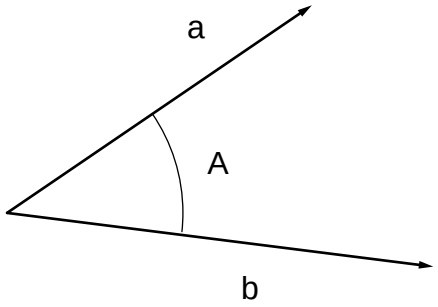
$b = (b_x, b_y, b_z)$

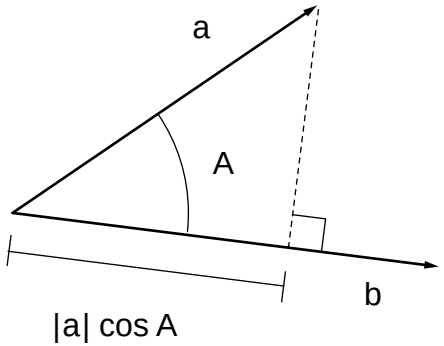
Dot Product
(Inner Product)
(Scalar Product)

$a \cdot b = a_x \times b_x + a_y \times b_y + a_z \times b_z$

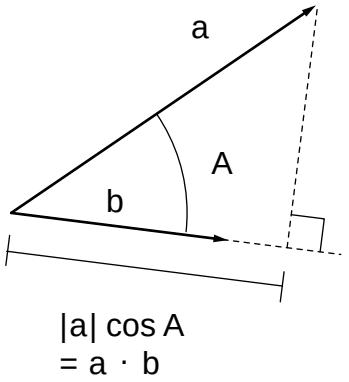
Geometrical Interpretation

$a \cdot b = |a|\,|b| \cos A$

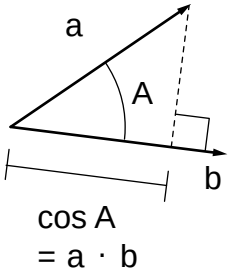


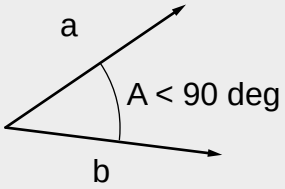


if $|b| = 1$

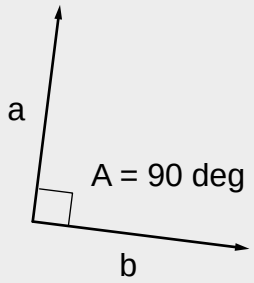


if $|a| = 1,\ |b| = 1$

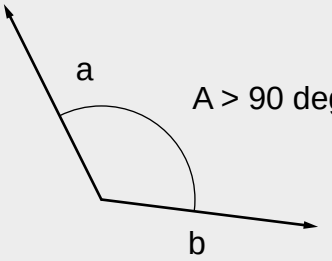




$0 < a \cdot b$



$a \cdot b = 0$



$a \cdot b < 0$

VECTOR OPERATION : CROSS PRODUCT

Definition

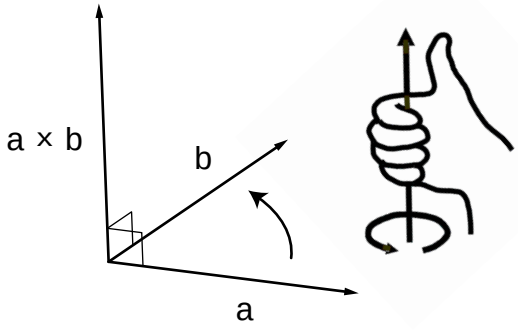
$a = (a_x, a_y, a_z)$ $b = (b_x, b_y, b_z)$

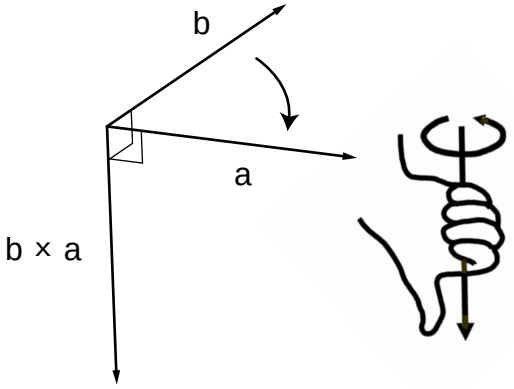
Cross Product
(Outer Product)
(Vector Product)

$$a \times b = \begin{pmatrix} a_y \times b_z - a_z \times b_y \\ a_z \times b_x - a_x \times b_z \\ a_x \times b_y - a_y \times b_x \end{pmatrix}$$

Geometrical Interpretation

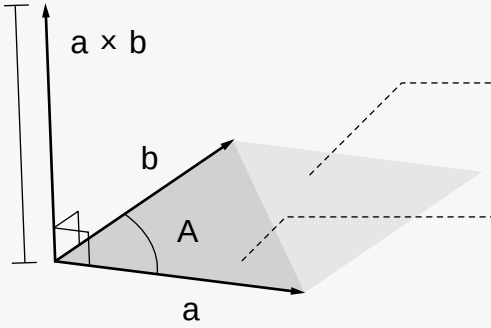
- Perpendicular Direction





- Length & Area

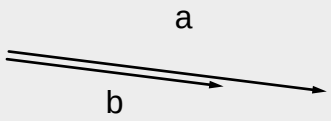
$|a \times b| = |a|\,|b| \sin A$



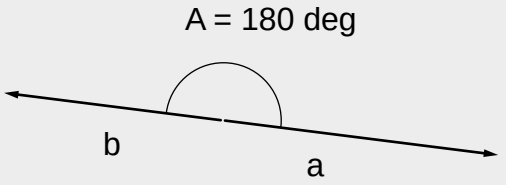
Area of Parallelogram = $|a|\,|b| \sin A = |a \times b|$

Area of Triangle = $1/2 \times |a|\,|b| \sin A = |a \times b| / 2$

- Parallel

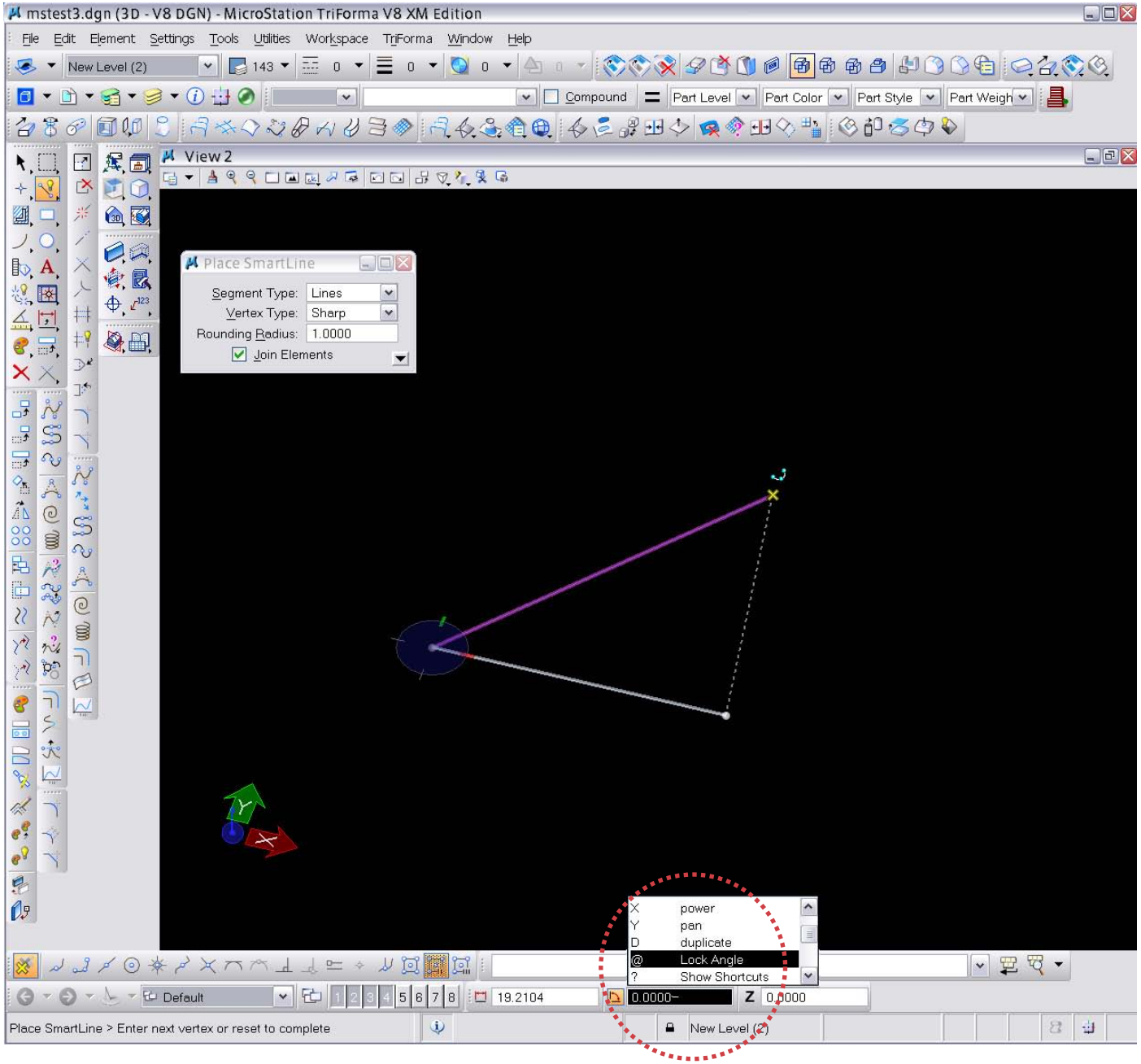


$|a \times b| = 0$



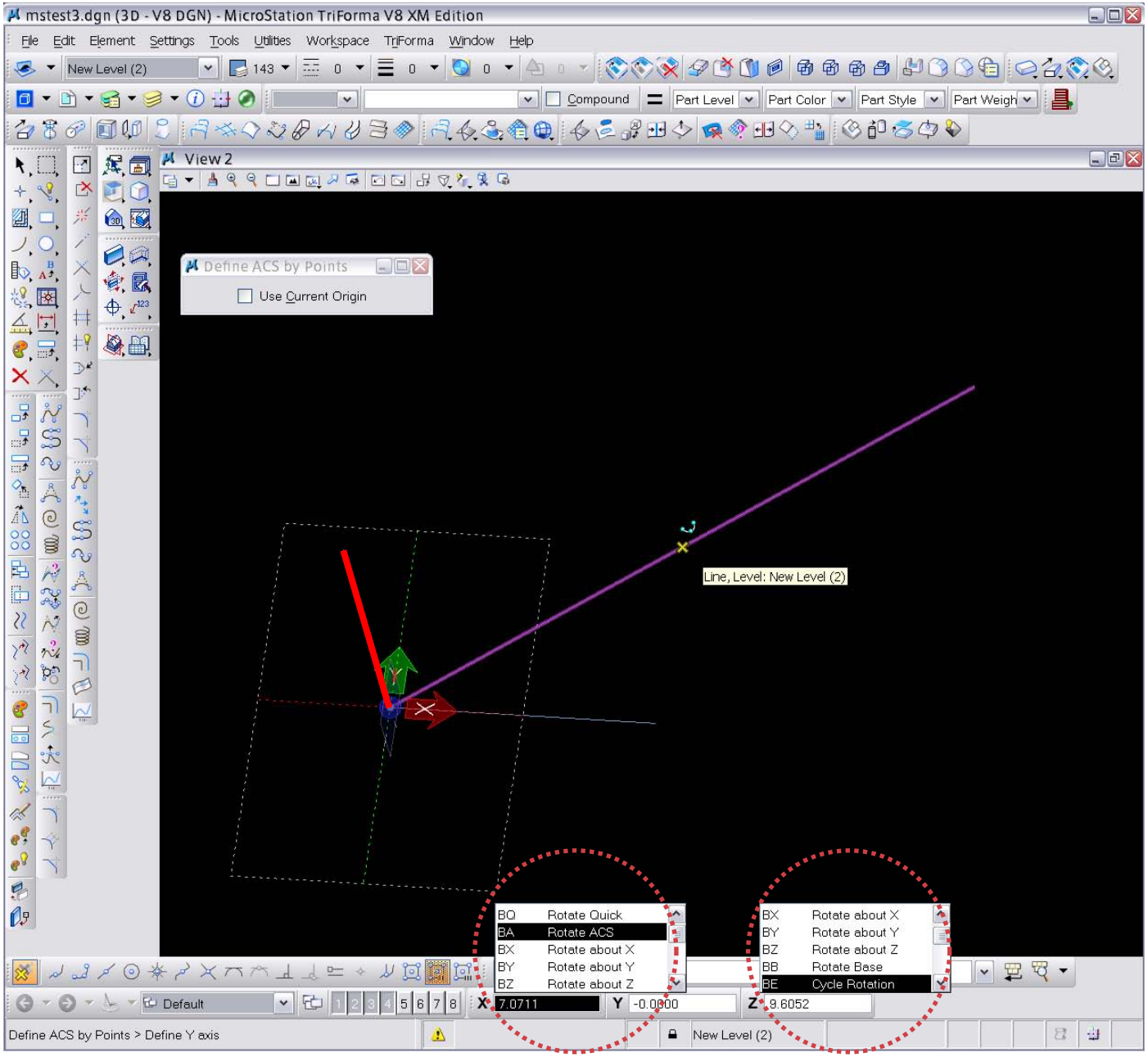
$|a \times b| = 0$

OPERATIONS IN MICROSTATION



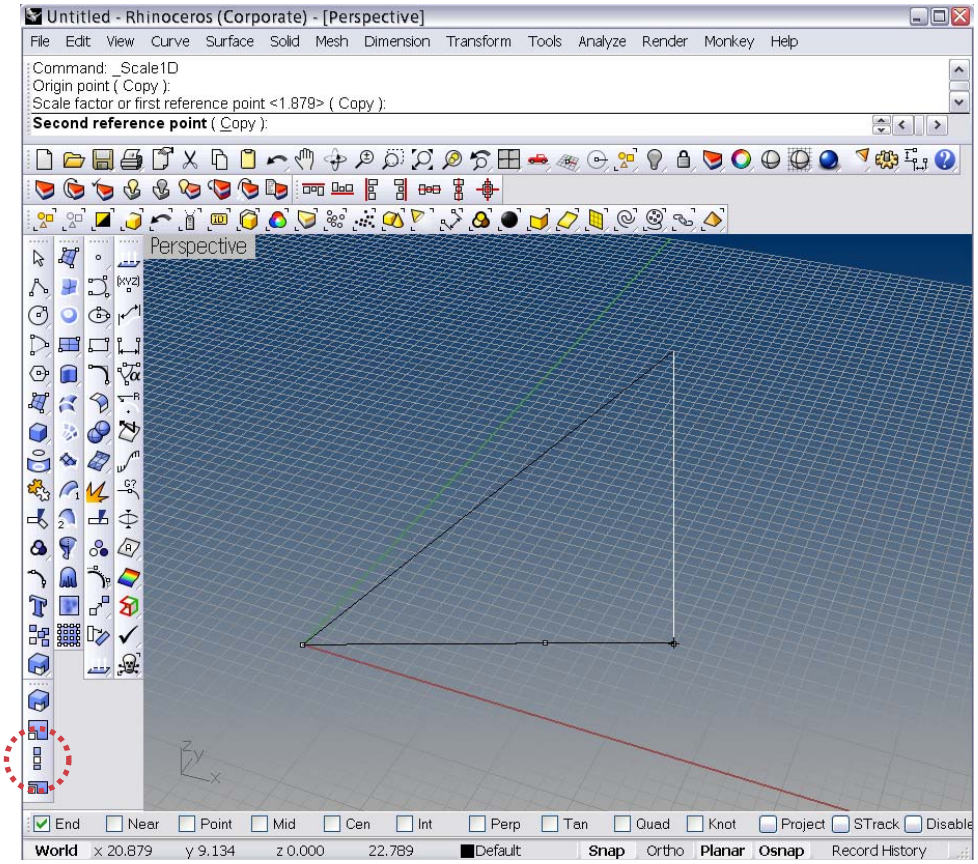
Dot Product as Line Projection

OPERATIONS IN MICROSTATION

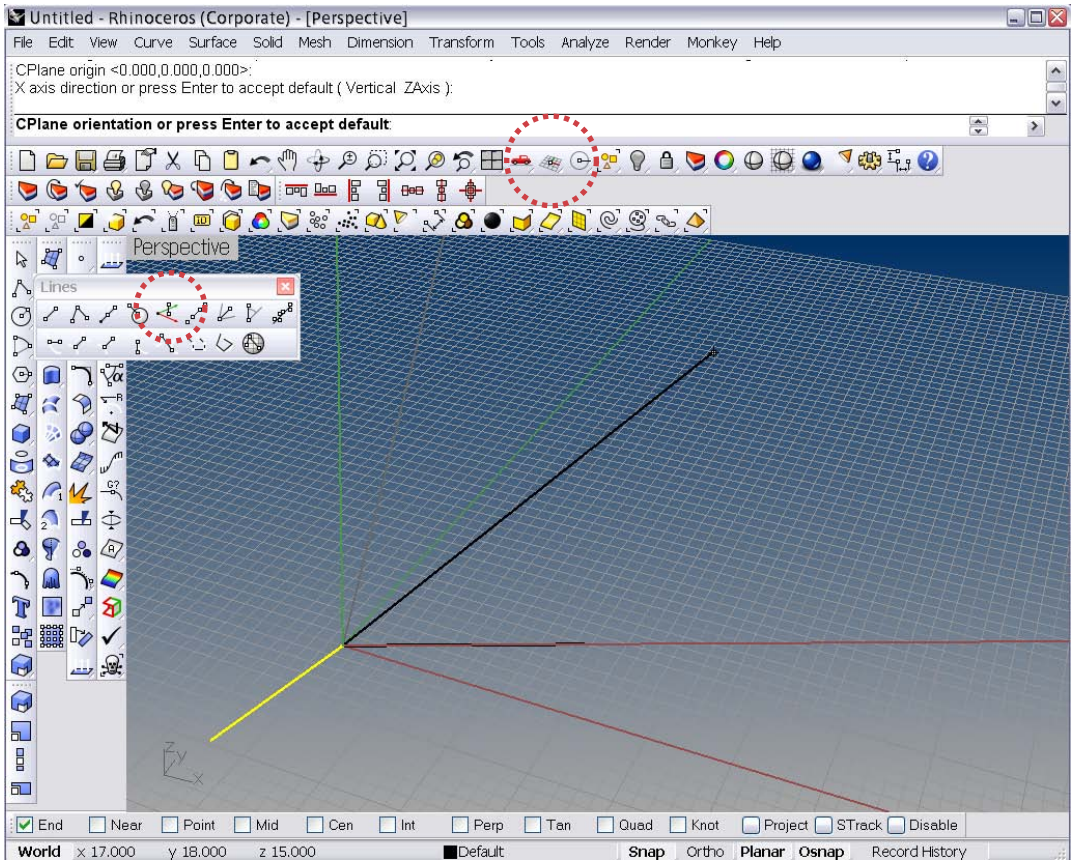


Cross Product as Perpendicular Line

OPERATIONS IN RHINOCEROS

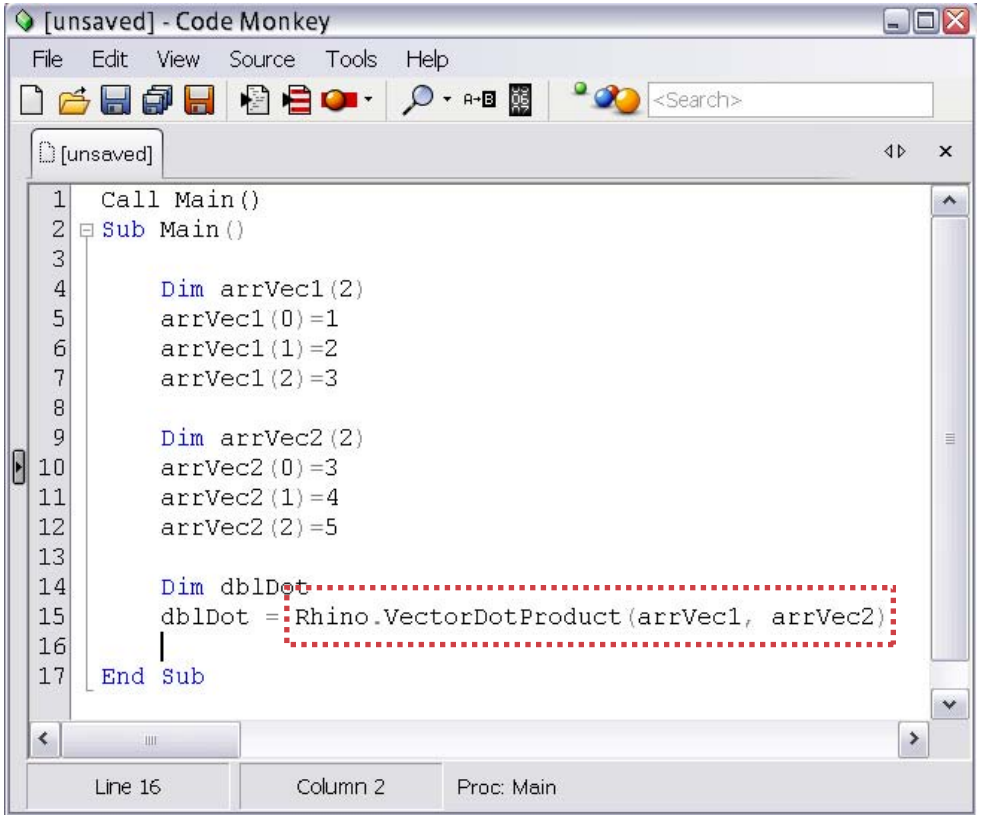


Dot Product as Line Projection

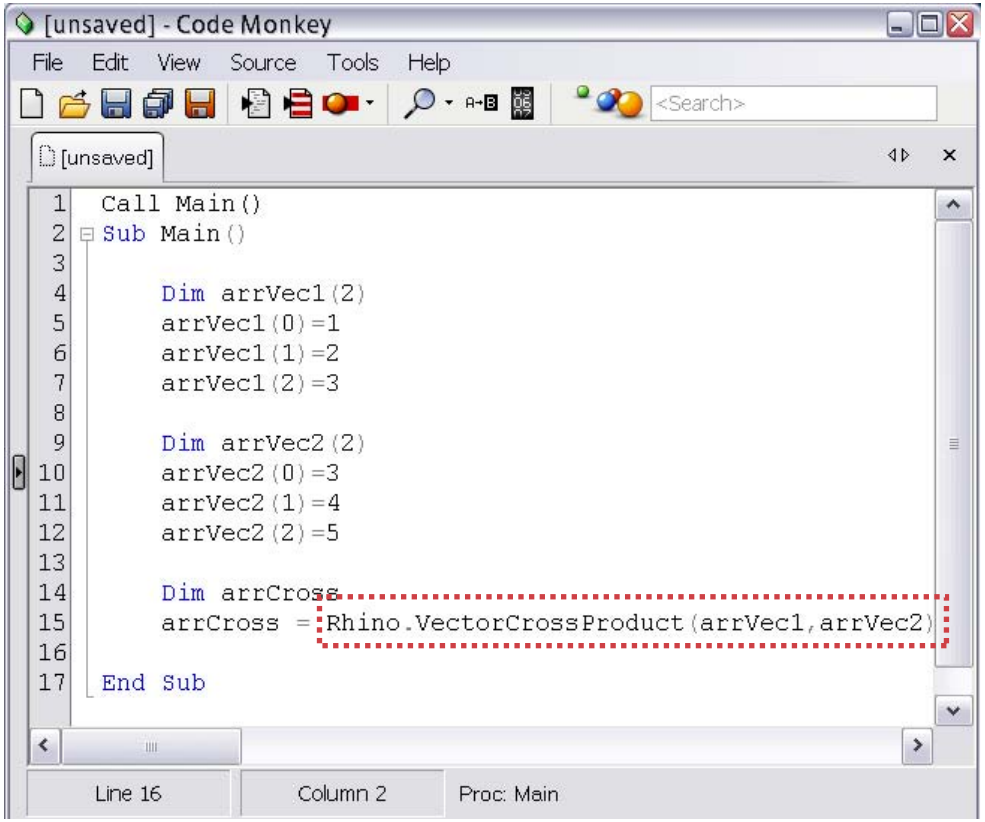


Cross Product as Perpendicular Line

OPERATIONS IN RHINOSCRIPT

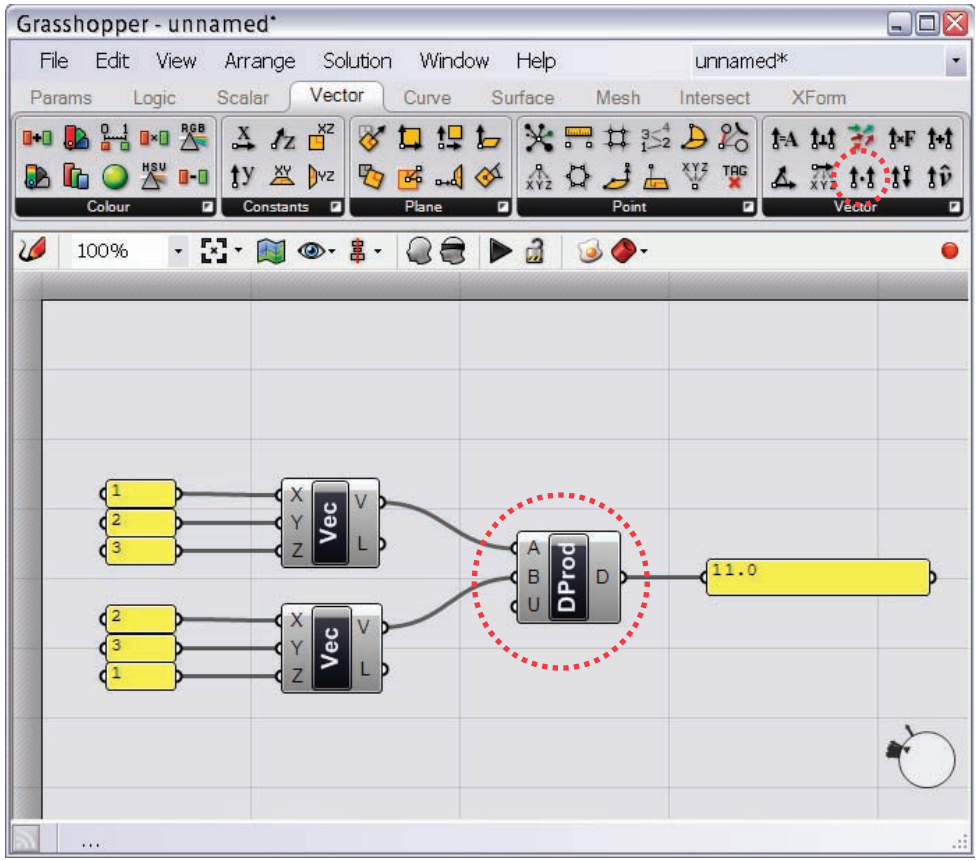


Dot Product

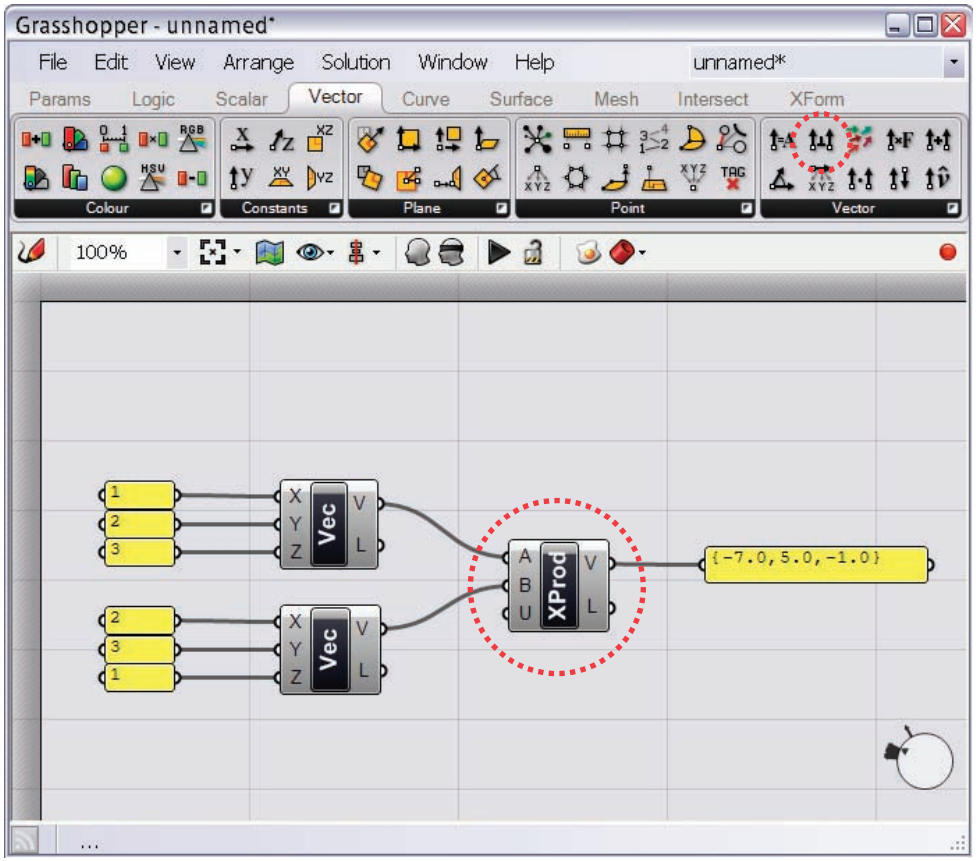


Cross Product

OPERATIONS IN GRASSHOPPER

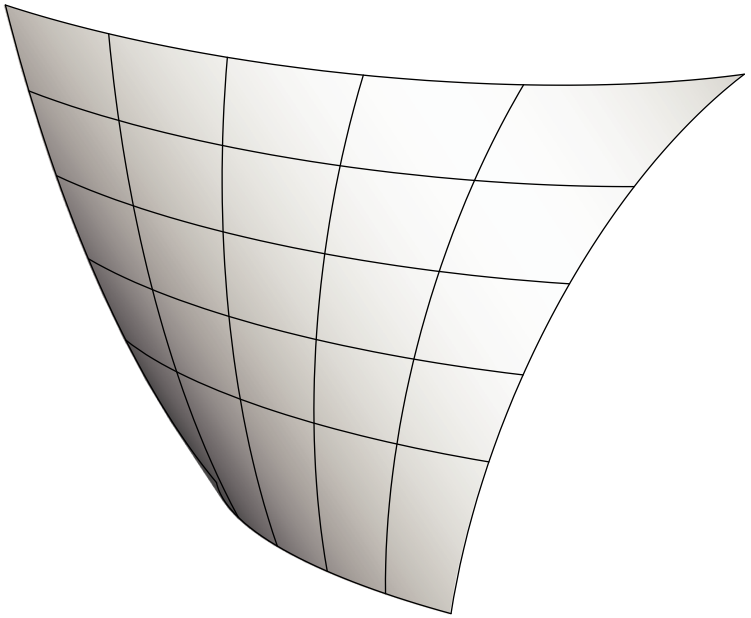


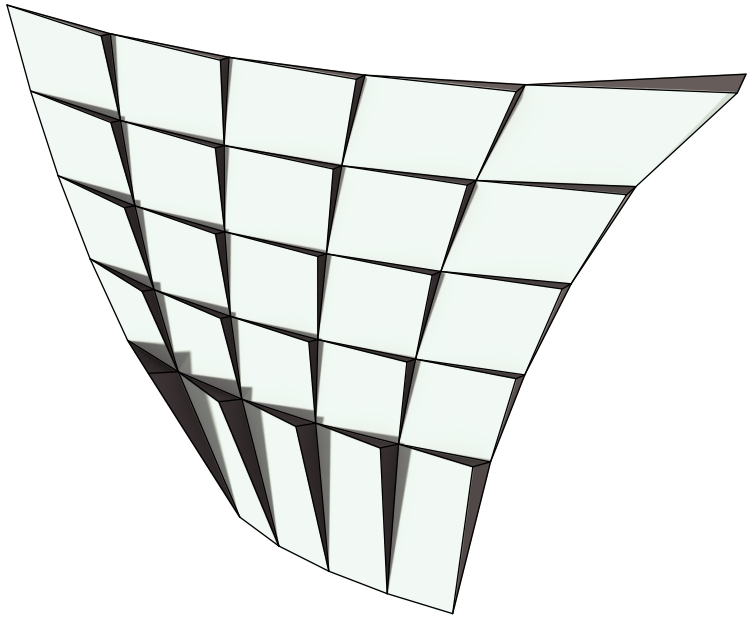
Dot Product

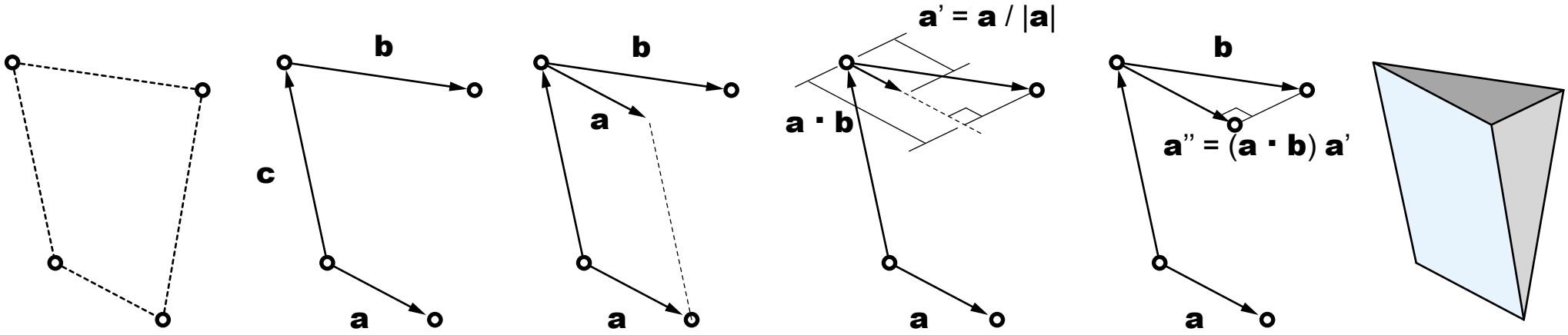


Cross Product

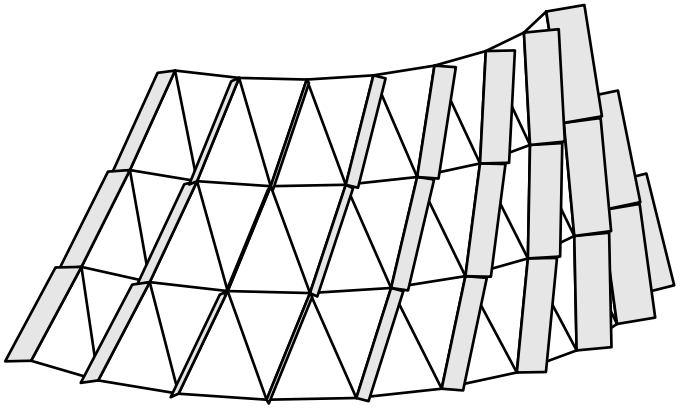
EXAMPLE OF VECTOR : DOT PRODUCT

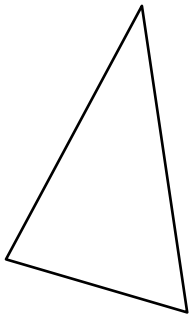


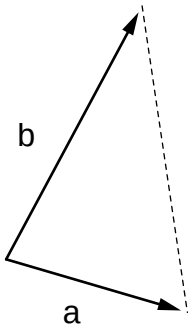


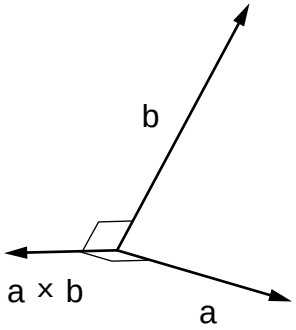


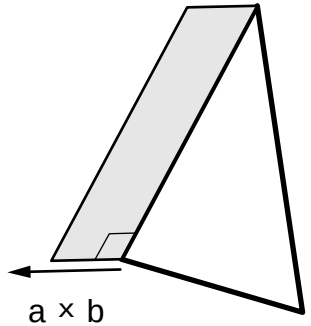
EXAMPLE OF VECTOR : CROSS PRODUCT



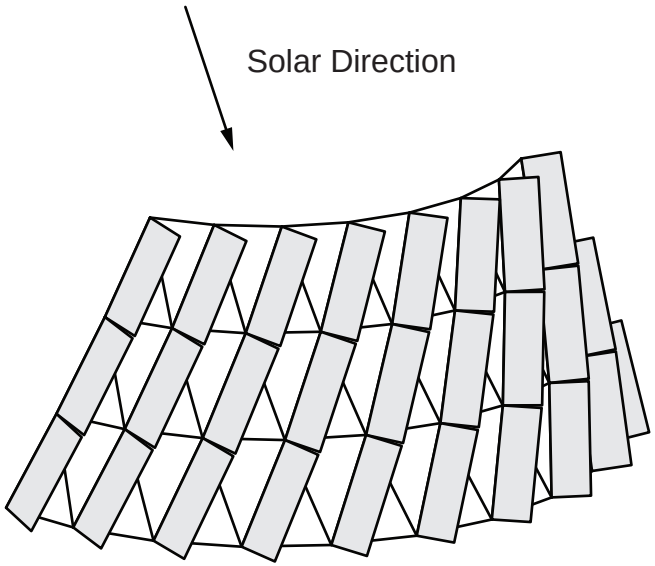


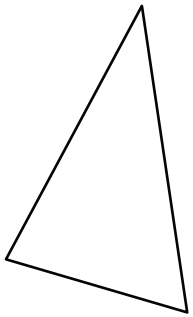


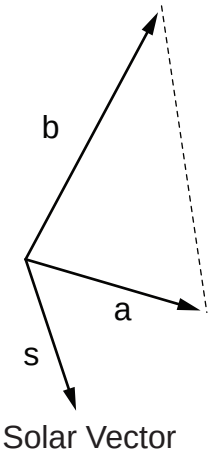


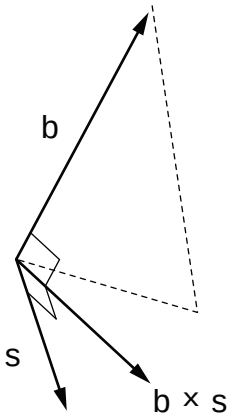


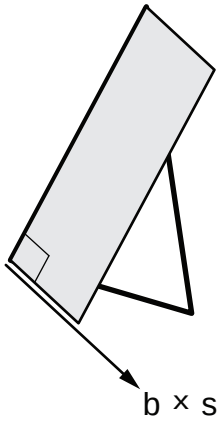
EXAMPLE OF VECTOR : CROSS PRODUCT



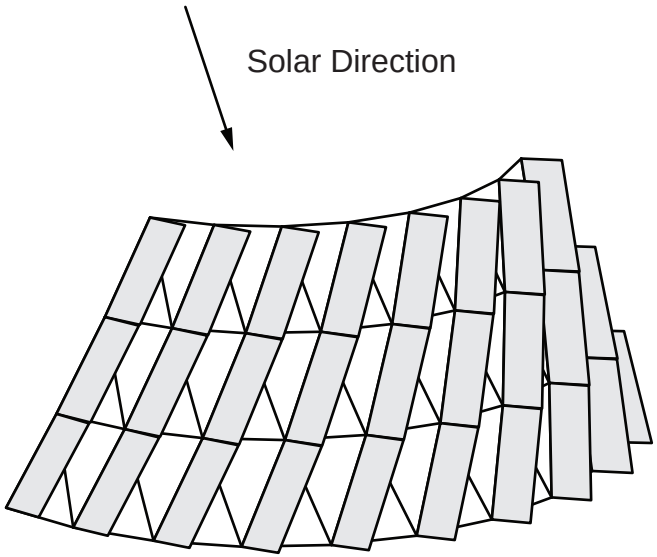


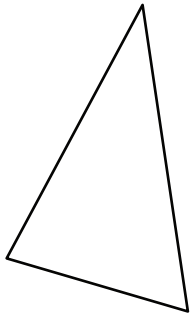


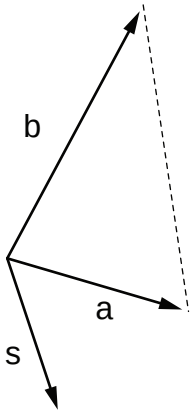




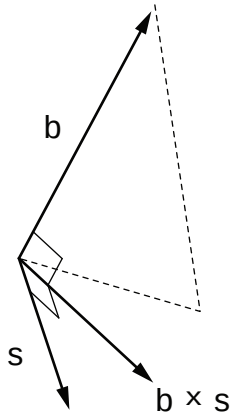
EXAMPLE OF VECTOR : CROSS PRODUCT

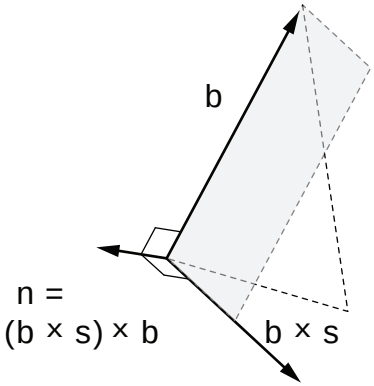


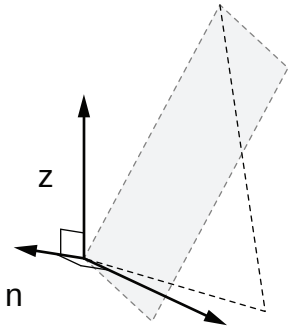




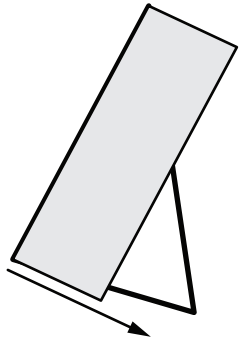
Solar Vector





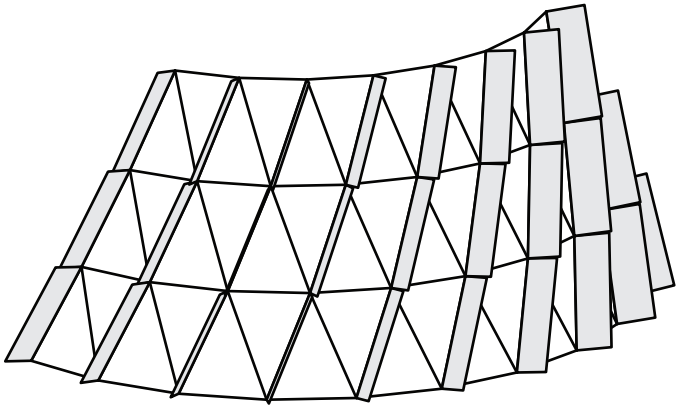


z × n =
z × ((b × s) × b)



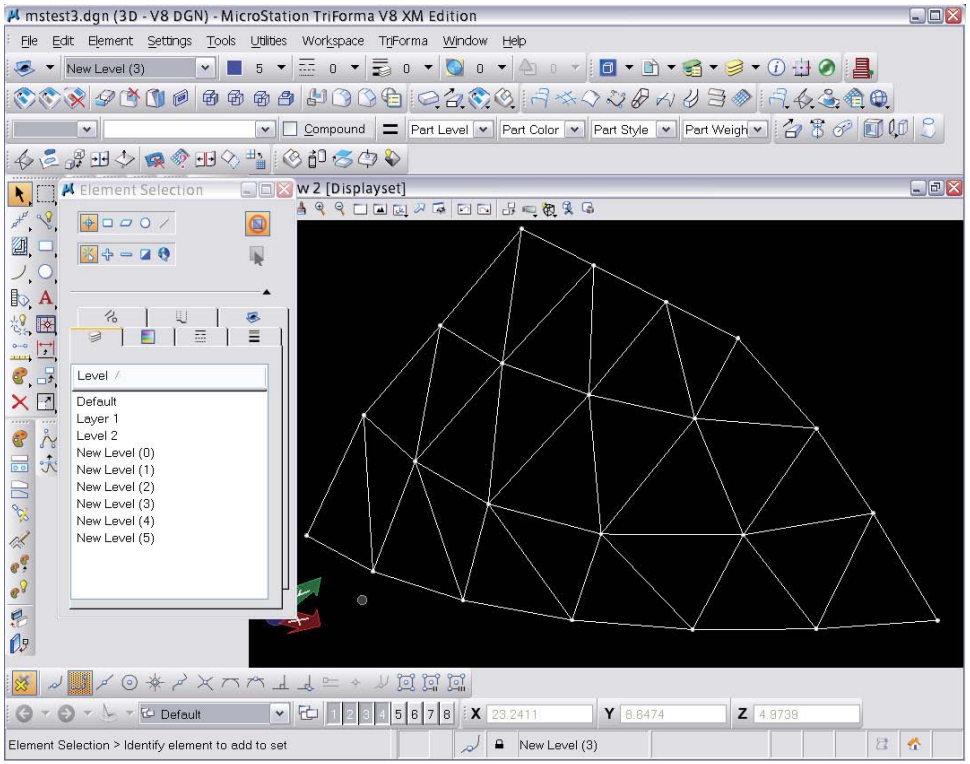
z × ((b × s) × b)

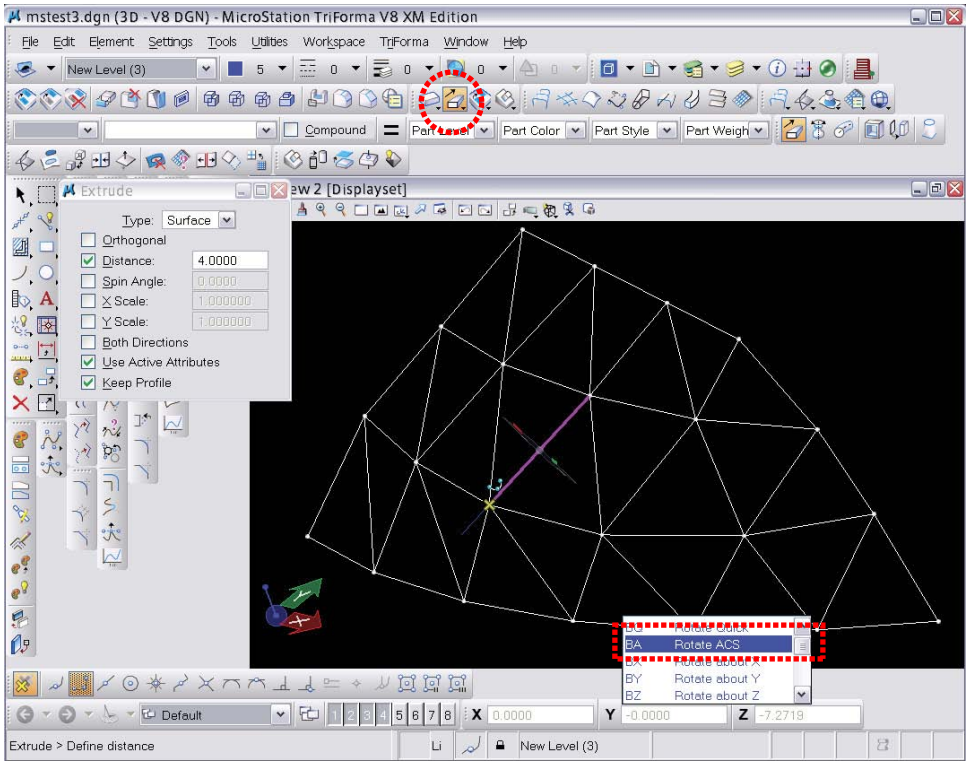
POINT AND VECTOR EXERCISE

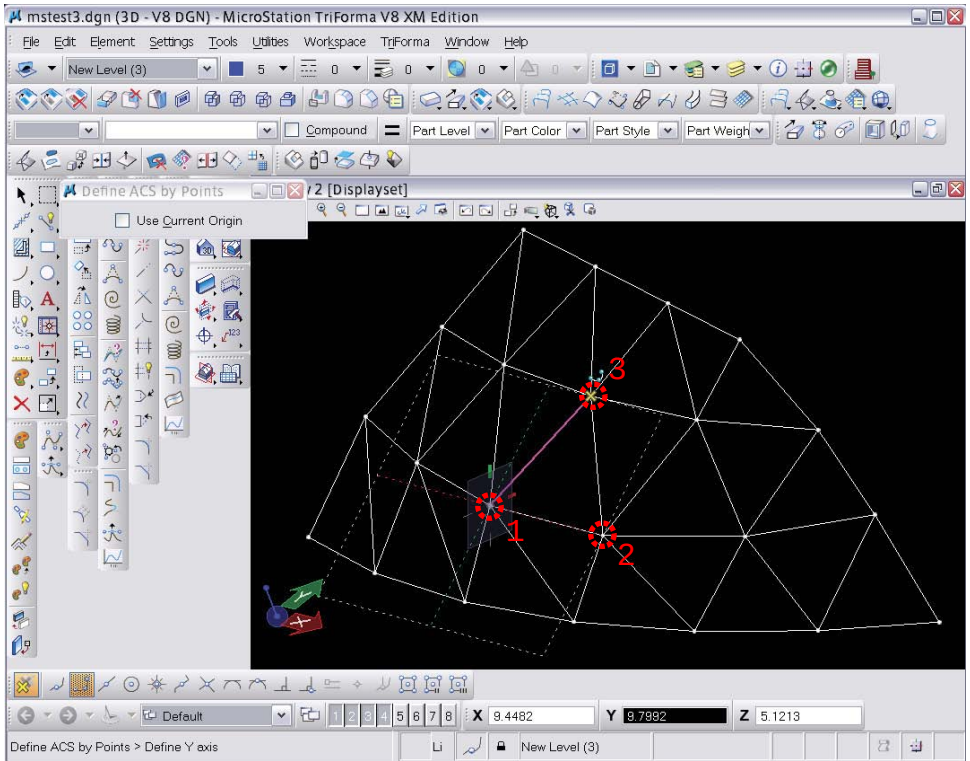


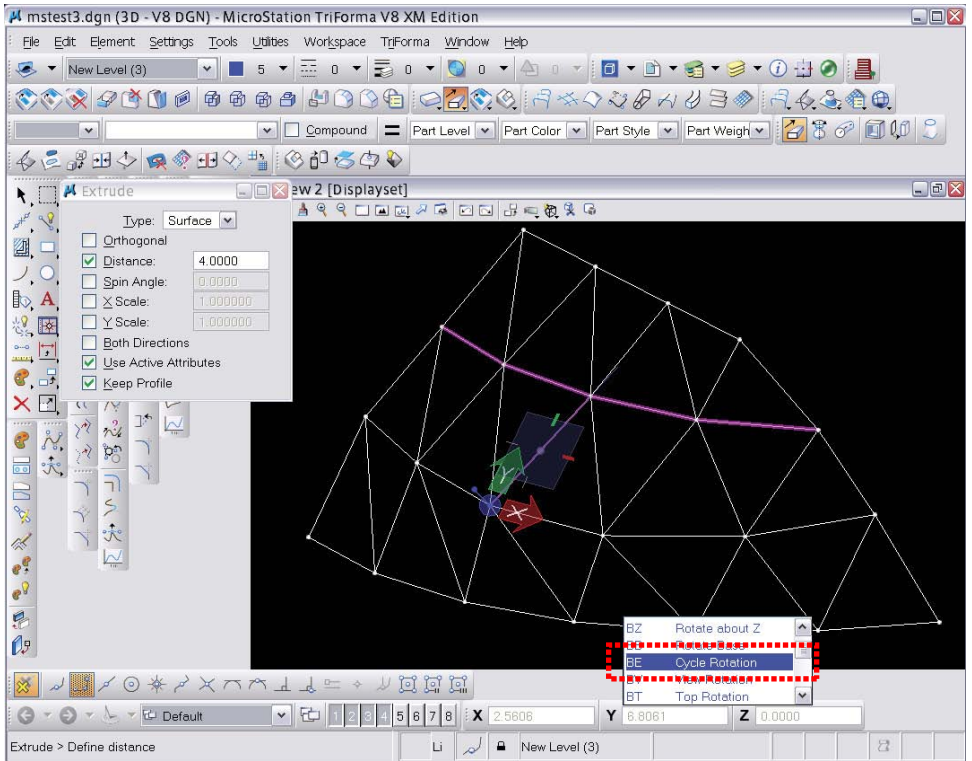
Make small diagrid and model fins perpendicular to the diagrid on any platform you choose.

POINT AND VECTOR EXERCISE

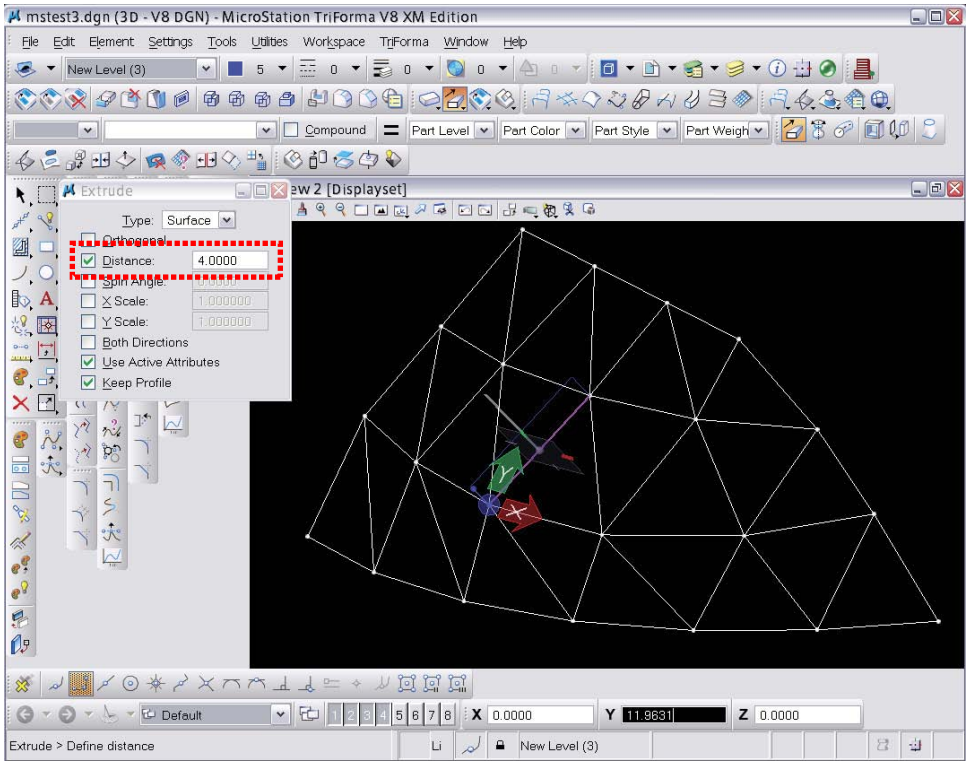


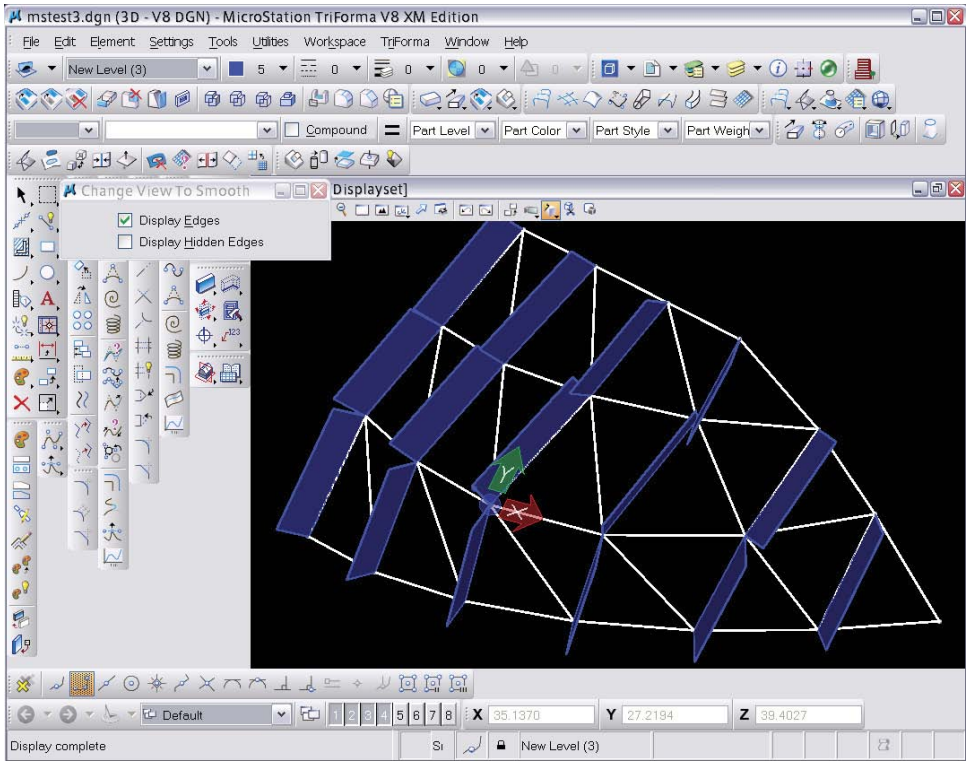




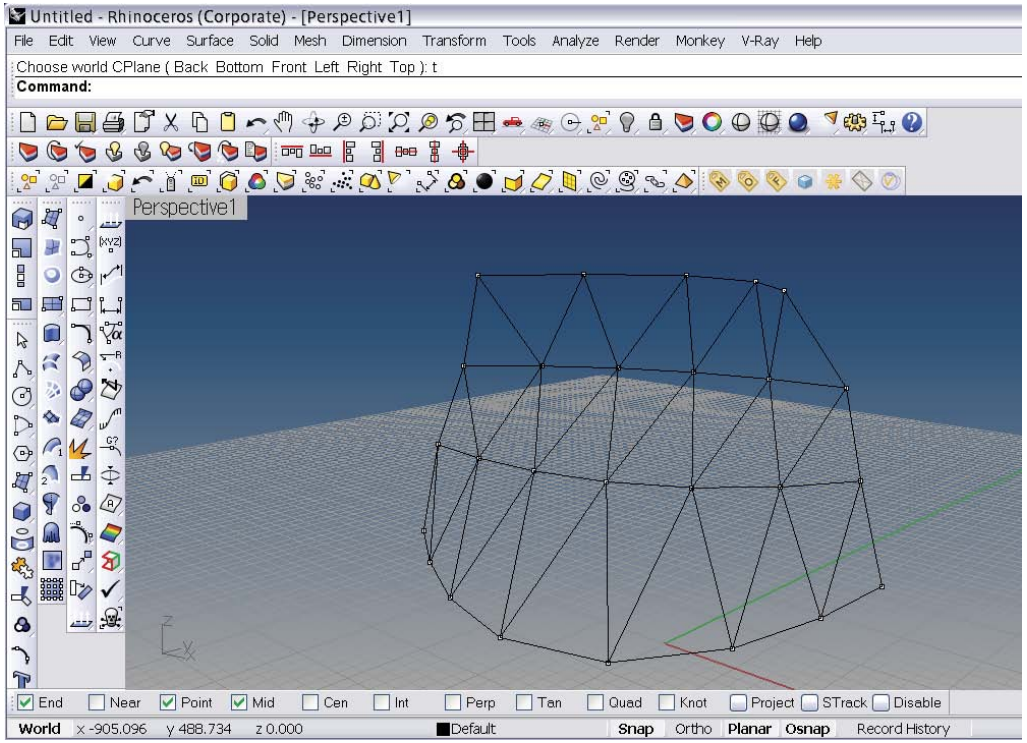


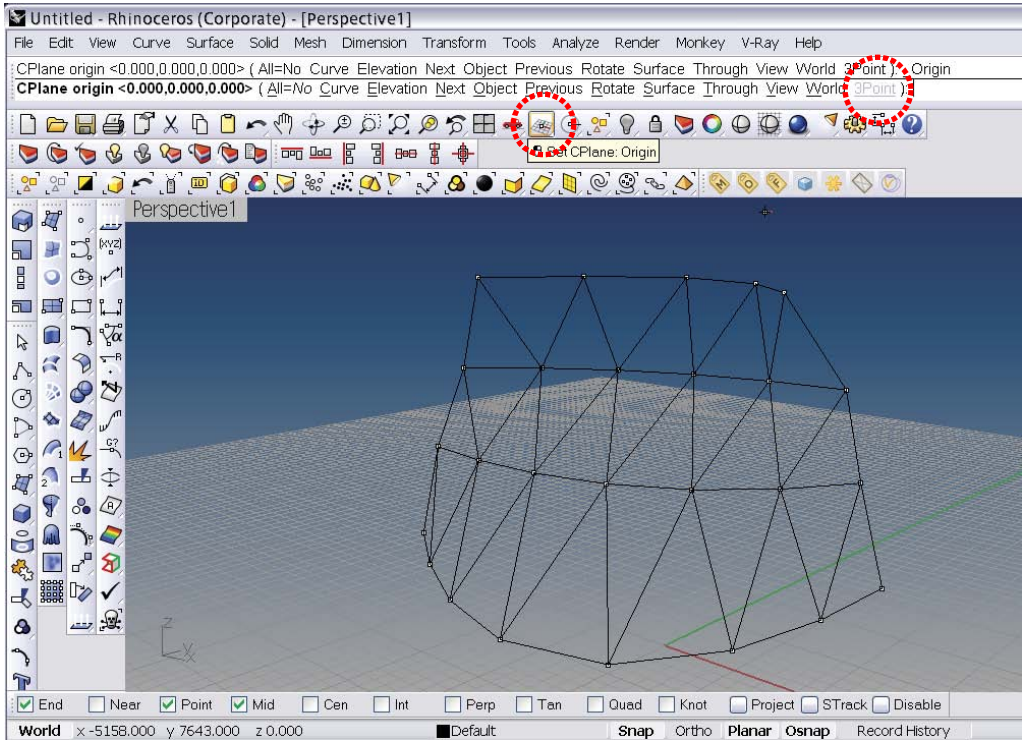
POINT AND VECTOR EXERCISE

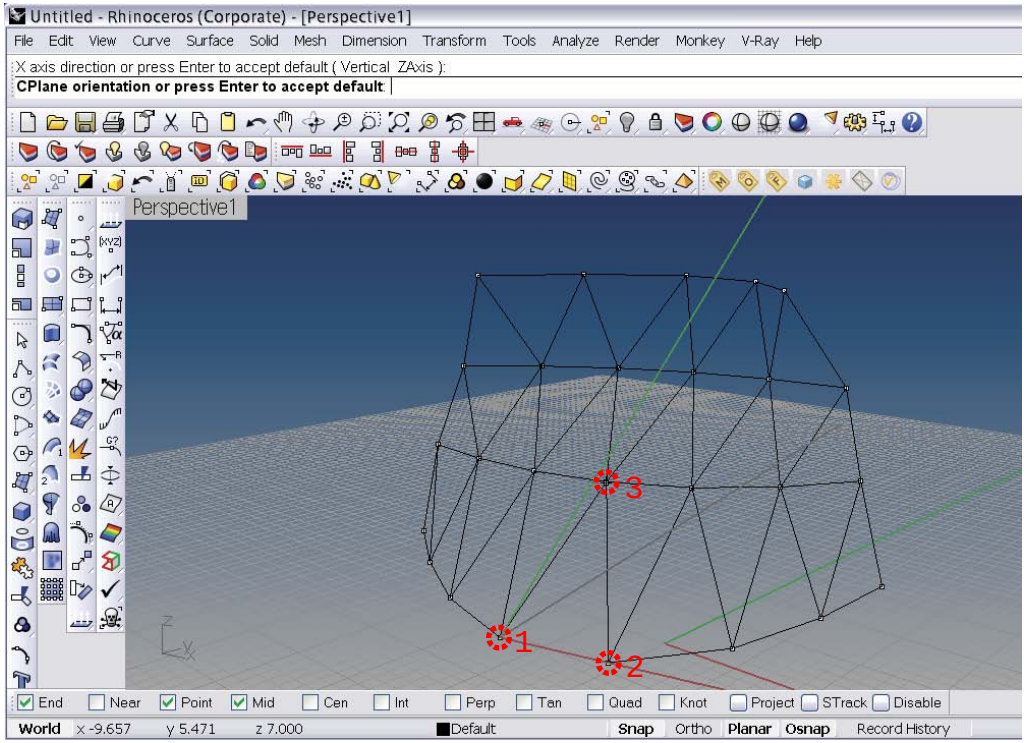


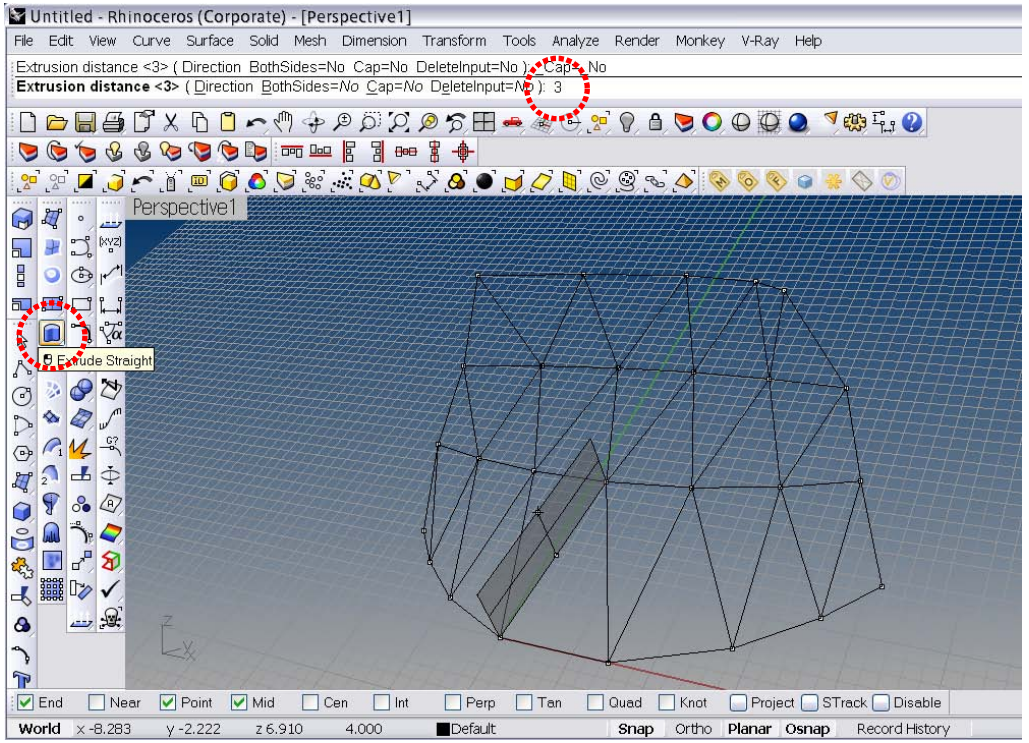


POINT AND VECTOR EXERCISE









POINT AND VECTOR EXERCISE