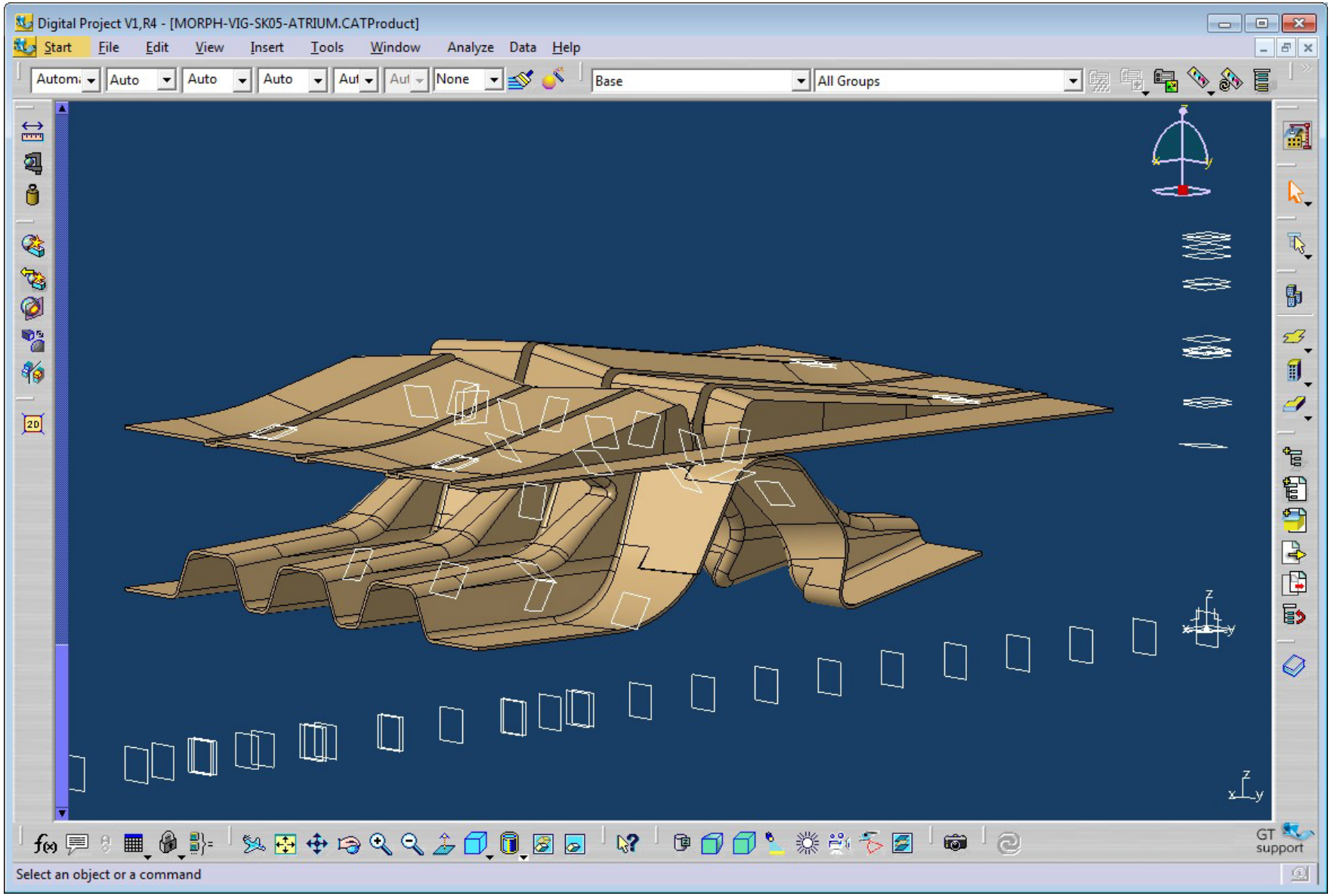
MORPH SEMINAR 2010

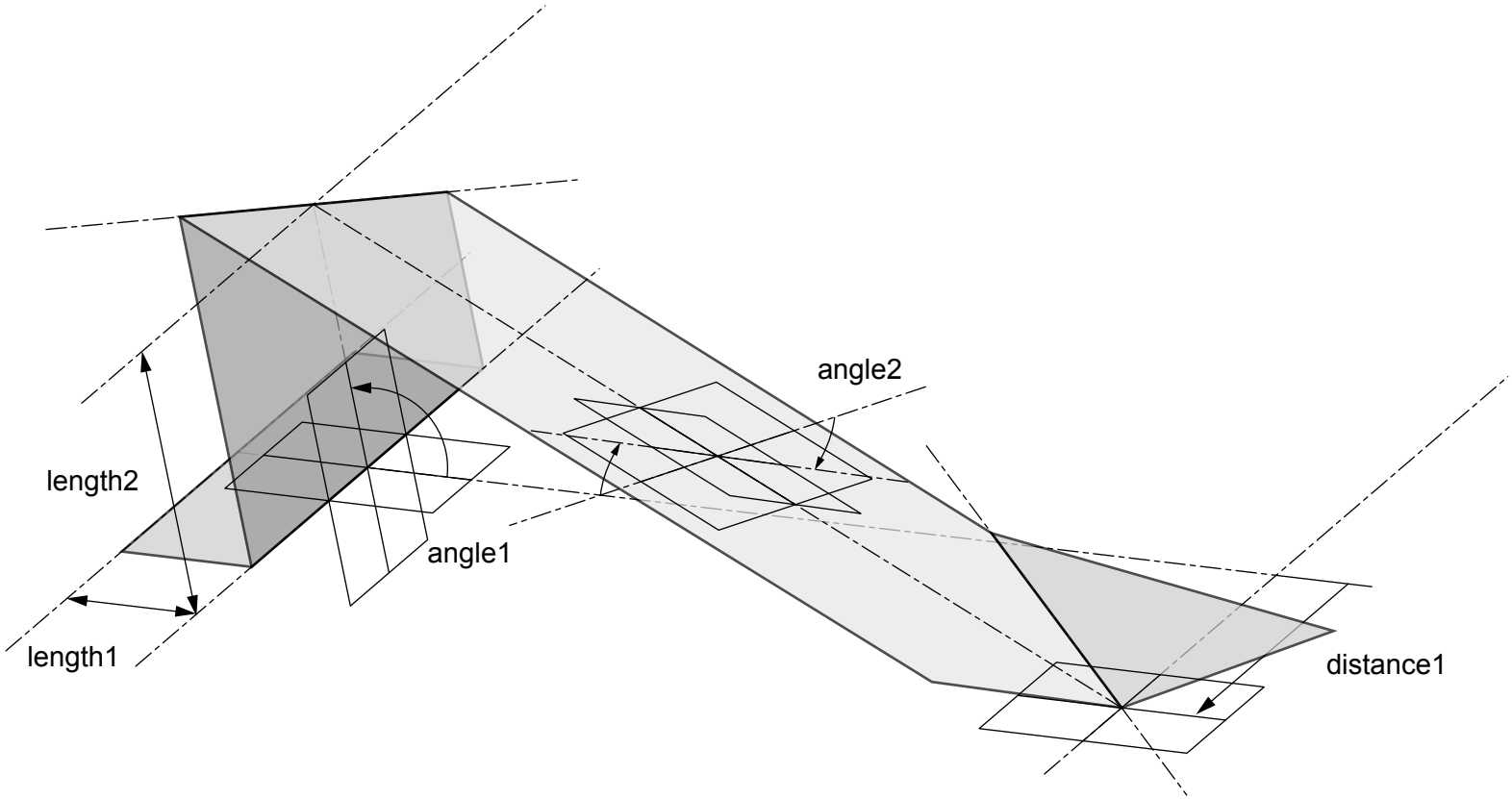
MATHEMATICS FOR SYSTEMATIC MODELING

WEEK4 PLANE

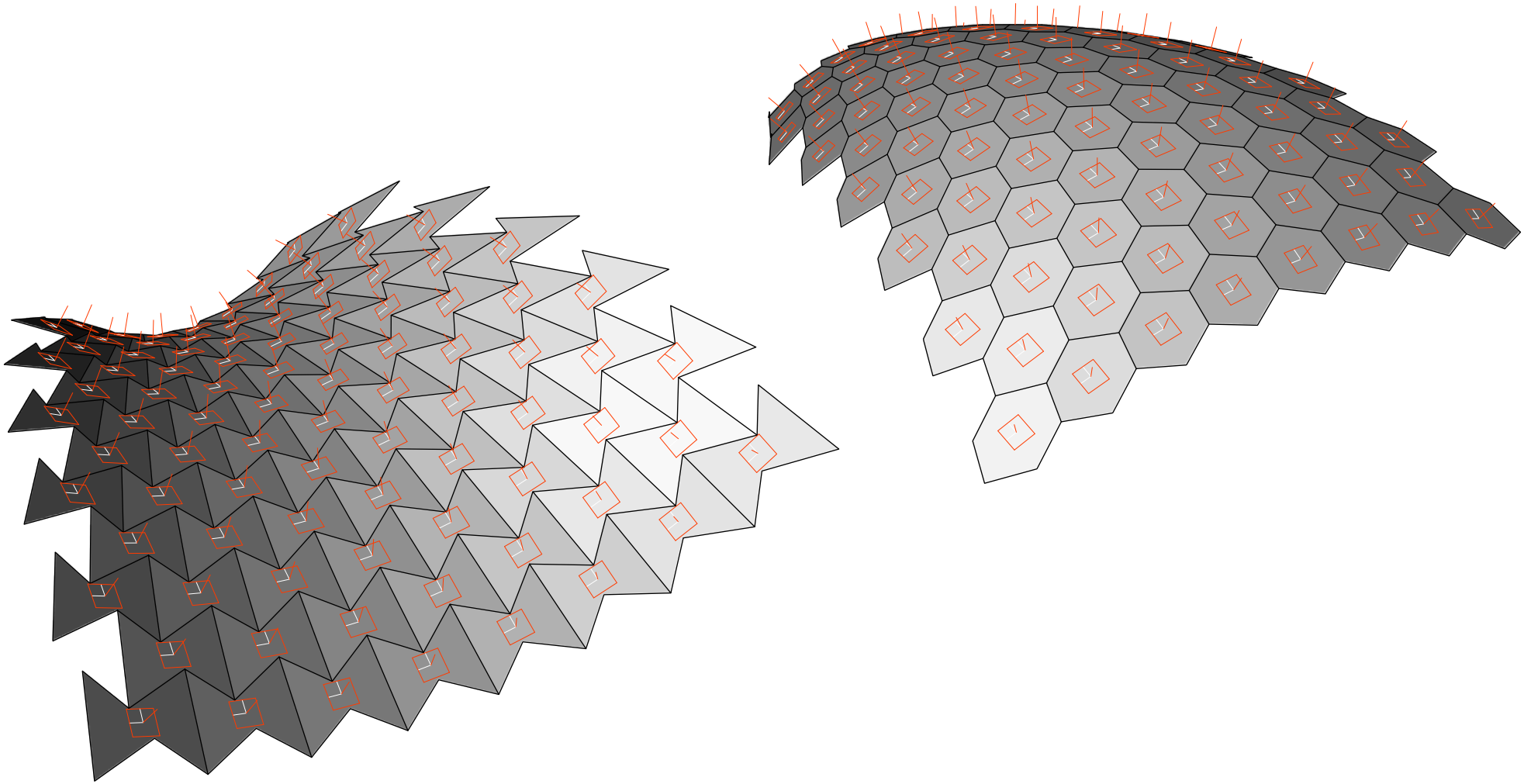
PLANES AS CONTROL GEOMETRY



PLANES FOR FOLDING GEOMETRY

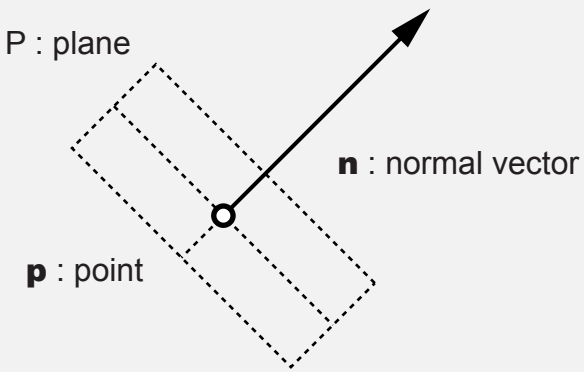


PLANES AND PANELIZATION

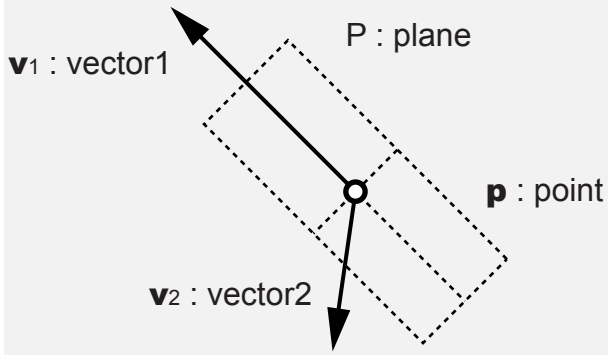


PLANE : DEFINITION

PLANE BY A POINT AND NORMAL VECTOR

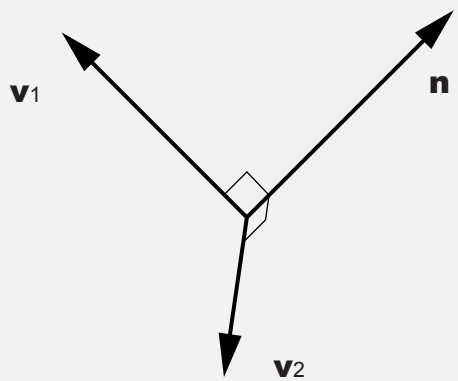


PLANE BY A POINT AND TWO VECTORS



$\mathbf{v}_1 \neq k\,\mathbf{v}_2$

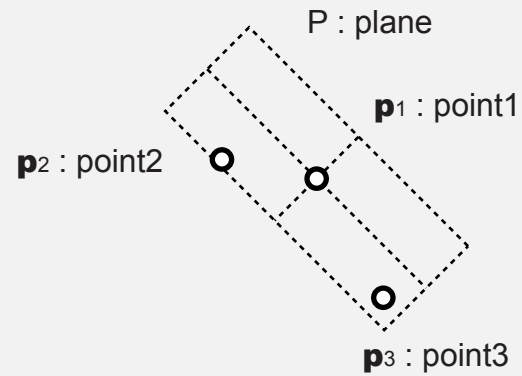
($\mathbf{v}_1$ and $\mathbf{v}_2$ cannot be parallel)



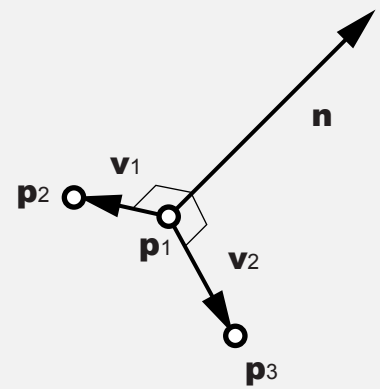
$\mathbf{v}_1 \times \mathbf{v}_2 = k\,\mathbf{n}$ (cross vector)

($\mathbf{v}_1$ and $\mathbf{v}_2$ are perpendicular to $\mathbf{n}$ and the length doesn't matter)

PLANE BY 3 POINTS



(3 points cannot be on a line)



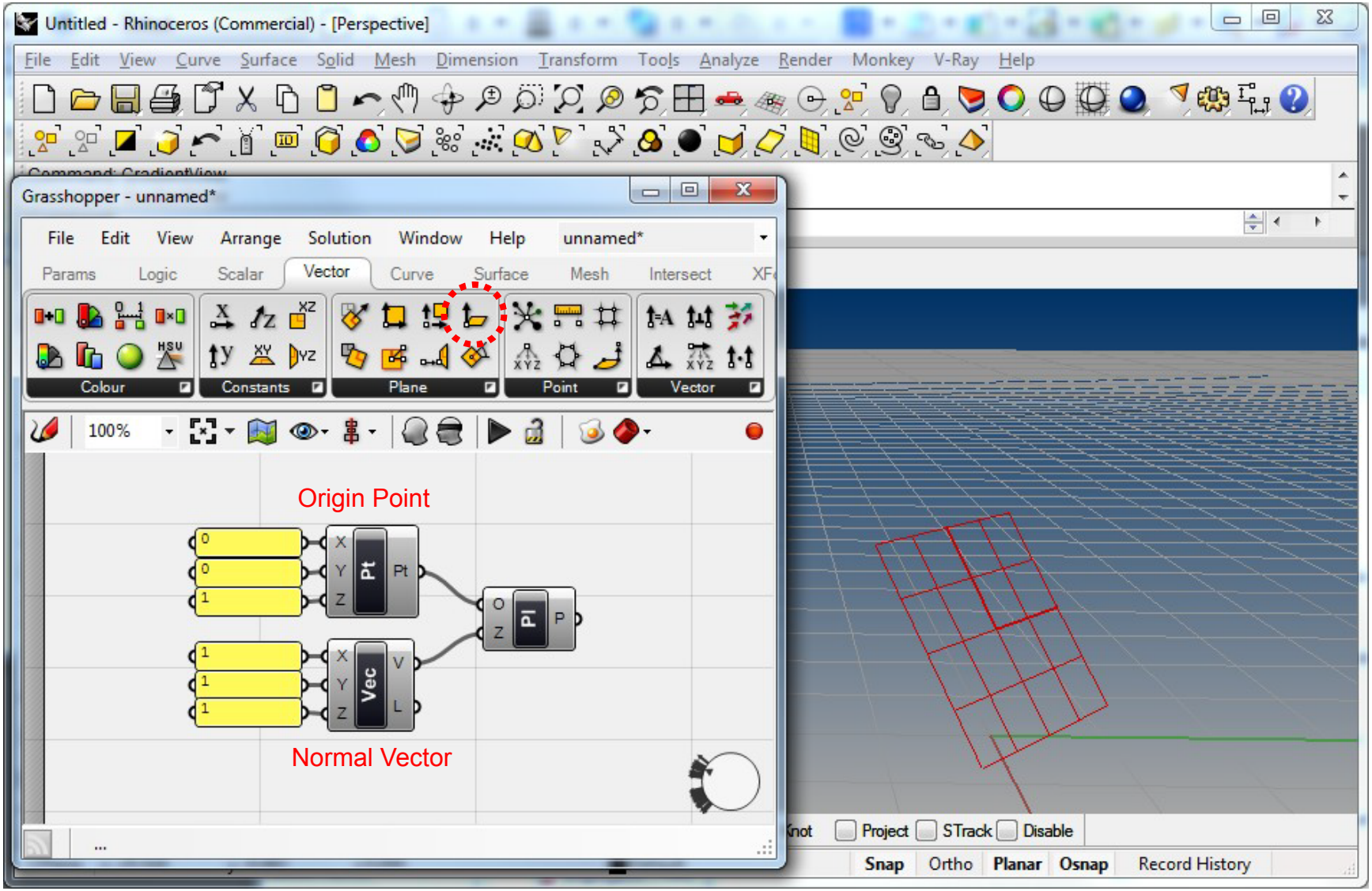
$\mathbf{v}_1 = \mathbf{p}_2 - \mathbf{p}_1$

$\mathbf{v}_2 = \mathbf{p}_3 - \mathbf{p}_1$

$(\mathbf{p}_2 - \mathbf{p}_1) \times (\mathbf{p}_3 - \mathbf{p}_1) = k\,\mathbf{n}$

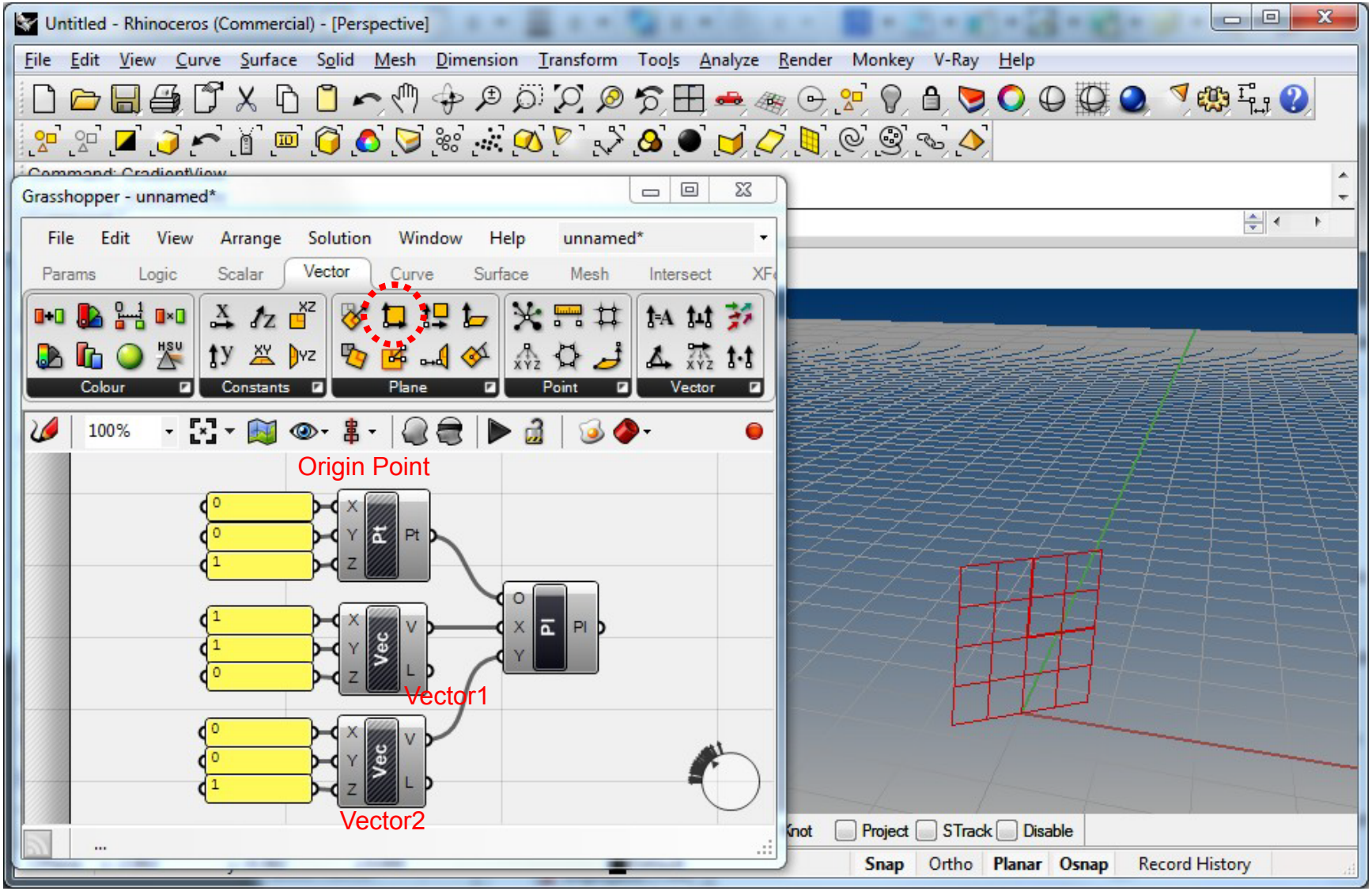
PLANE : DEFINITION IN GRASSHOPPER

PLANE BY A POINT AND NORMAL VECTOR



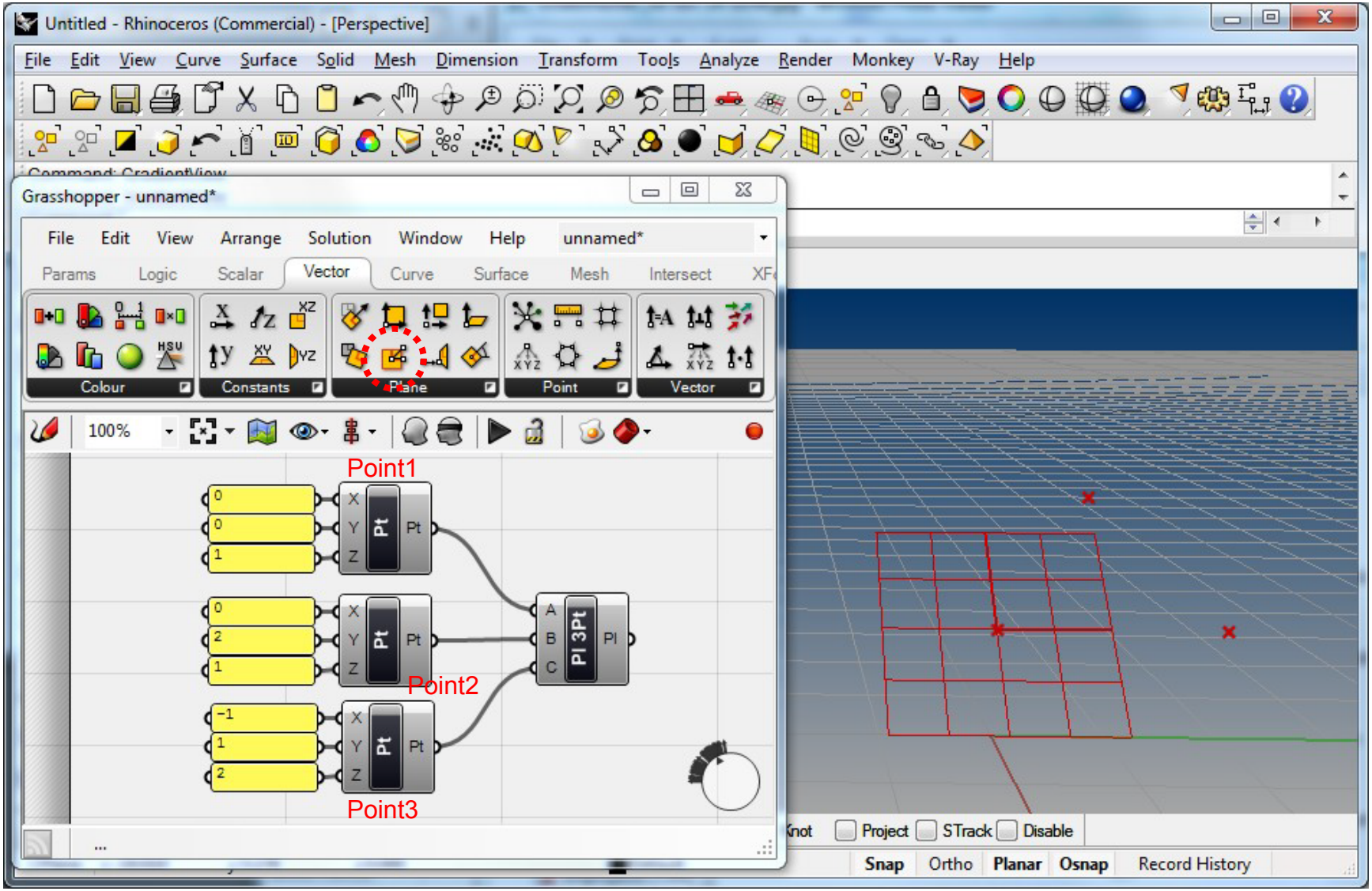
PLANE : DEFINITION IN GRASSHOPPER

PLANE BYA POINT AND TWO VECTORS



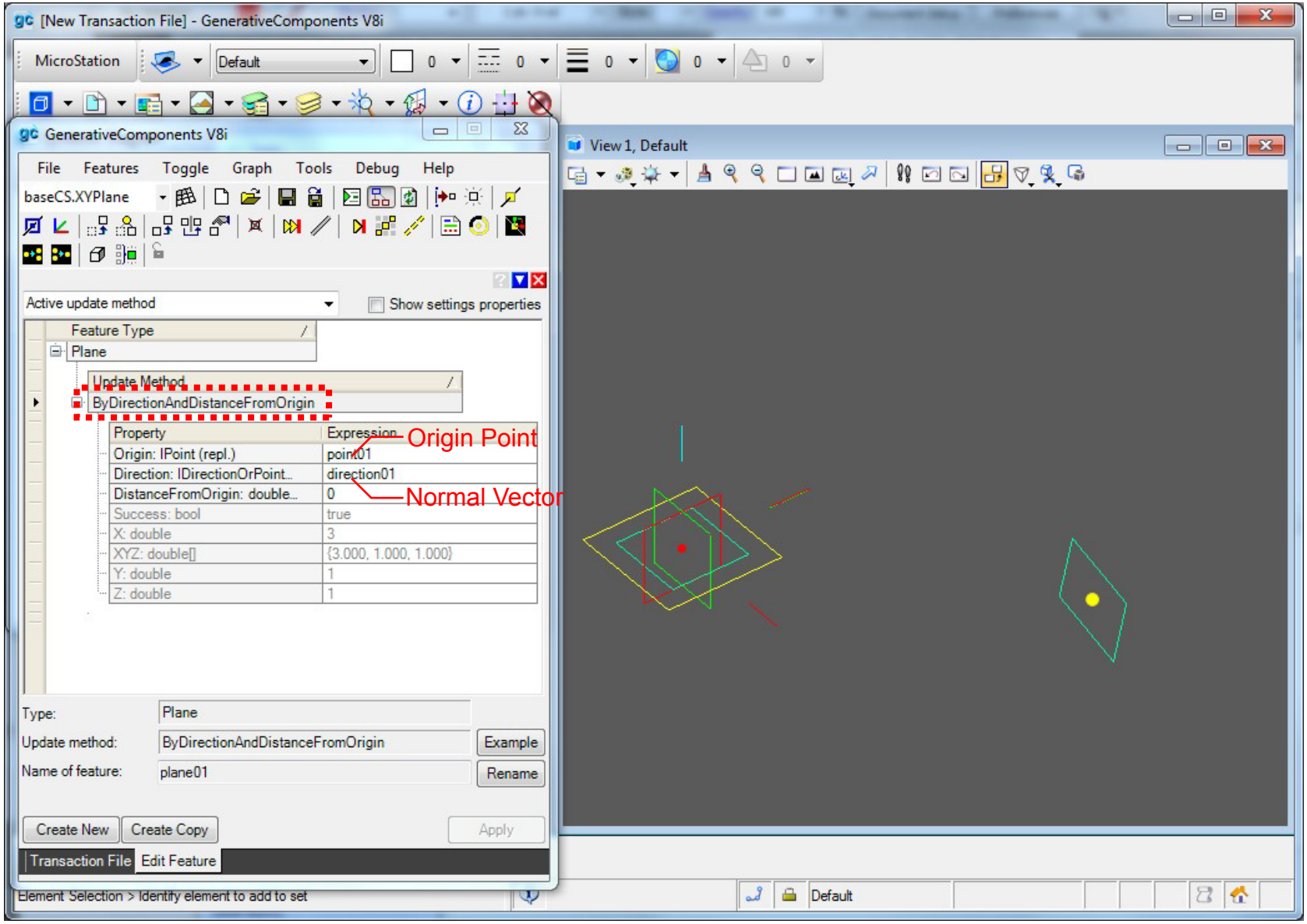
PLANE : DEFINITION IN GRASSHOPPER

PLANE BY 3 POINTS



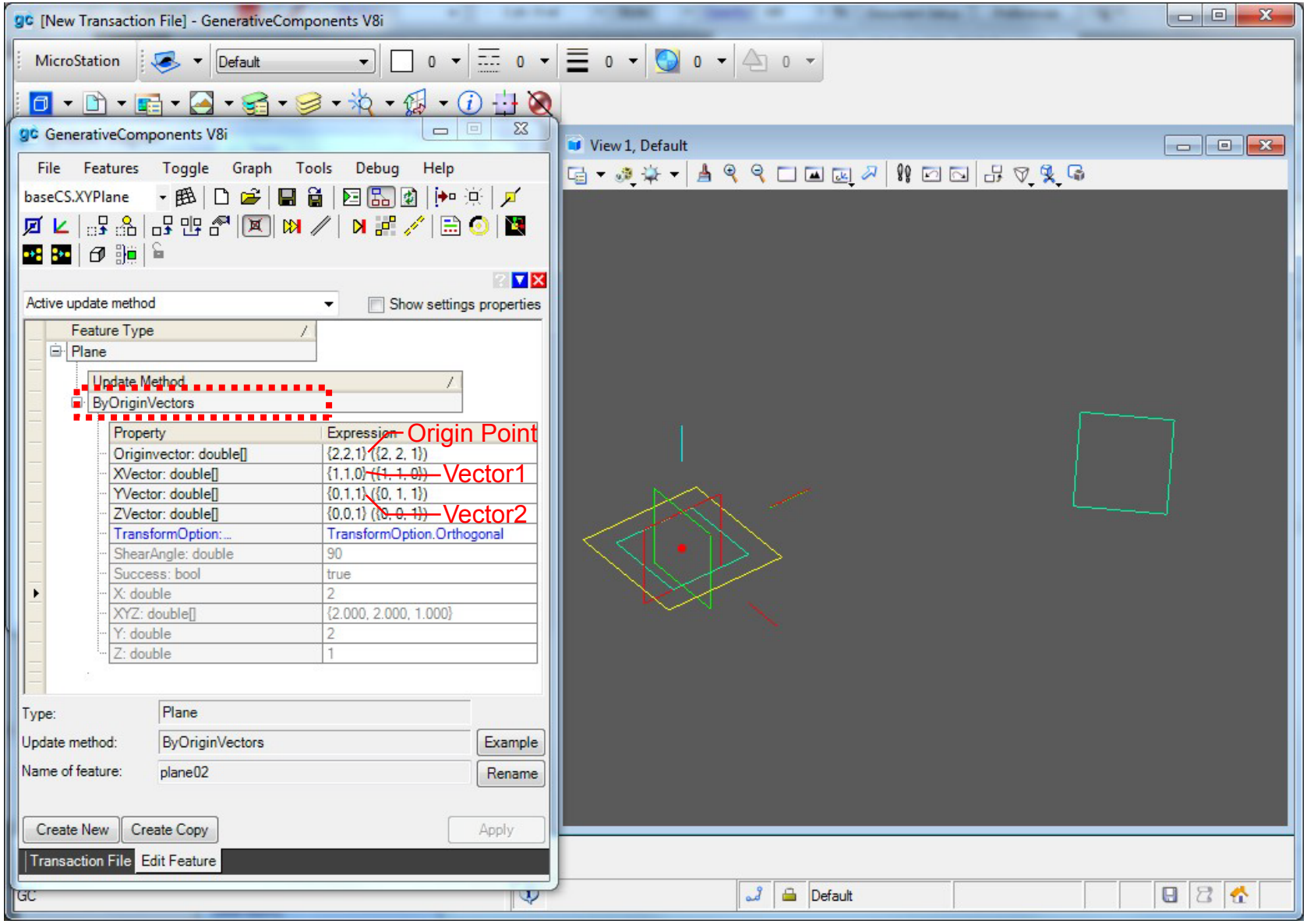
PLANE : DEFINITION IN GENERATIVE COMPONENTS

PLANE BY A POINT AND NORMAL VECTOR



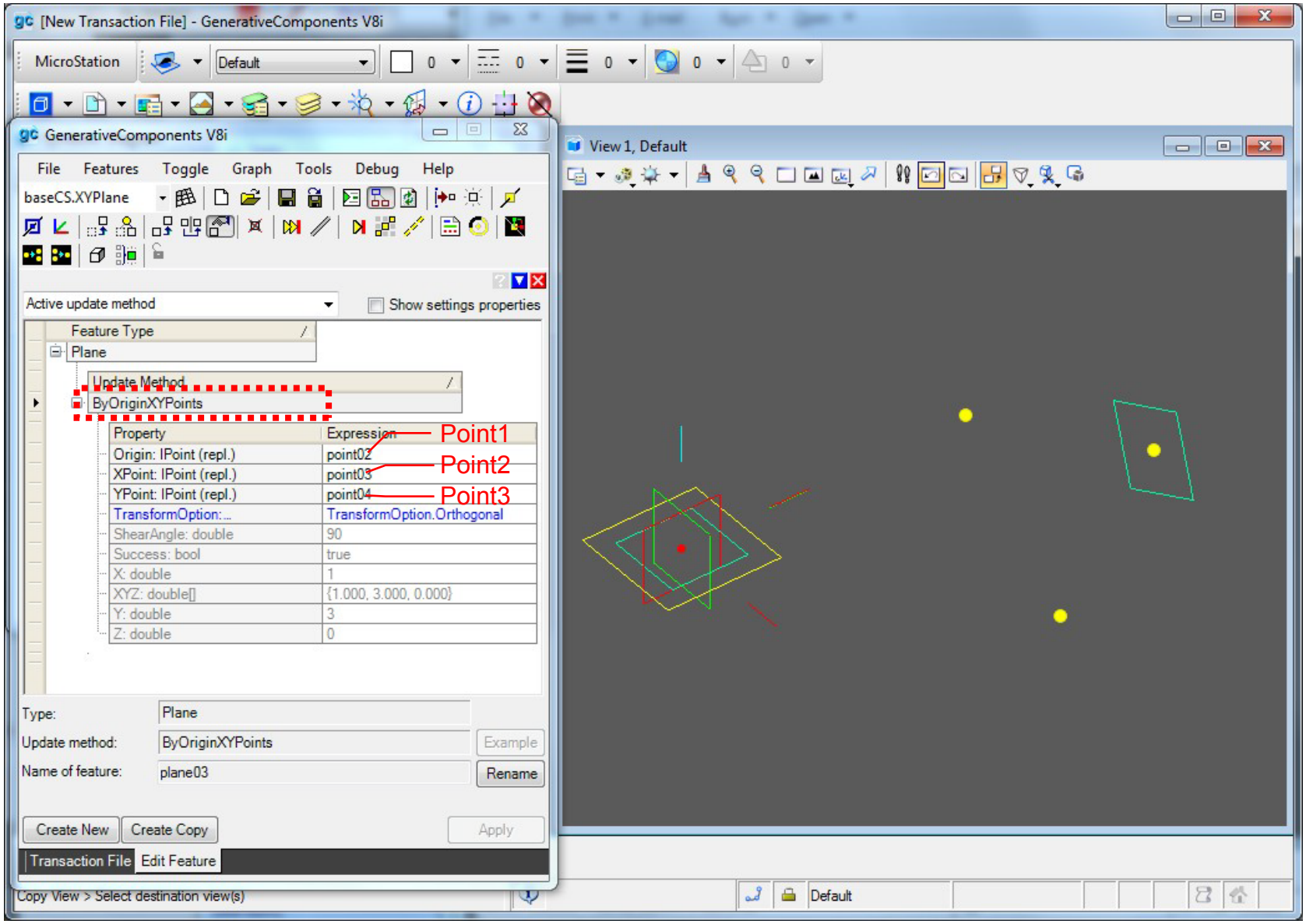
PLANE : DEFINITION IN GENERATIVE COMPONENTS

PLANE BY A POINT AND TWO VECTORS



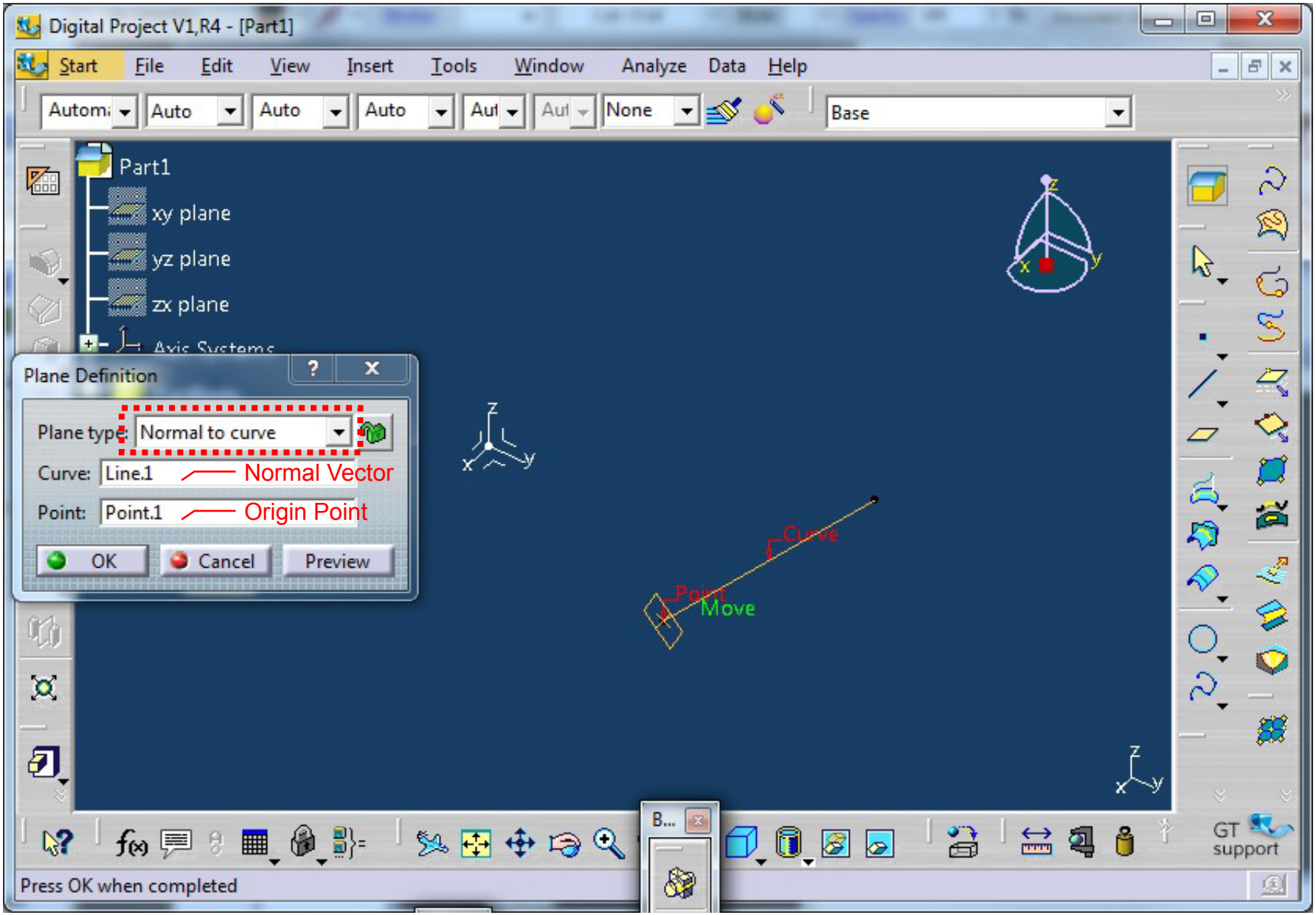
PLANE : DEFINITION IN GENERATIVE COMPONENTS

PLANE BY 3 POINTS



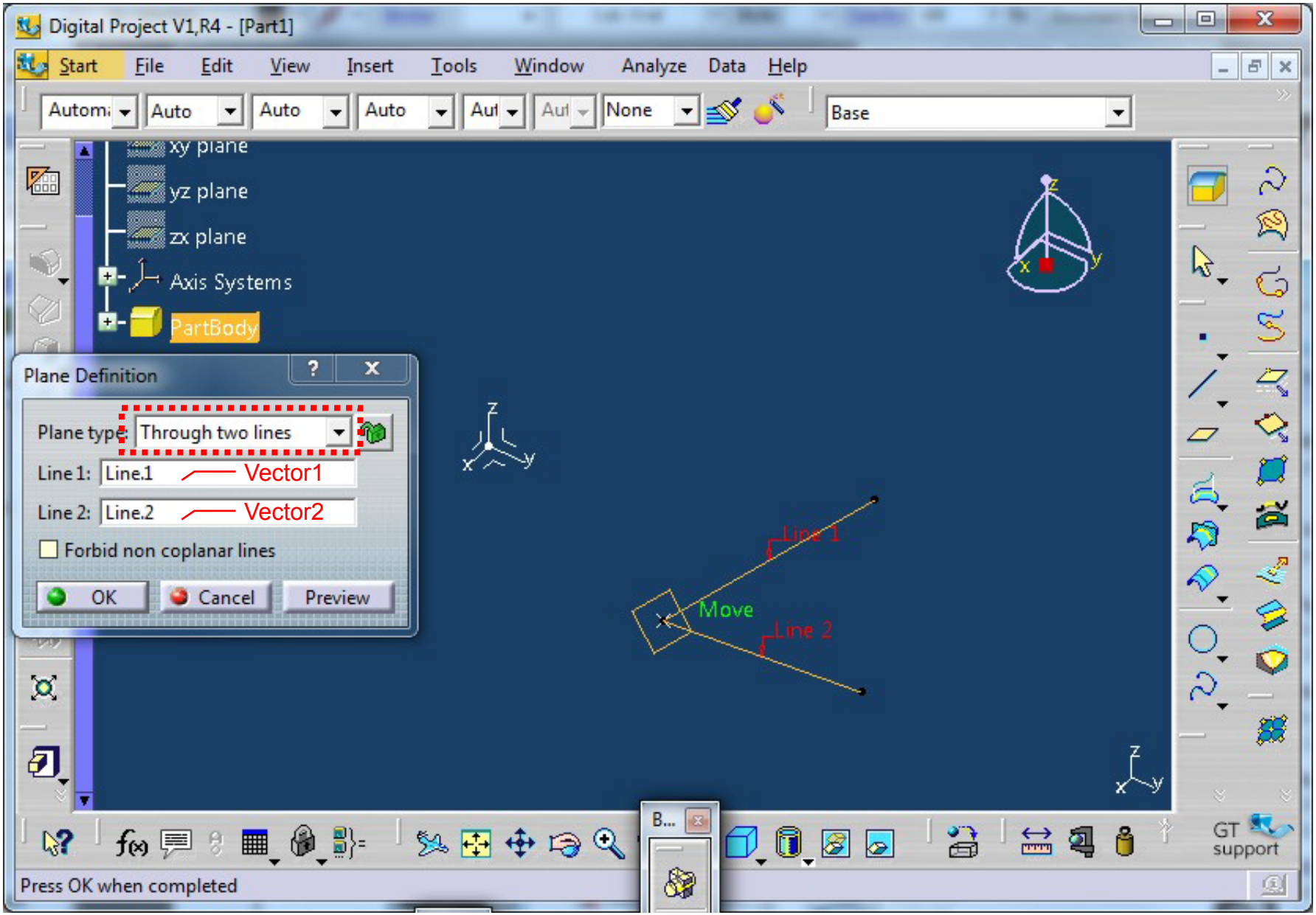
PLANE : DEFINITION IN DIGITAL PROJECT

PLANE BY A POINT AND NORMAL VECTOR



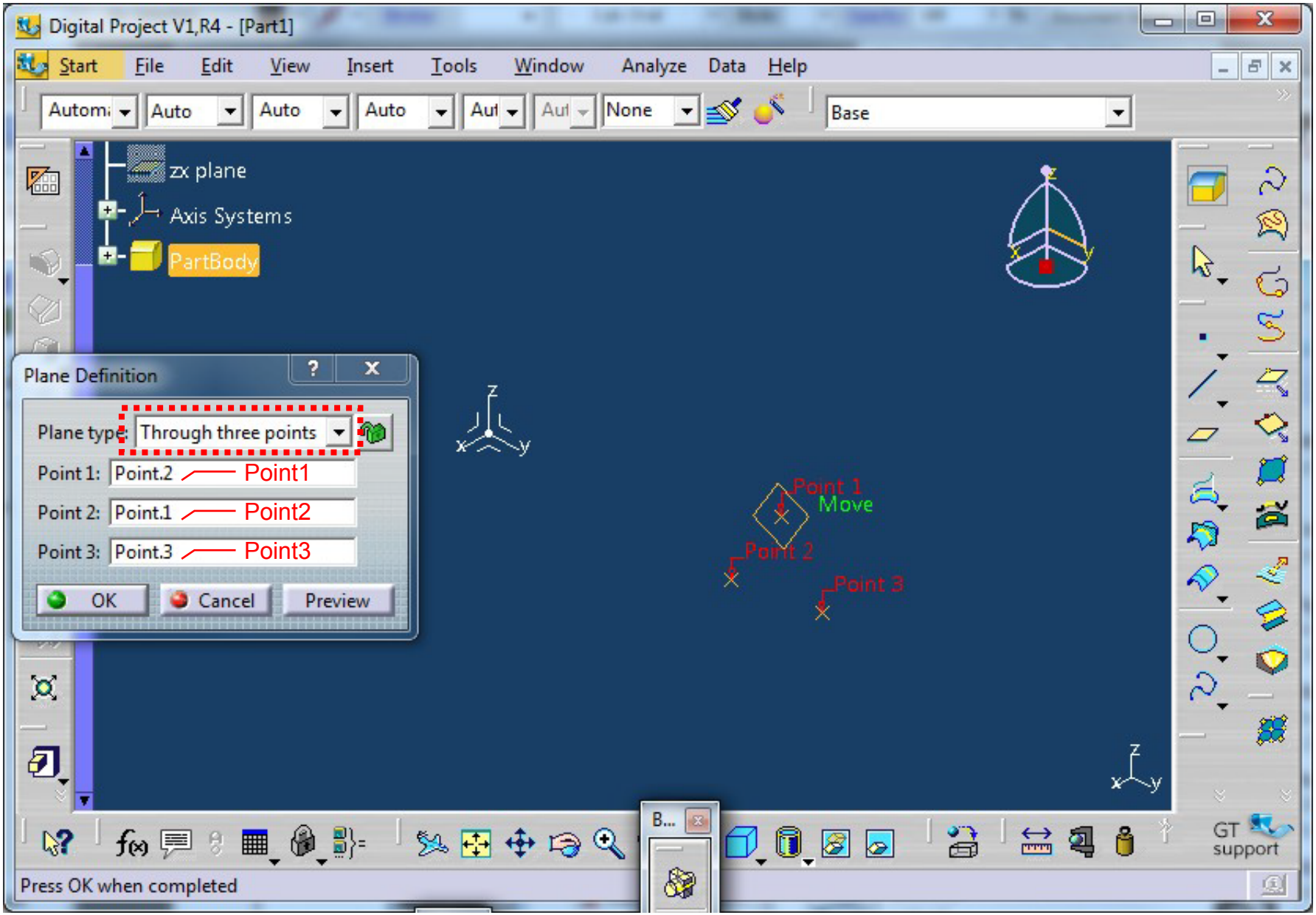
PLANE : DEFINITION IN DIGITAL PROJECT

PLANE BY A POINT AND TWO VECTORS



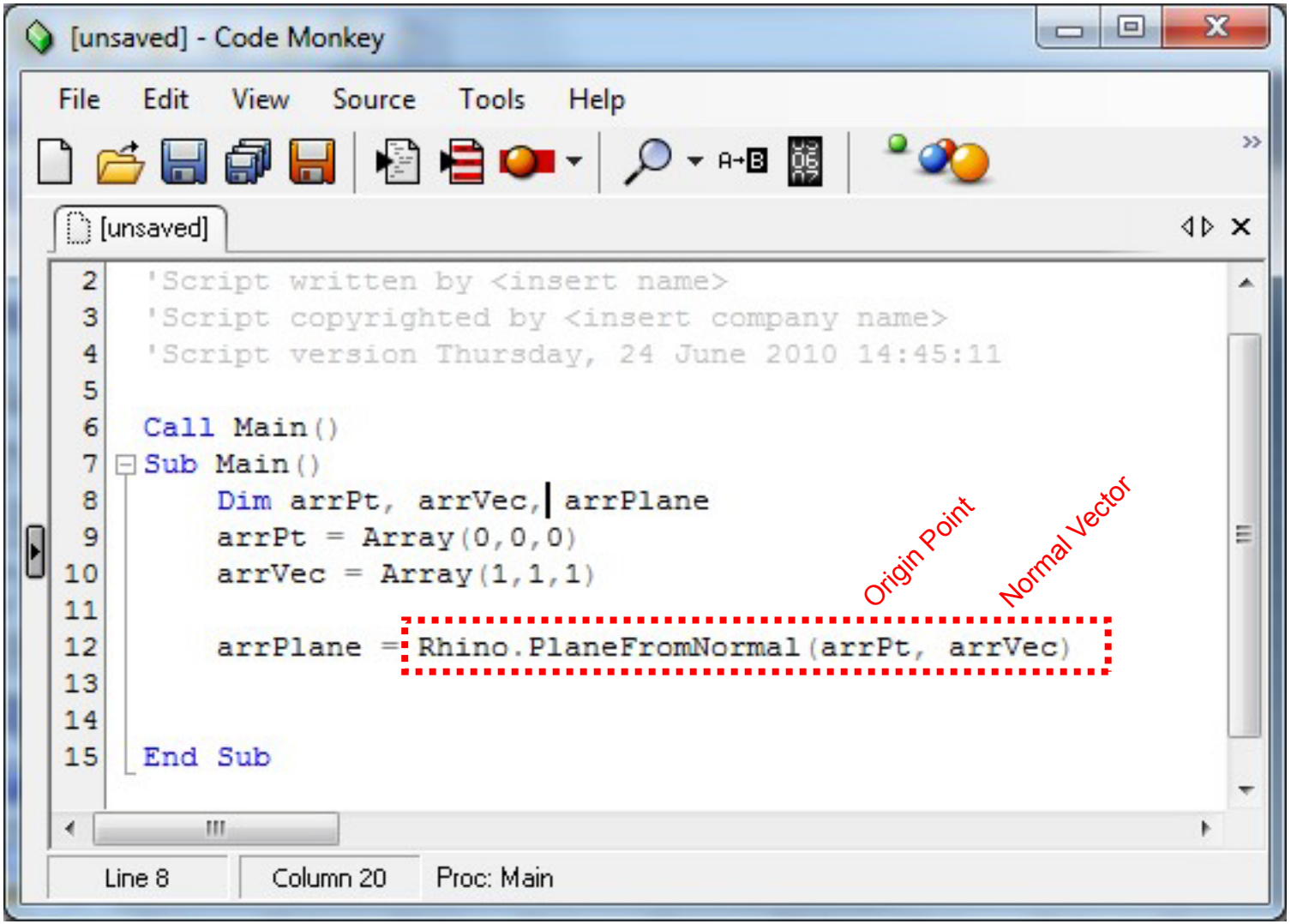
PLANE : DEFINITION IN DIGITAL PROJECT

PLANE BY 3 POINTS



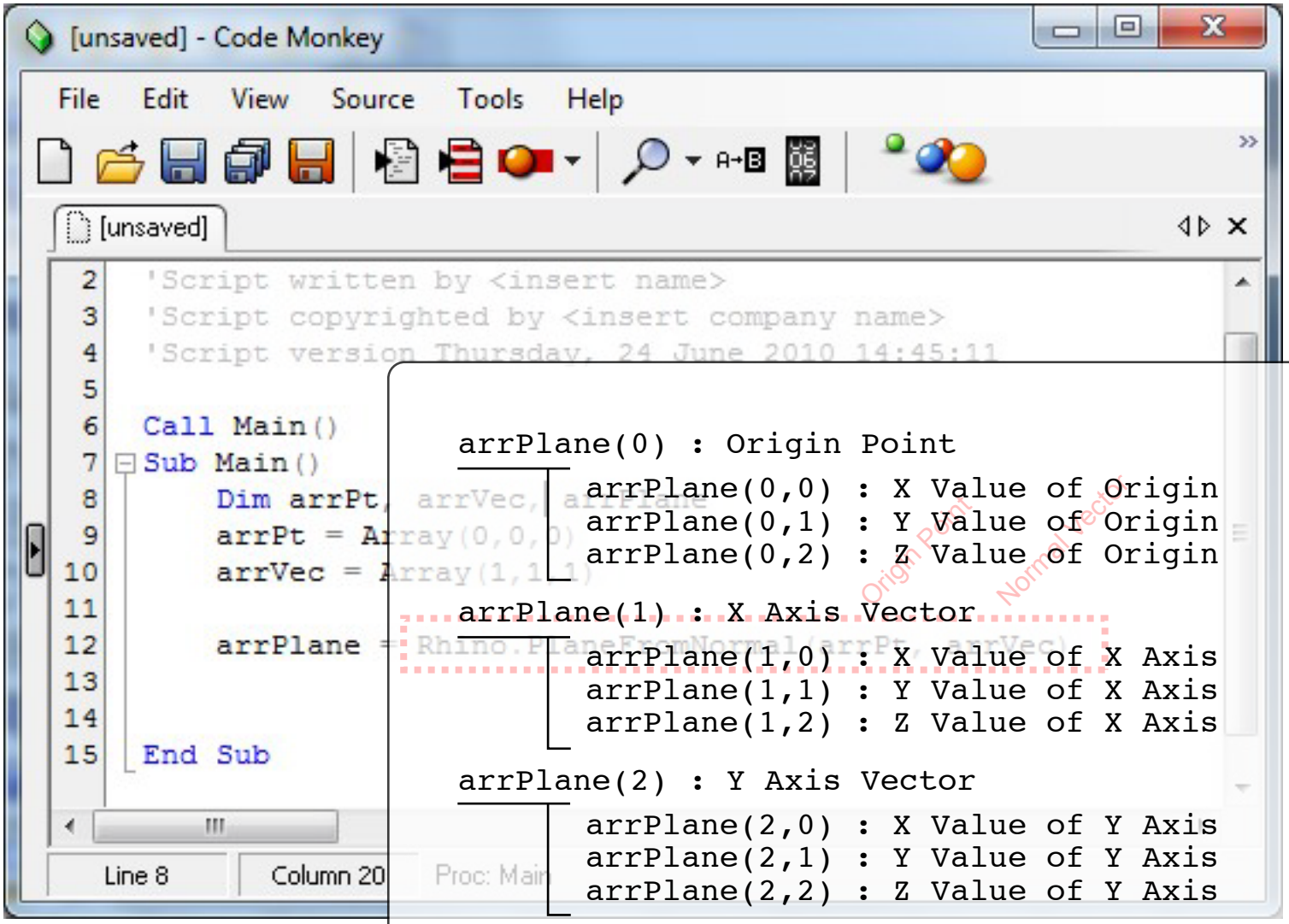
PLANE : DEFINITION IN RHINOSCRIPT

PLANE BY A POINT AND NORMAL VECTOR



PLANE : DEFINITION IN RHINOSCRIPT

PLANE BY A POINT AND NORMAL VECTOR




```
arrPlane(0) : Origin Point
    arrPlane(0,0) : X Value of Origin
    arrPlane(0,1) : Y Value of Origin
    arrPlane(0,2) : Z Value of Origin

arrPlane(1) : X Axis Vector
    arrPlane(1,0) : X Value of X Axis
    arrPlane(1,1) : Y Value of X Axis
    arrPlane(1,2) : Z Value of X Axis

arrPlane(2) : Y Axis Vector
    arrPlane(2,0) : X Value of Y Axis
    arrPlane(2,1) : Y Value of Y Axis
    arrPlane(2,2) : Z Value of Y Axis

arrPlane(3) : Z Axis Vector
    arrPlane(3,0) : X Value of Z Axis
    arrPlane(3,1) : Y Value of Z Axis
    arrPlane(3,2) : Z Value of Z Axis
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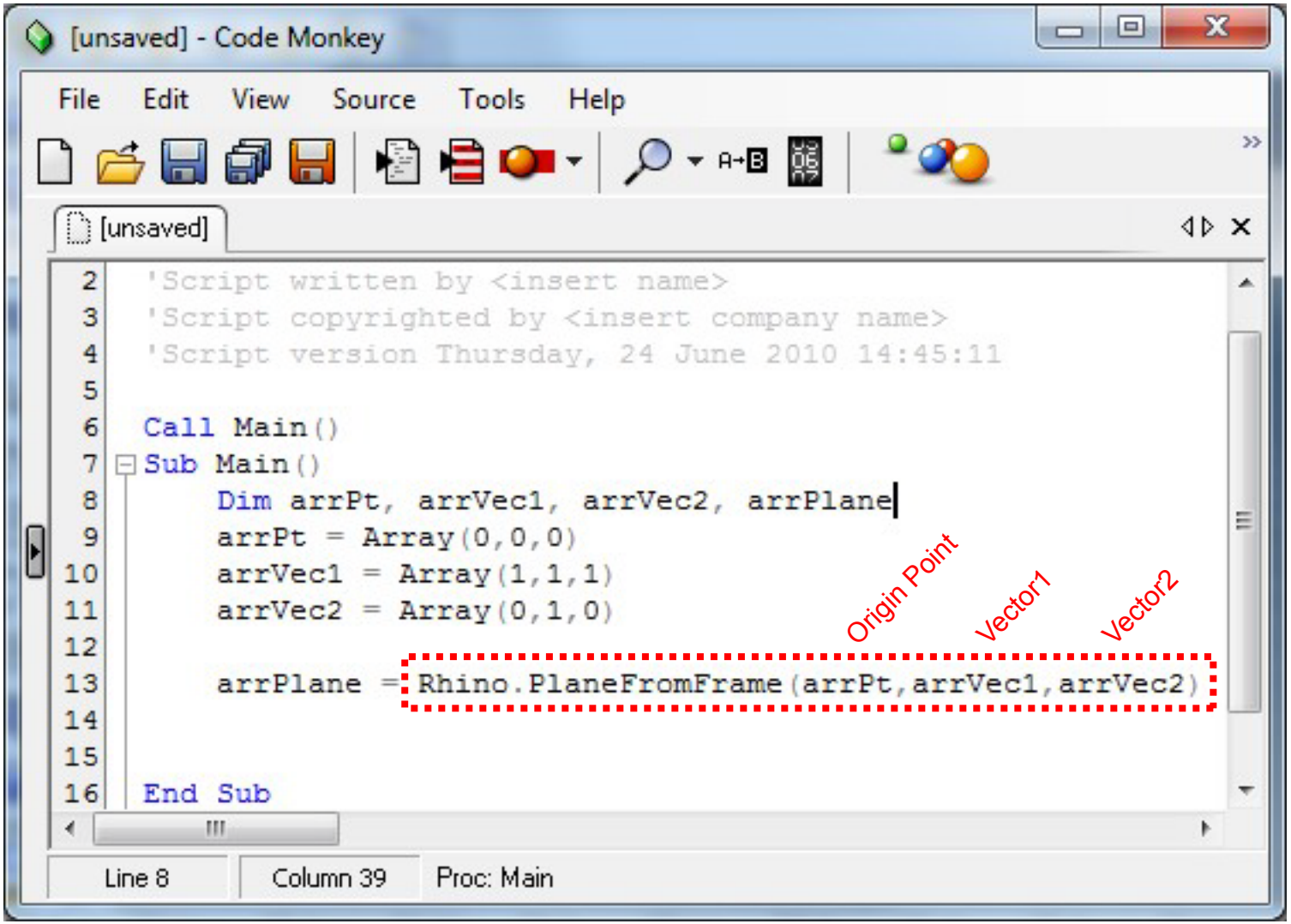
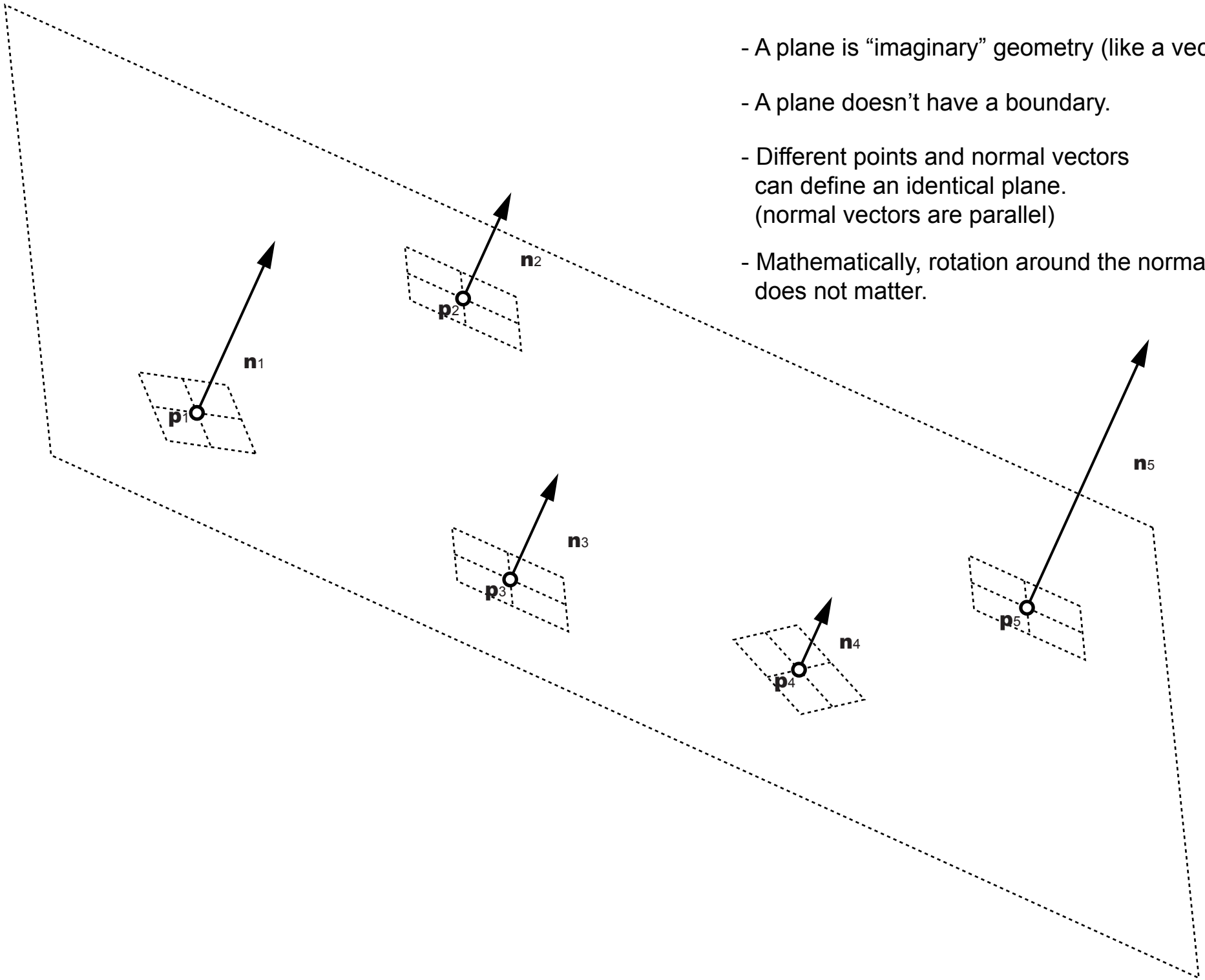
PLANE : DEFINITION IN RHINOSCRIPT

PLANE BY A POINT AND TWO VECTORS



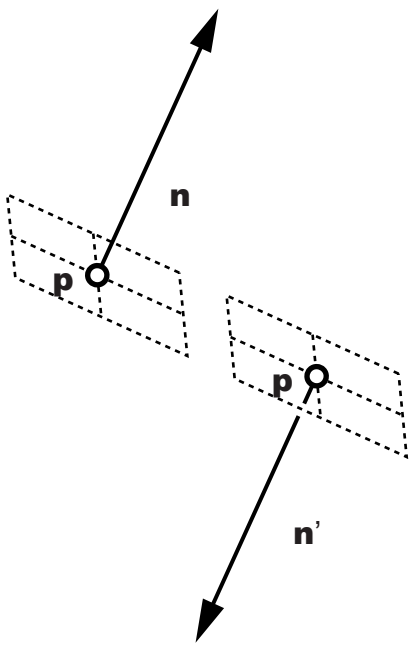
INFINITE PLANE



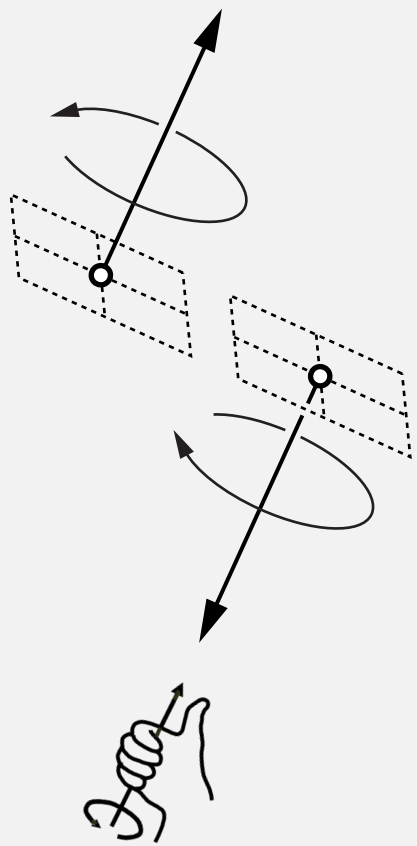
- A plane is “imaginary” geometry (like a vector).
- A plane doesn’t have a boundary.
- Different points and normal vectors can define an identical plane. (normal vectors are parallel)
- Mathematically, rotation around the normal does not matter.

DIRECTION OF PLANE

In some cases direction of a plane (direction of normal vector) matters

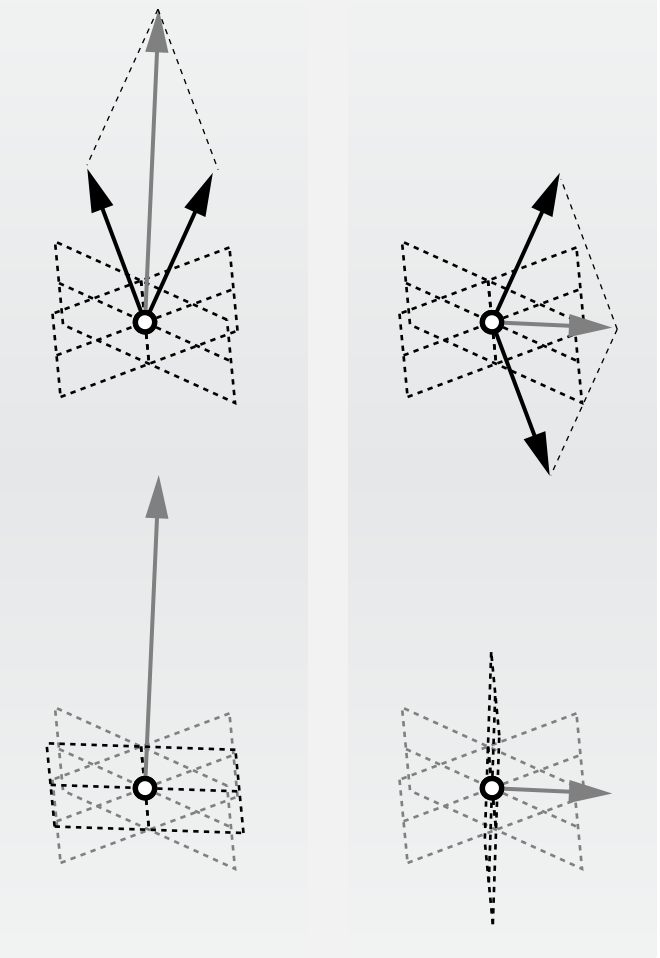


Case 1) Rotation on a plane



Positive direction of rotation on a plane is defined by right-hand thumb rule.

Case 2) Bisector plane

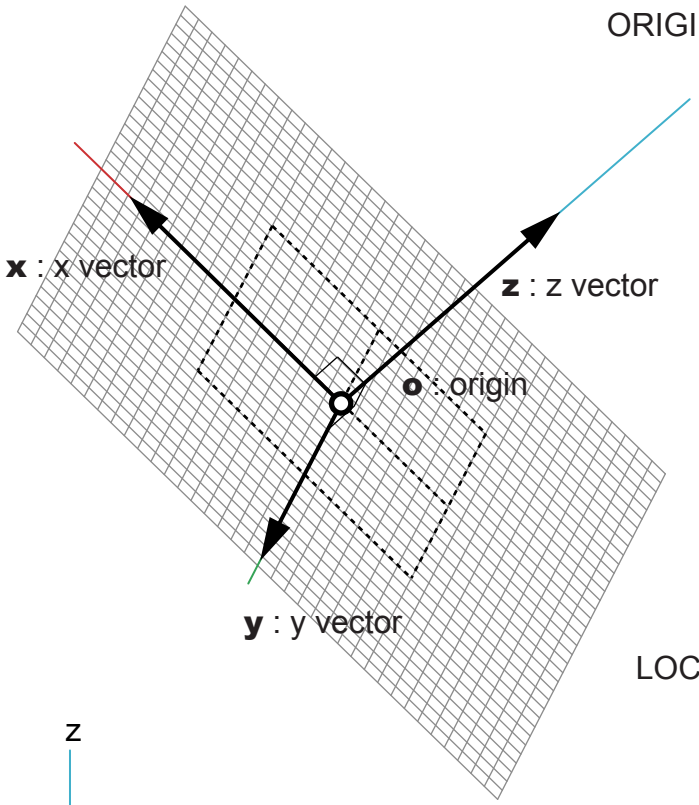


Bisector plane is calculated by bisector vector of normal vectors.

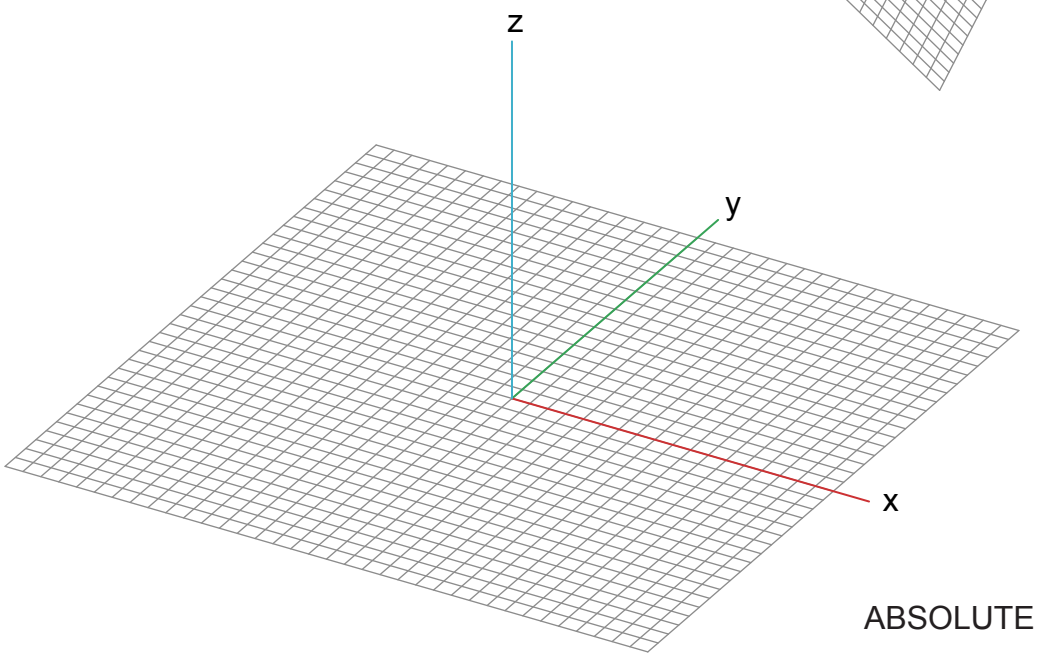
COORDINATE SYSTEM (AXIS SYSTEM)

Some platforms use coordinates system to define local coordinates

COORDINATES SYSTEM BY ORIGIN AND XYZ VECTOR

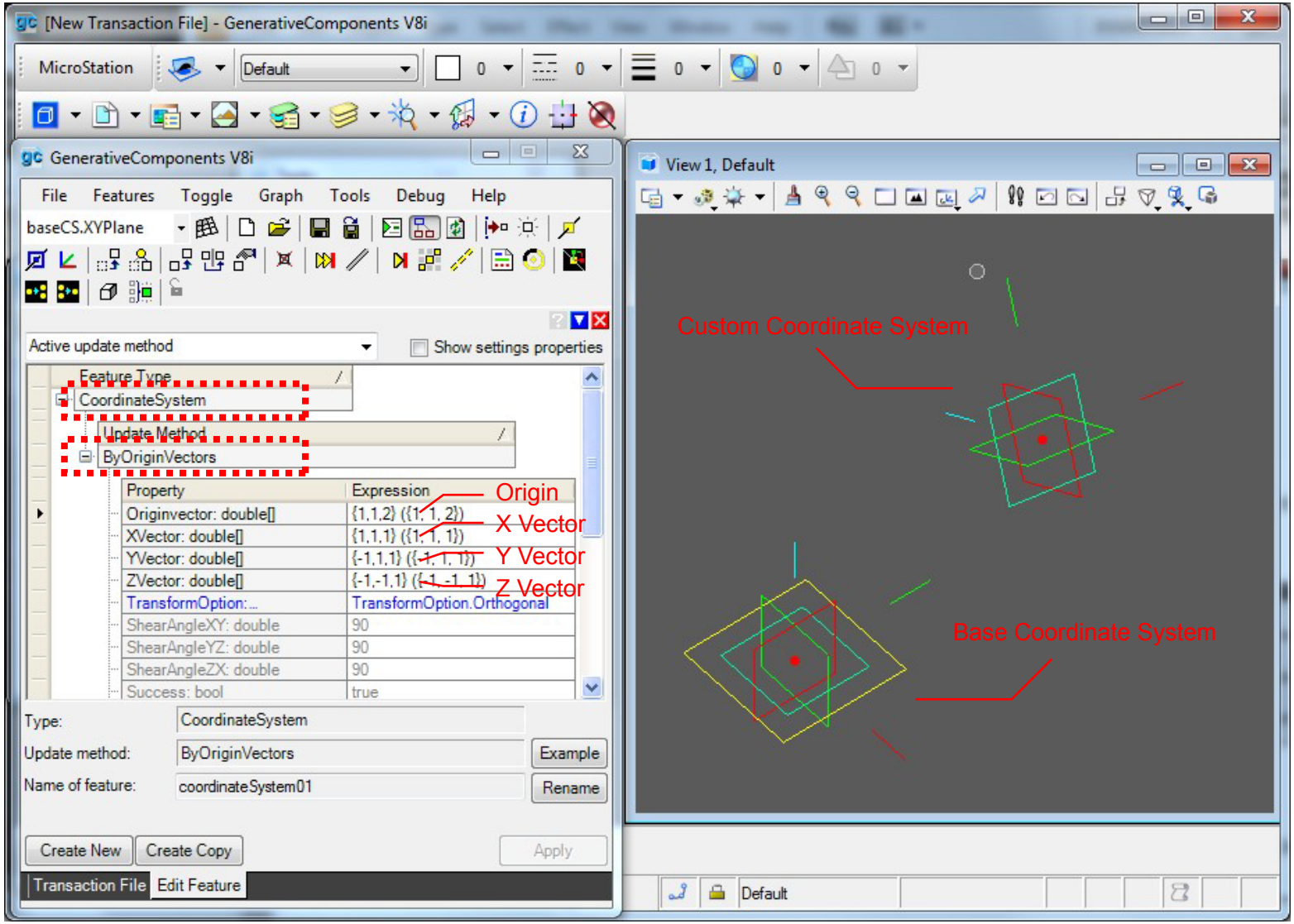


LOCAL COORDINATES

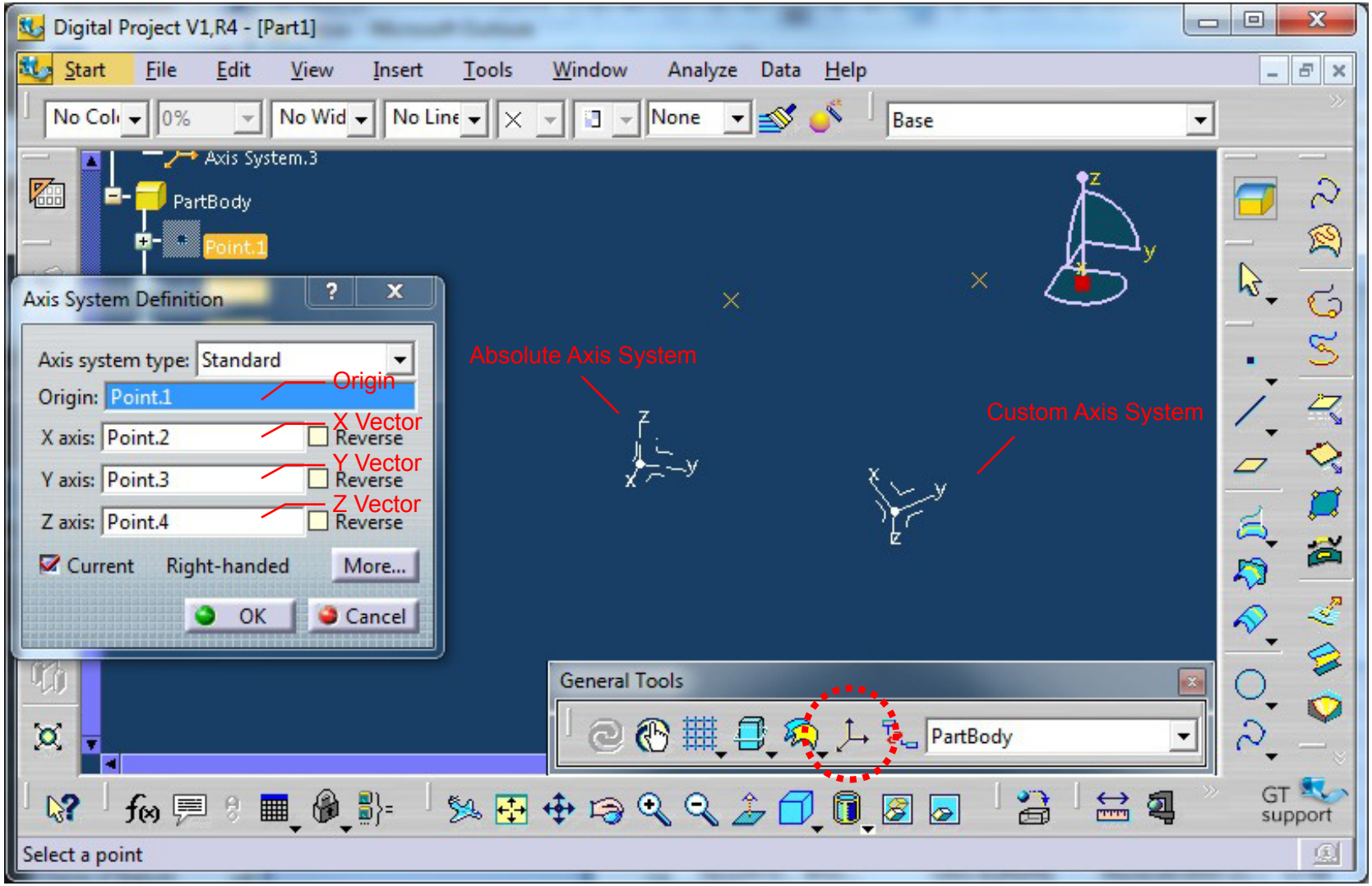


ABSOLUTE (WORLD/GLOBAL) COORDINATES

COORDINATE SYSTEM IN GENERATIVE COMPONENTS

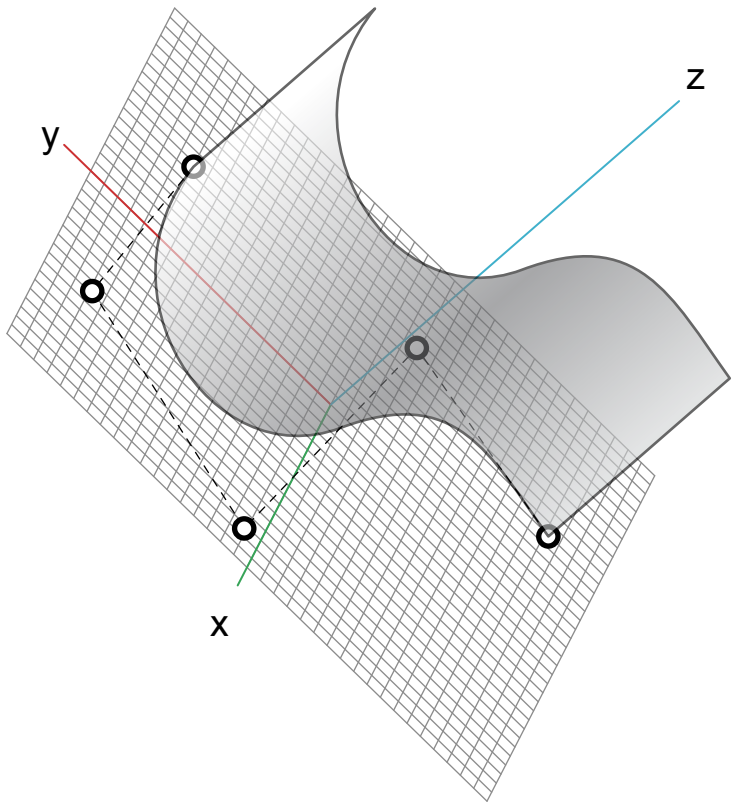


COORDINATE SYSTEM IN DIGITAL PROJECT

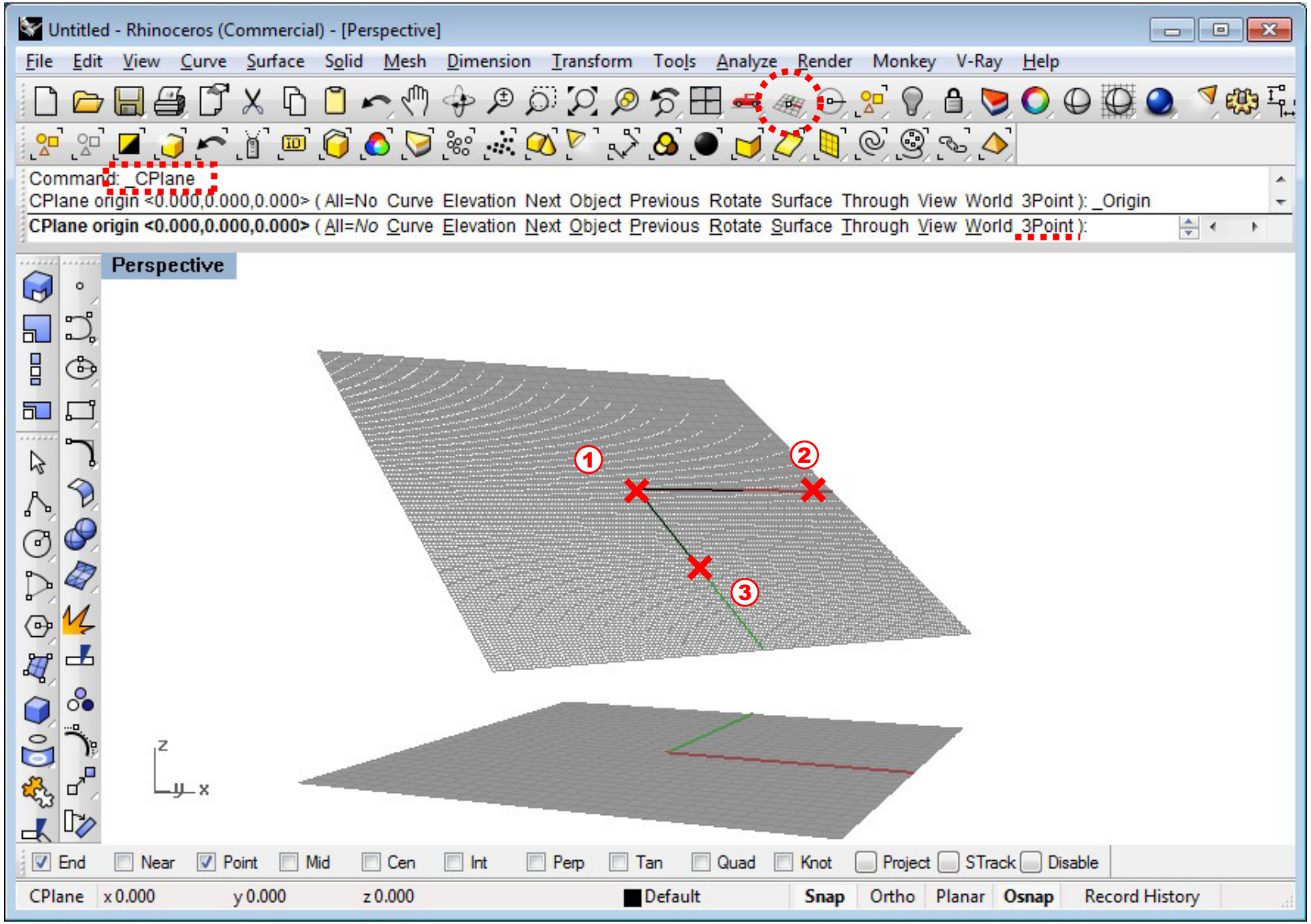


CONSTRUCTION PLANE / AUXILIARY COORDINATE SYSTEM

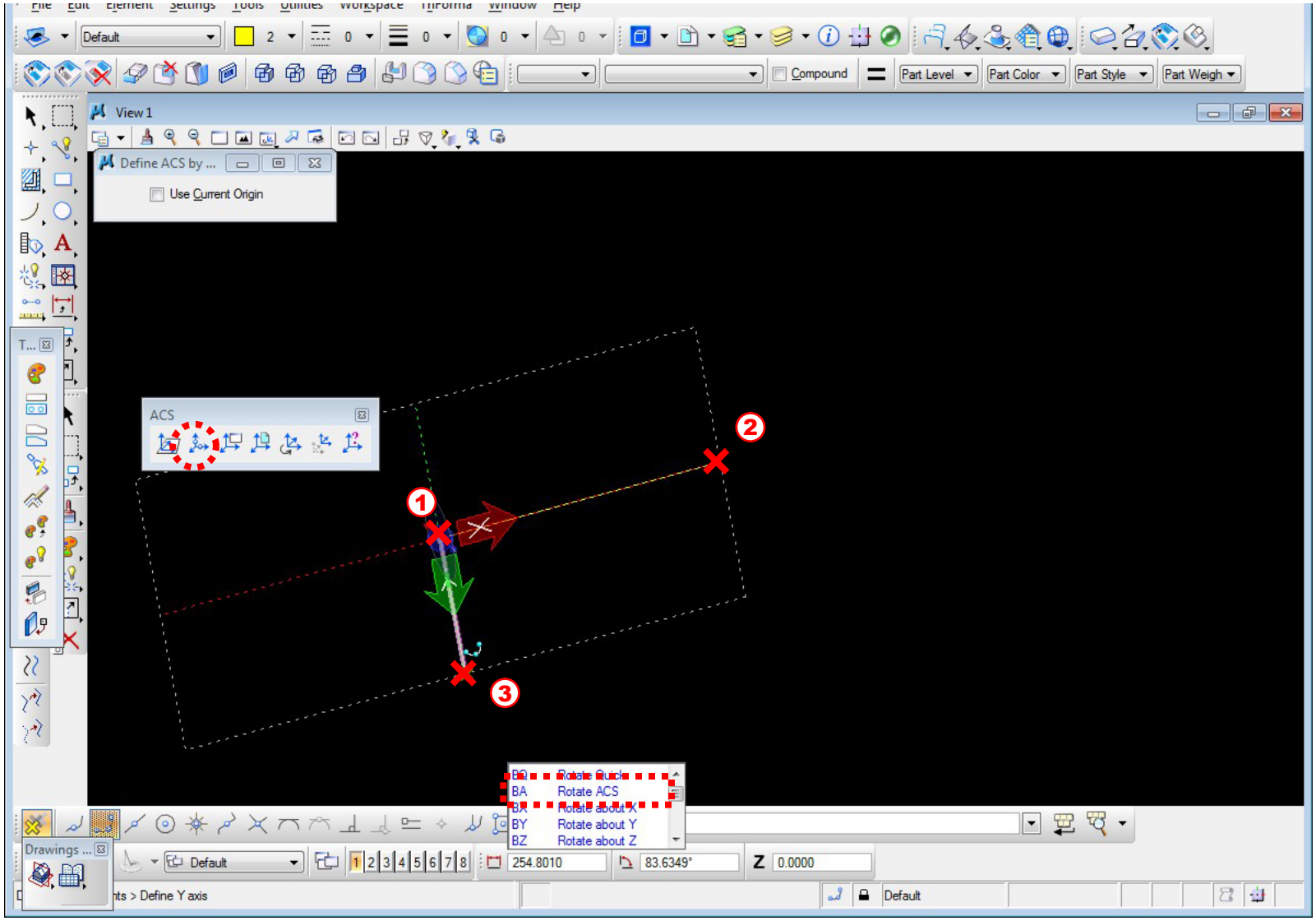
Some platforms provide construction plane (auxiliary coordinate system) to assist 2D based manual modeling.



CONSTRUCTION PLANE IN RHINOCEROS



CONSTRUCTION PLANE (ACS) IN MICROSTATION

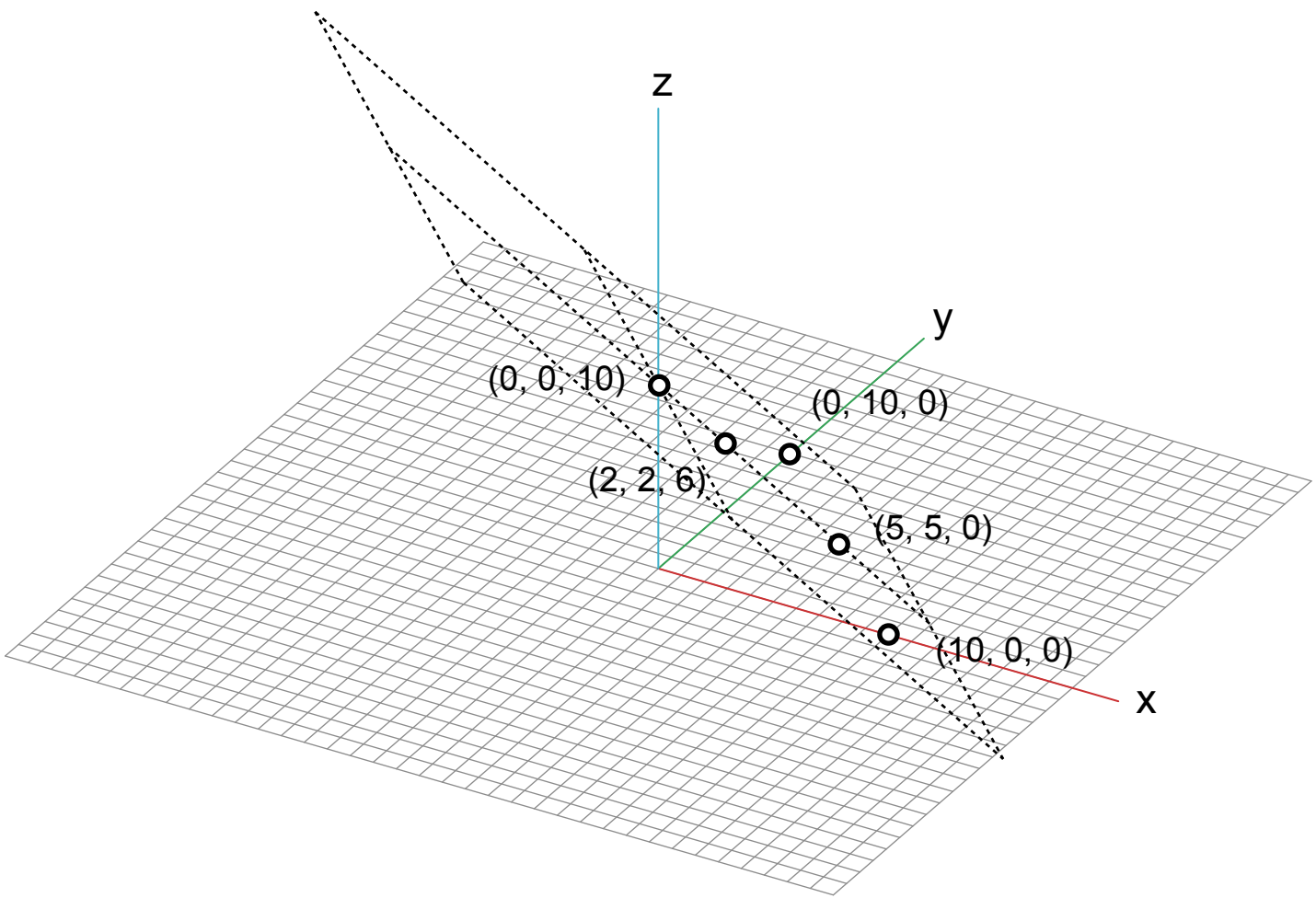


EQUATION OF PLANE

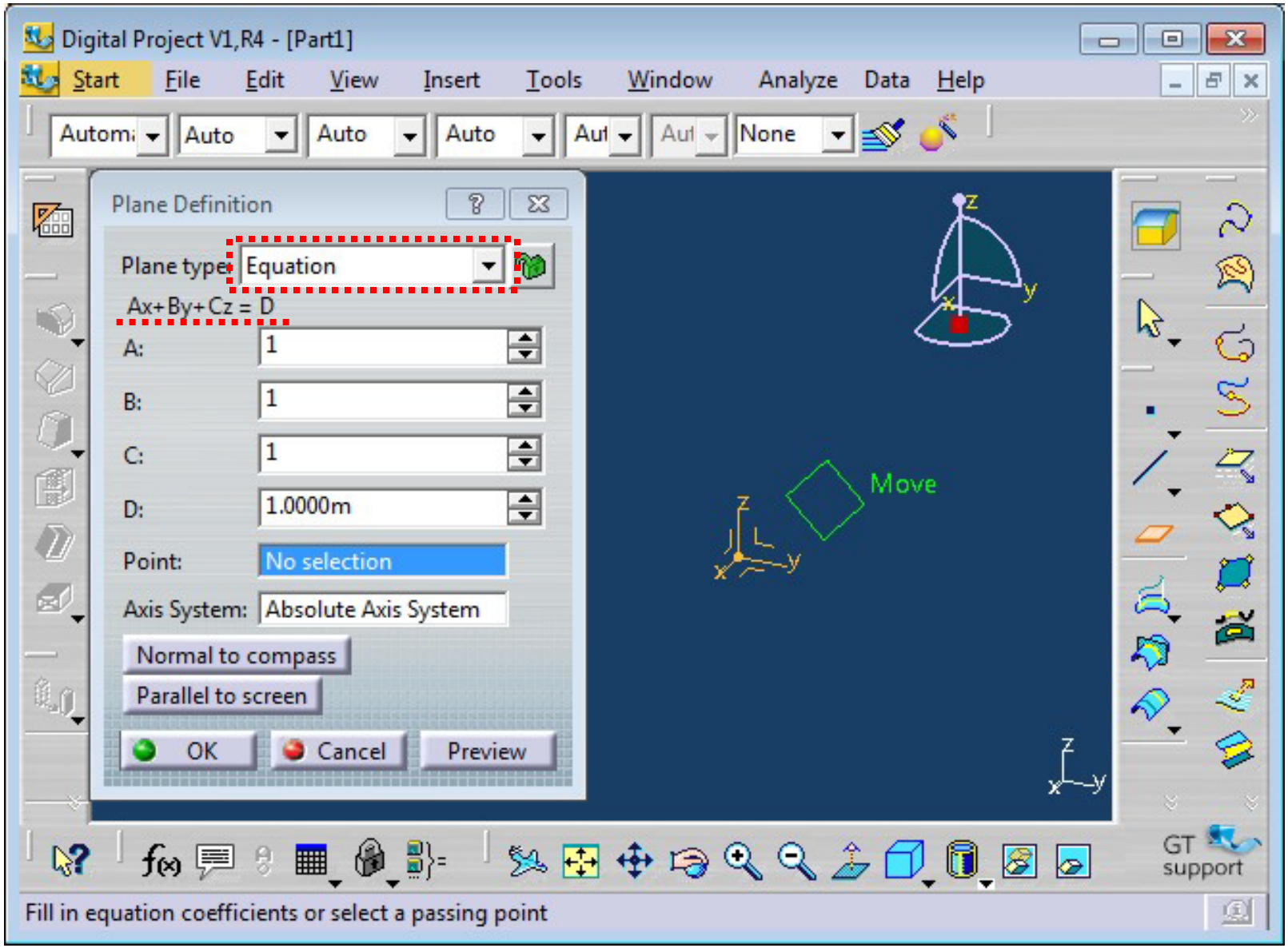
Polynomial Equation

$$a\,x + b\,y + c\,z + d = 0$$

ex) $x + y + z - 10 = 0$



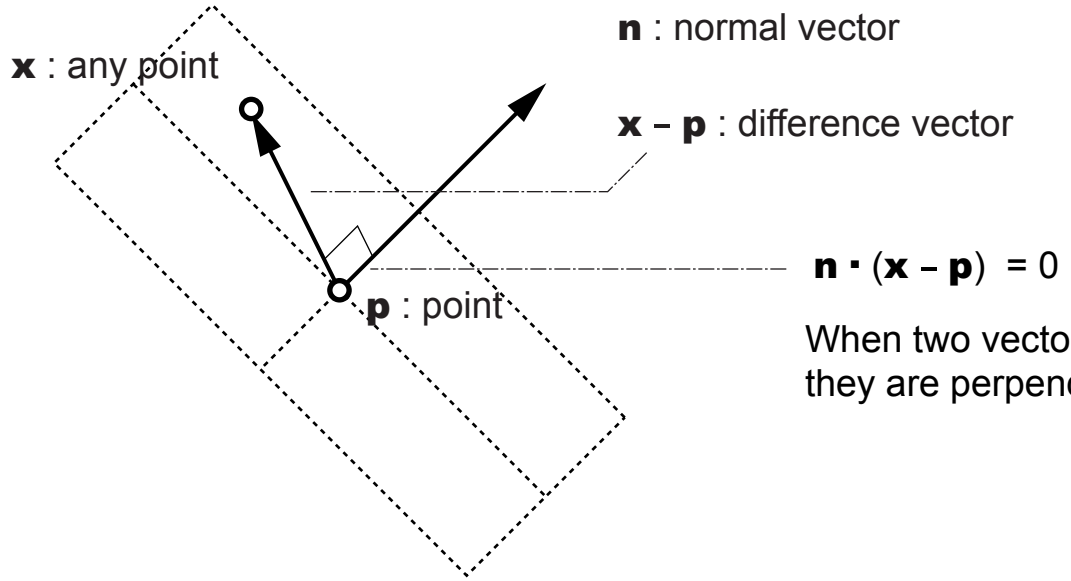
EQUATION OF PLANE



EQUATION OF PLANE

Vector Equation

$$\mathbf{n} \cdot (\mathbf{x} - \mathbf{p}) = 0$$



$\mathbf{n} \cdot (\mathbf{x} - \mathbf{p}) = 0$

When two vectors' dot product is zero, they are perpendicular.

EQUATION OF PLANE

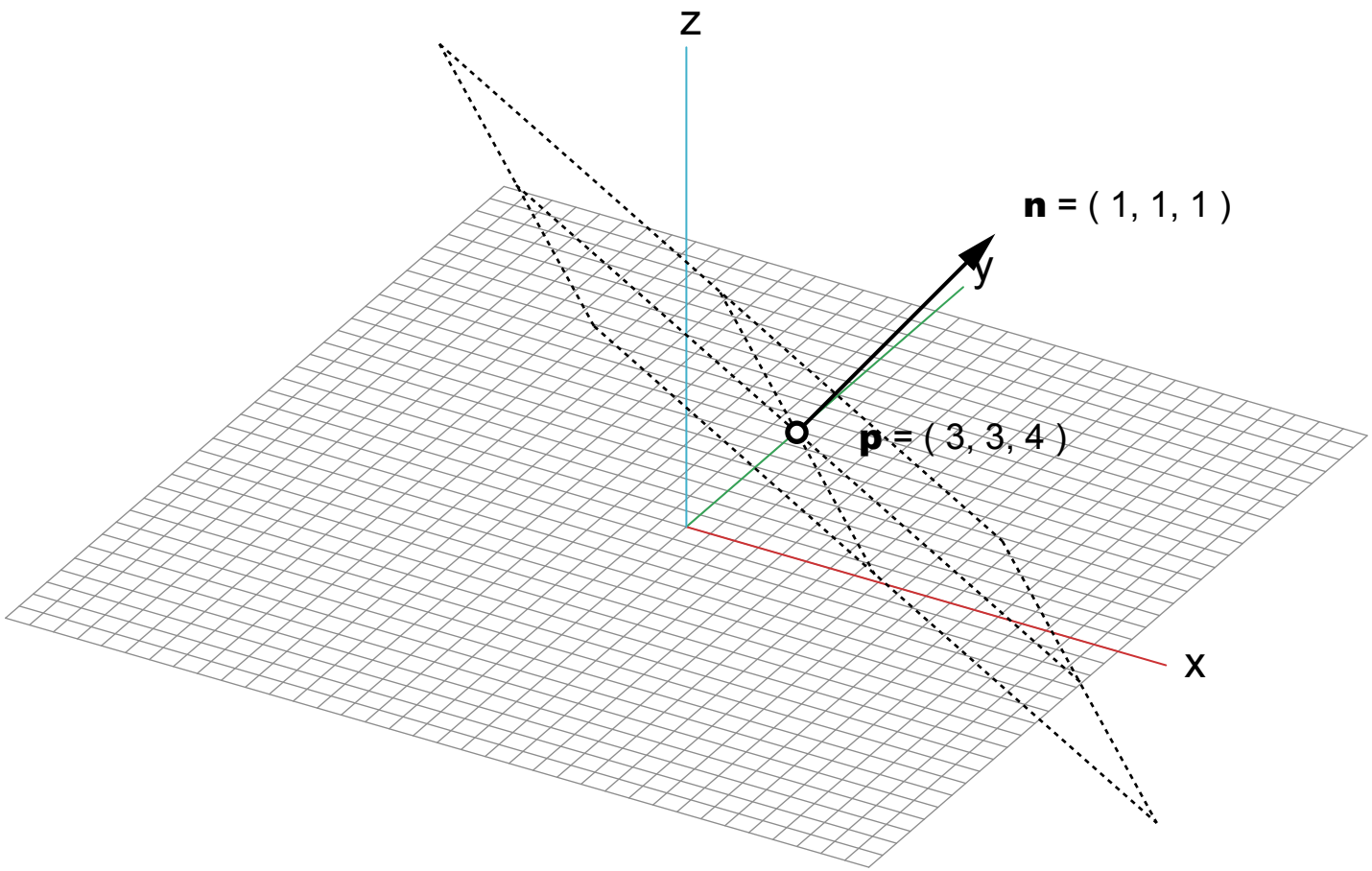
Vector Equation

$$\mathbf{n} \cdot (\mathbf{x} - \mathbf{p}) = 0$$

ex)

$\mathbf{n} = (1, 1, 1)$

$\mathbf{p} = (3, 3, 4)$



EQUATION OF PLANE

Relationship of Polynomial and Vector Equation

ex) $\mathbf{n} = (1, 1, 1)$ $\mathbf{p} = (3, 3, 4)$

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cdot \left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix} \right) = 0$$

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} x - 3 \\ y - 3 \\ z - 4 \end{pmatrix} = 0$$

$$1 \times (x - 3) + 1 \times (y - 3) + 1 \times (z - 4) = 0$$

$$x - 3 + y - 3 + z - 4 = 0$$

$$\underline{x + y + z - 10 = 0}$$

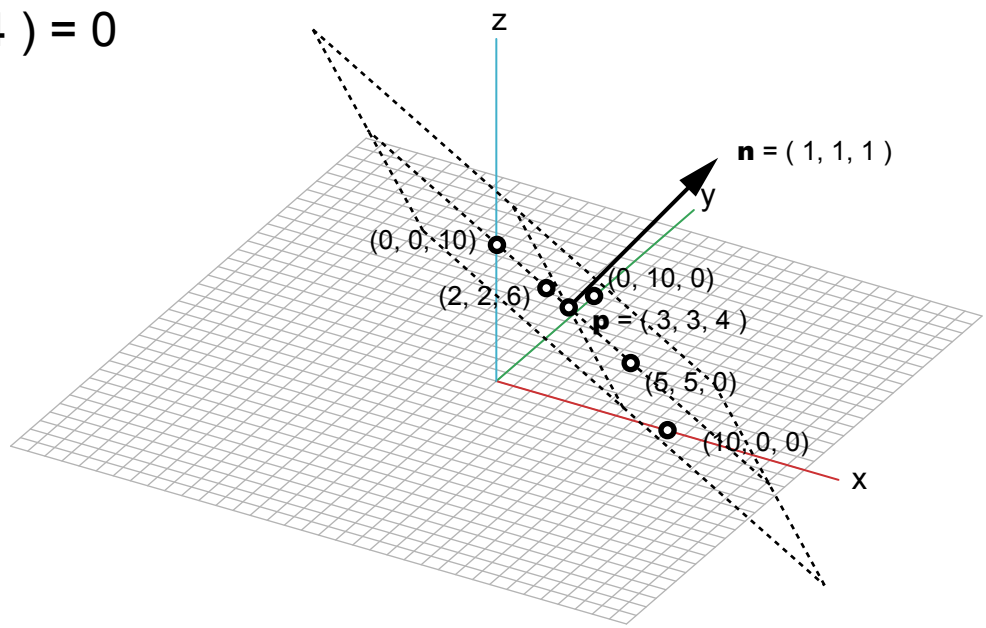
PLANE EQUATION

$$\mathbf{n} \cdot (\mathbf{x} - \mathbf{p}) = 0$$

DOT PRODUCT

$$\mathbf{a} = (a_x, a_y, a_z) \quad \mathbf{b} = (b_x, b_y, b_z)$$

$$\mathbf{a} \cdot \mathbf{b} = a_x \times b_x + a_y \times b_y + a_z \times b_z$$



EQUATION OF PLANE

Relationship of Polynomial and Vector Equation

$\mathbf{n} = (n_x, n_y, n_z)$ $\quad \mathbf{p} = (p_x, p_y, p_z)$

$$\begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix} \cdot \left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} \right) = 0$$

$$\begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix} \cdot \begin{pmatrix} x - p_x \\ y - p_y \\ z - p_z \end{pmatrix} = 0$$

$$n_x (x - p_x) + n_y (y - p_y) + n_z (z - p_z) = 0$$

$$\underline{n_x}\, x + \underline{n_x}\, y + \underline{n_z}\, z \ \underline{- n_x p_x - n_y p_y - n_z p_z} = 0$$

$$a\, x + b\, y + c\, z + d = 0$$

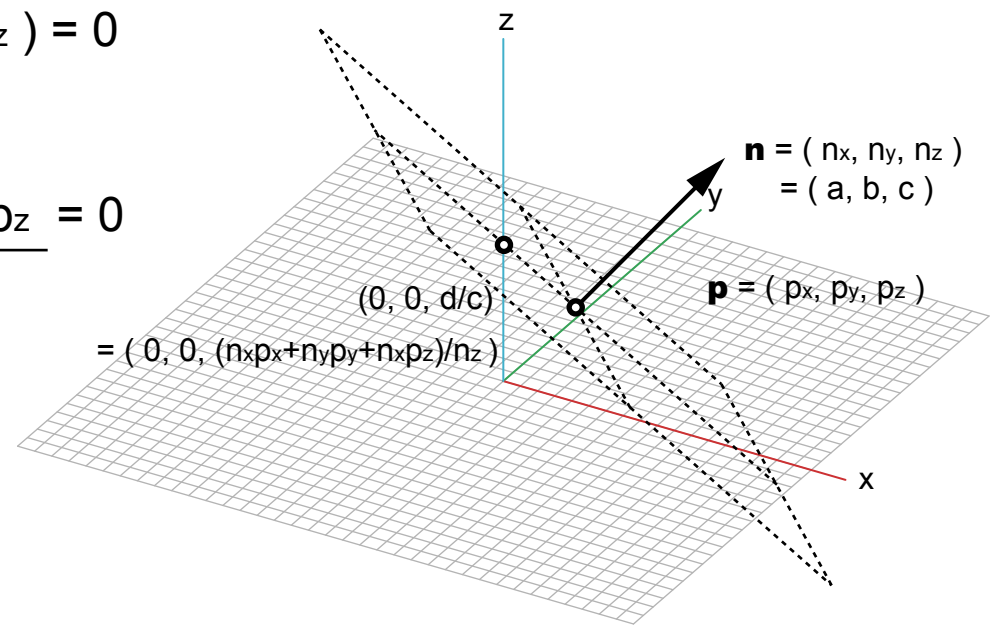
PLANE EQUATION

$$\mathbf{n} \cdot (\mathbf{x} - \mathbf{p}) = 0$$

DOT PRODUCT

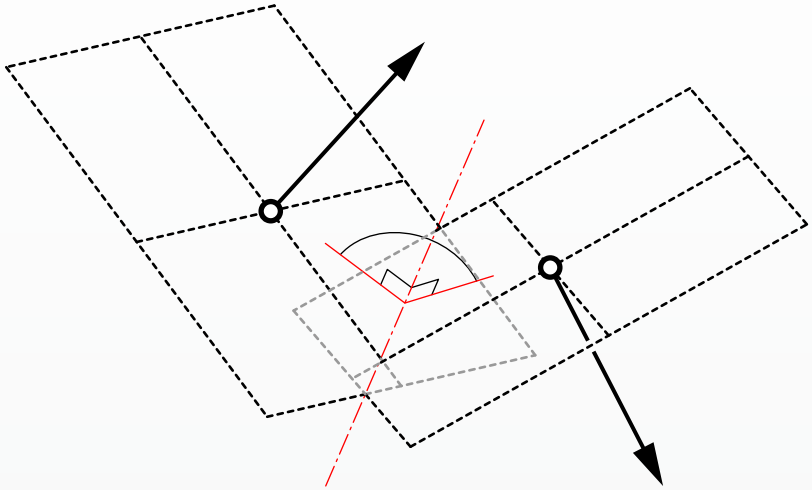
$\mathbf{a} = (a_x, a_y, a_z)$ $\quad \mathbf{b} = (b_x, b_y, b_z)$

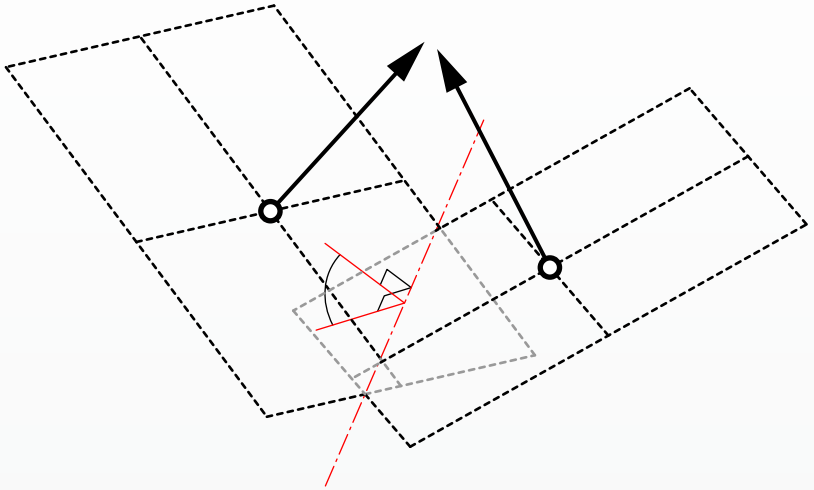
$$\mathbf{a} \cdot \mathbf{b} = a_x \times b_x + a_y \times b_y + a_z \times b_z$$



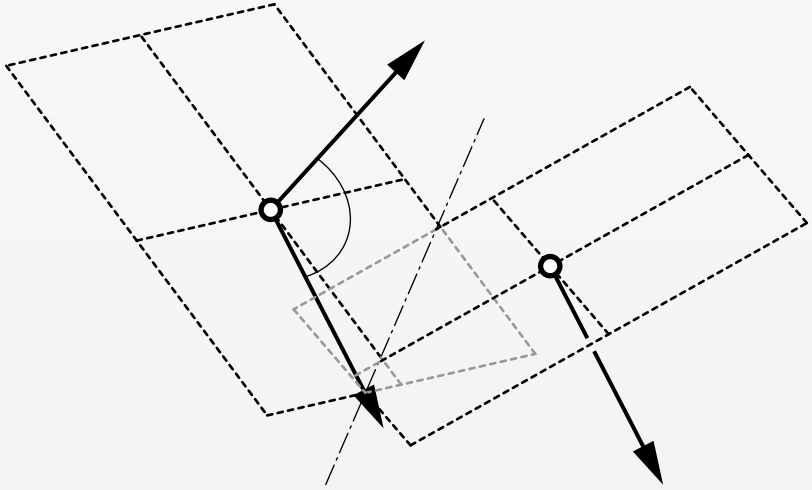
ANGLE OF PLANES

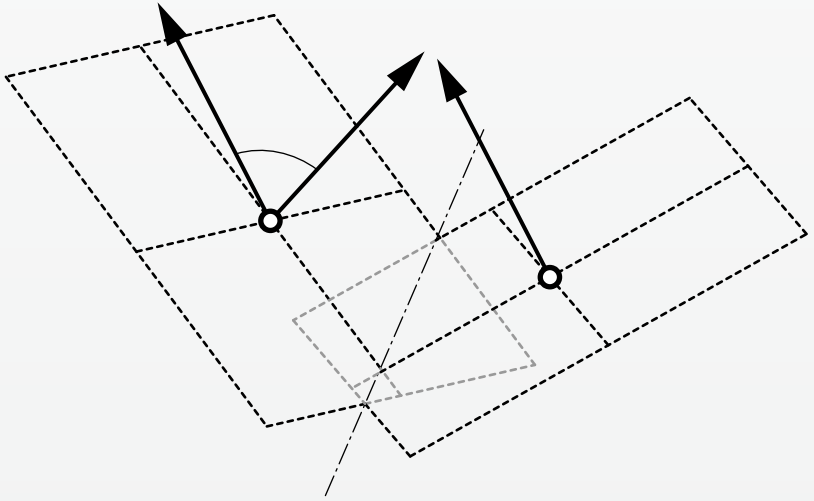
ANGLE BY PERPENDICULAR LINES TO INTERSECTION





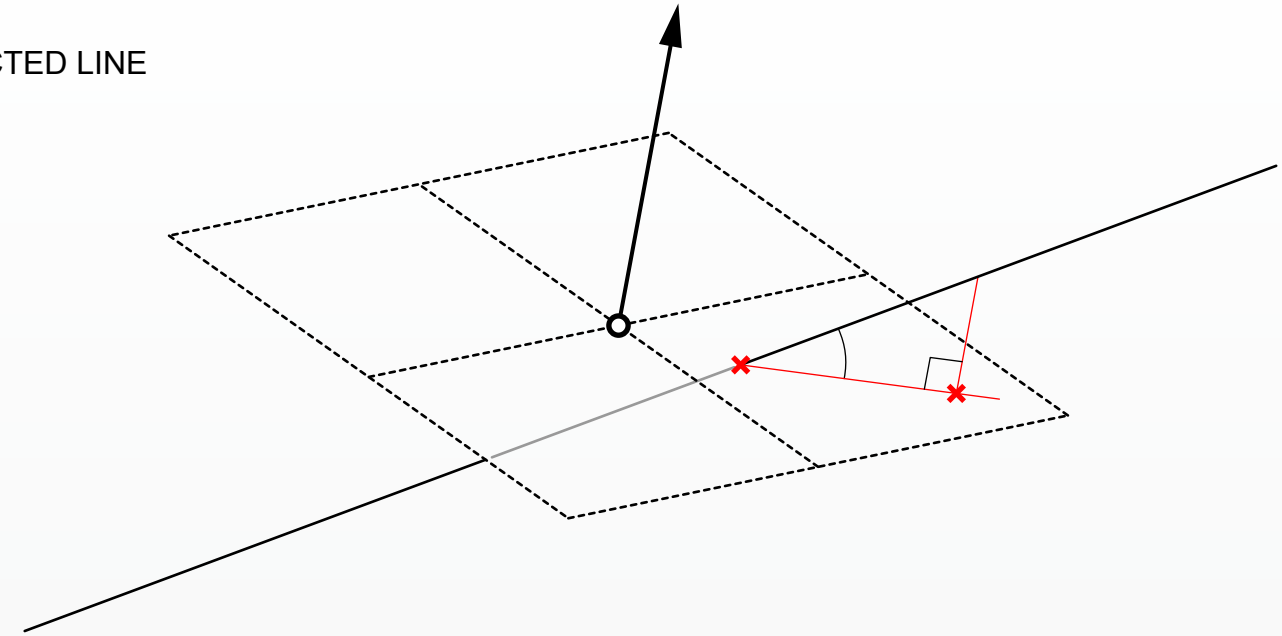
ANGLE BY NORMAL VECTORS



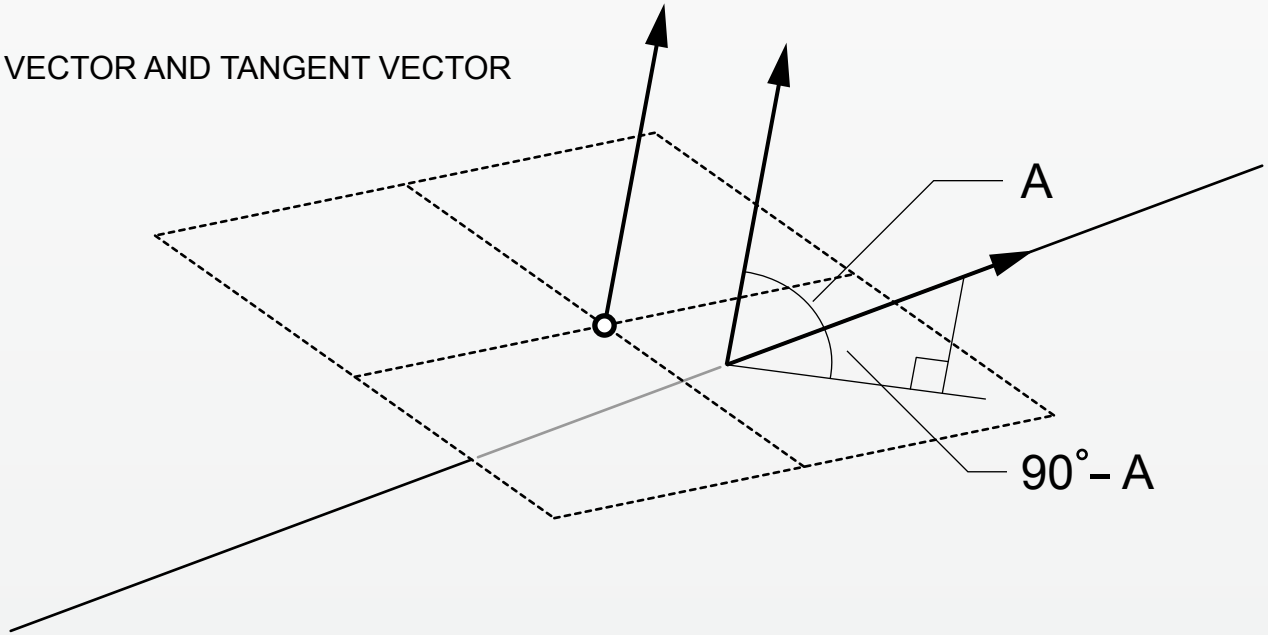


ANGLE OF PLANE AND LINE

ANGLE BY PROJECTED LINE

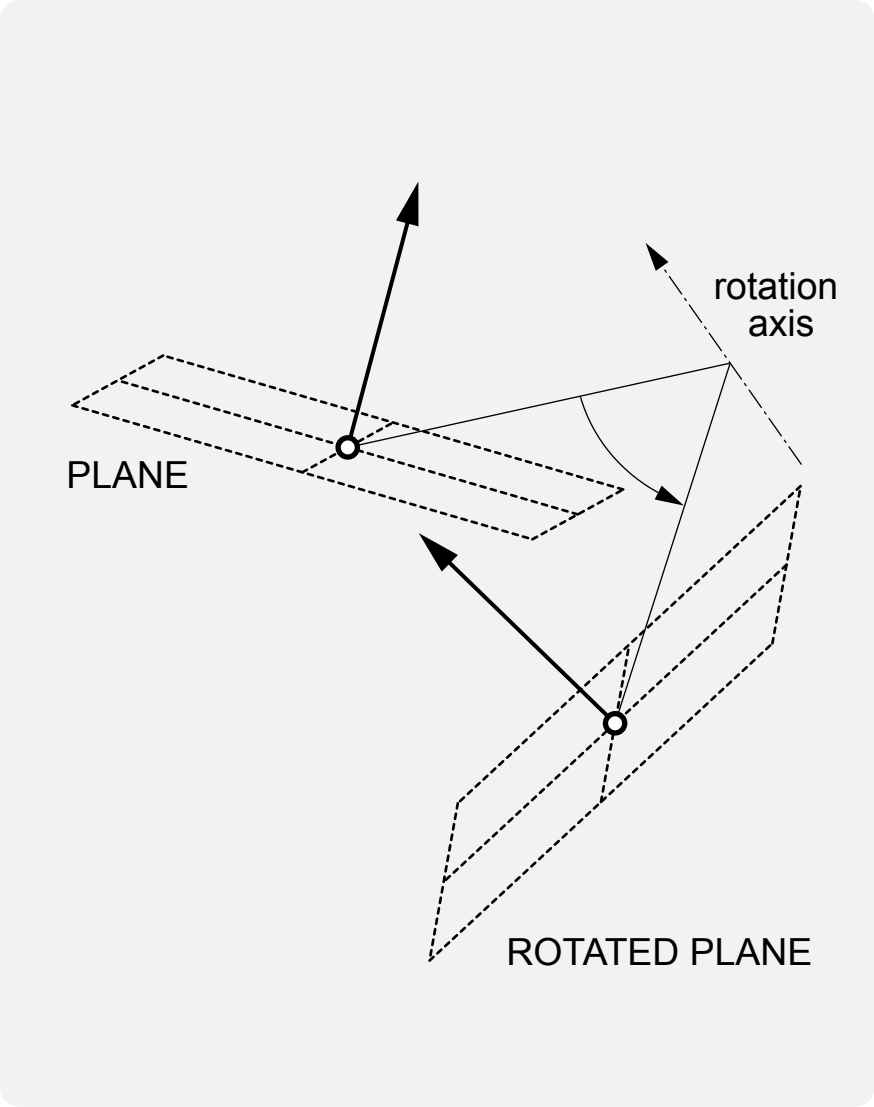


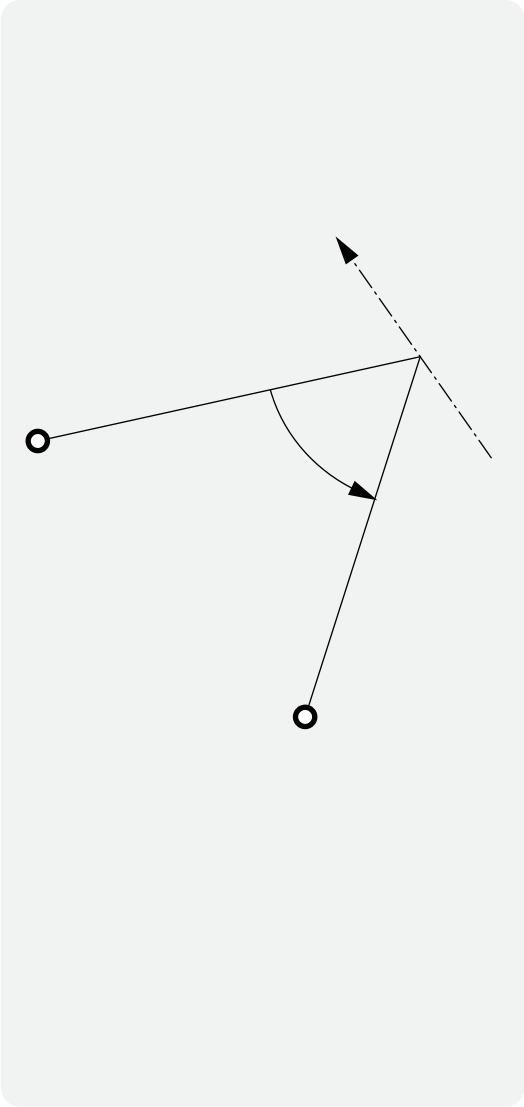
ANGLE BY NORMAL VECTOR AND TANGENT VECTOR

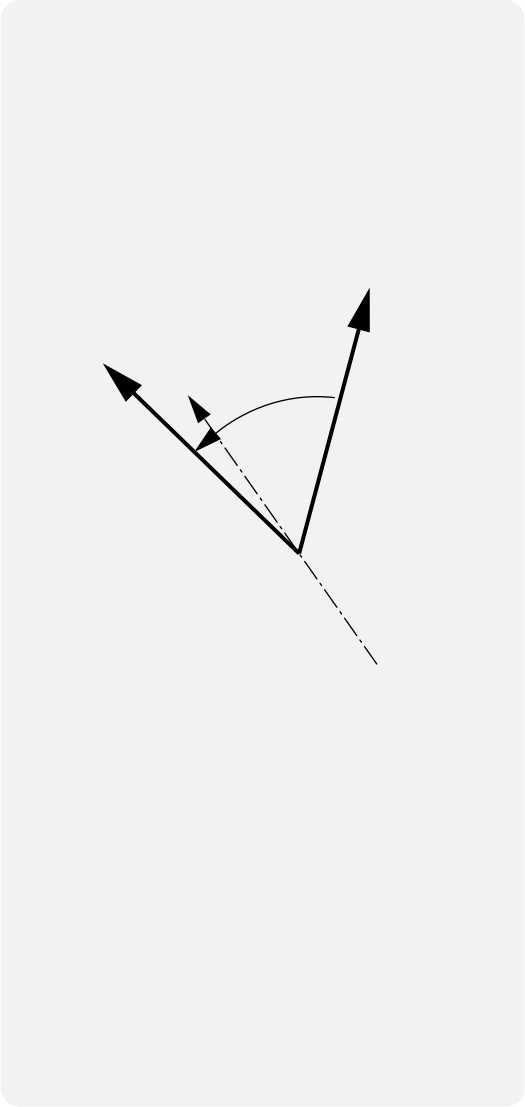


ROTATION OF PLANE

ROTATION OF PLANE = ROTATION OF POINT + ROTATION OF NORMAL VECTOR

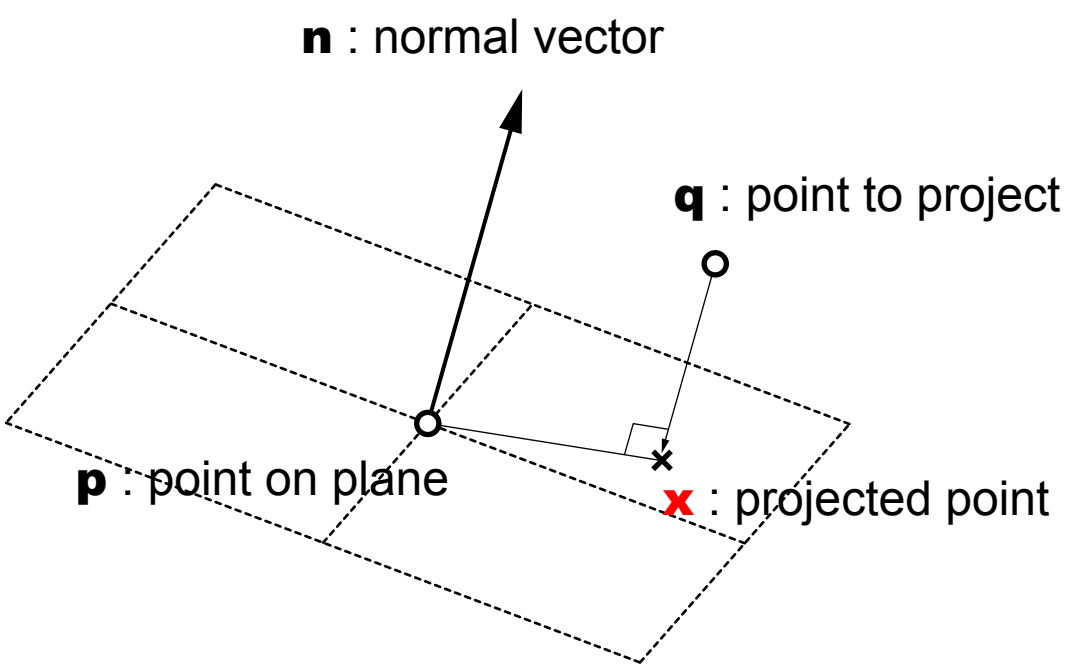






PROJECTION ONTO PLANE

PROJECTION IN NORMAL DIRECTION

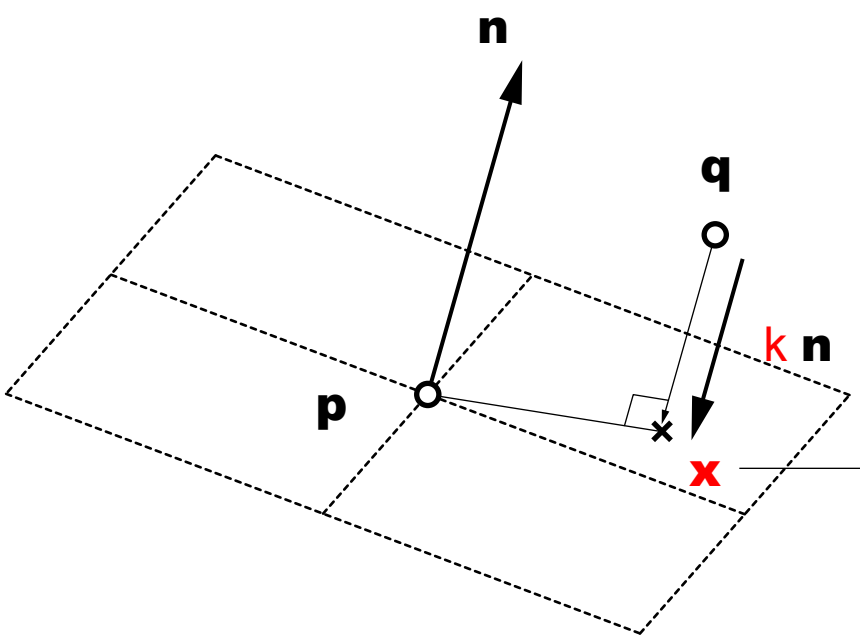


PLANE: $\mathbf{n} \cdot (\mathbf{x} - \mathbf{p}) = 0$

$\mathbf{x} = ?$

PROJECTION ONTO PLANE

PROJECTION IN NORMAL DIRECTION



PLANE: $\mathbf{n} \cdot (\mathbf{x} - \mathbf{p}) = 0$

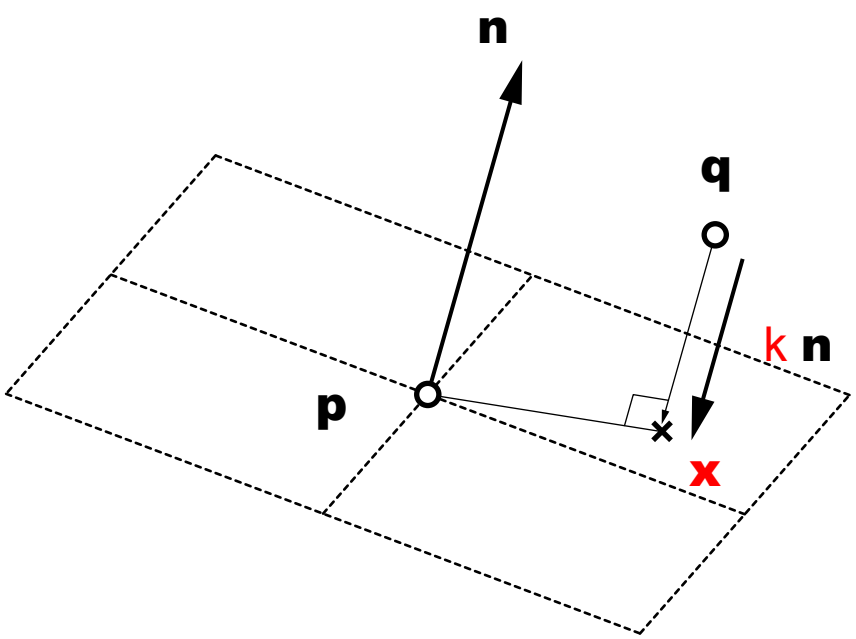
$\mathbf{x} = ?$

$\mathbf{x} = \mathbf{q} + k\,\mathbf{n}$

$k = ?$

PROJECTION ONTO PLANE

PROJECTION IN NORMAL DIRECTION



PLANE: $\mathbf{n} \cdot (\mathbf{x} - \mathbf{p}) = 0$

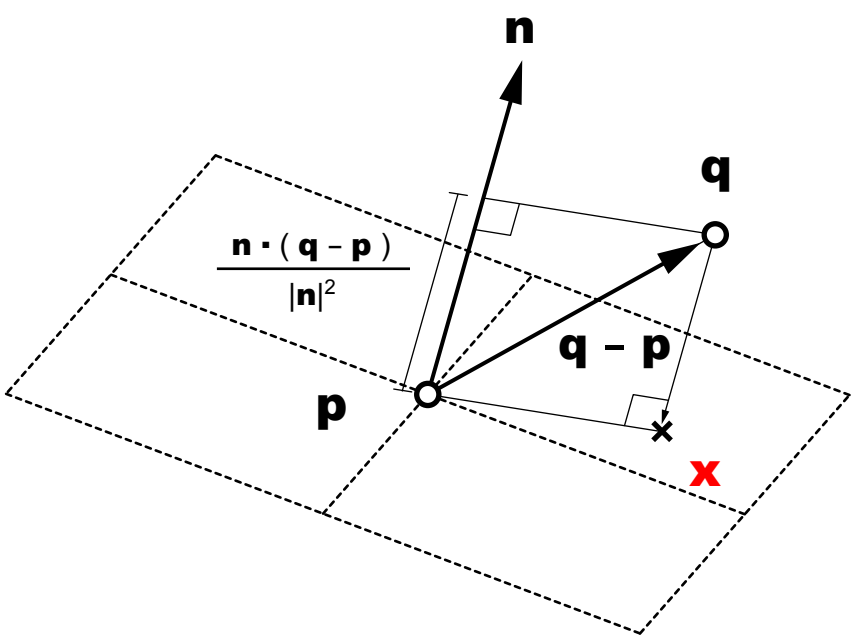
$\mathbf{x} = ?$

$\mathbf{x} = \mathbf{q} + k\,\mathbf{n}$

$\mathbf{n} \cdot ((\mathbf{q} + k\,\mathbf{n}) - \mathbf{p}) = 0$

PROJECTION ONTO PLANE

PROJECTION IN NORMAL DIRECTION



PLANE: $\mathbf{n} \cdot (\mathbf{x} - \mathbf{p}) = 0$

$\mathbf{x} = ?$

$\mathbf{x} = \mathbf{q} + k\,\mathbf{n}$

$\mathbf{n} \cdot ((\mathbf{q} + k\,\mathbf{n}) - \mathbf{p}) = 0$

$\mathbf{n} \cdot \mathbf{q} + k\,\mathbf{n} \cdot \mathbf{n} - \mathbf{n} \cdot \mathbf{p} = 0$

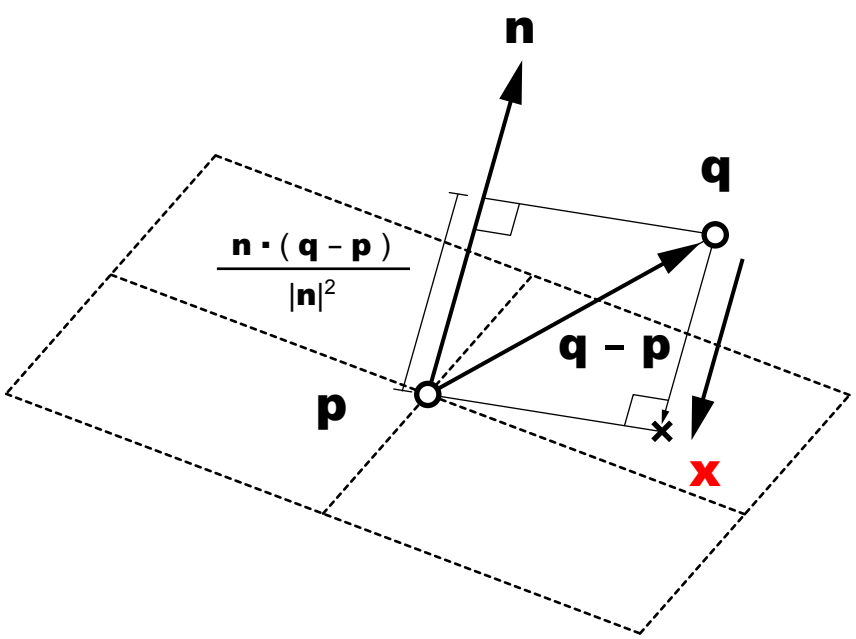
$k\,\mathbf{n} \cdot \mathbf{n} = \mathbf{n} \cdot \mathbf{p} - \mathbf{n} \cdot \mathbf{q}$

$k\,|\mathbf{n}|^2 = \mathbf{n} \cdot \mathbf{p} - \mathbf{n} \cdot \mathbf{q}$

$$k = \frac{\mathbf{n} \cdot (\mathbf{p} - \mathbf{q})}{|\mathbf{n}|^2}$$

PROJECTION ONTO PLANE

PROJECTION IN NORMAL DIRECTION



PLANE: $\mathbf{n} \cdot (\mathbf{x} - \mathbf{p}) = 0$

$\mathbf{x} = ?$

$\mathbf{x} = \mathbf{q} + k\,\mathbf{n}$

$\mathbf{n} \cdot ((\mathbf{q} + k\,\mathbf{n}) - \mathbf{p}) = 0$

$\mathbf{n} \cdot \mathbf{q} + k\,\mathbf{n} \cdot \mathbf{n} - \mathbf{n} \cdot \mathbf{p} = 0$

$k\,\mathbf{n} \cdot \mathbf{n} = \mathbf{n} \cdot \mathbf{p} - \mathbf{n} \cdot \mathbf{q}$

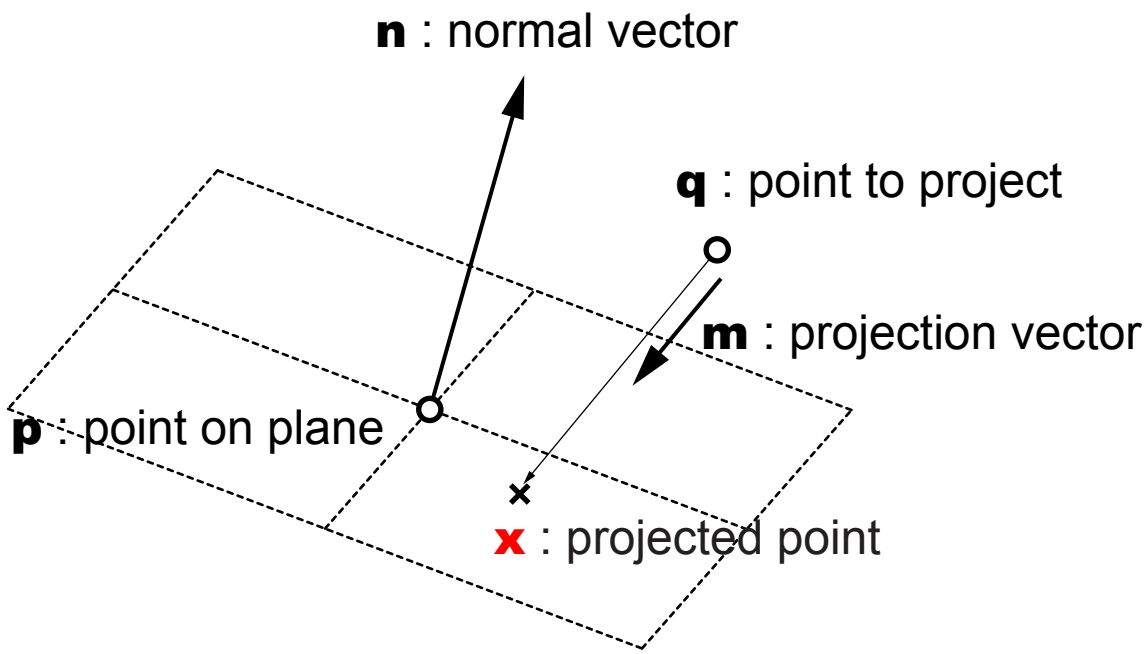
$k\,|\mathbf{n}|^2 = \mathbf{n} \cdot \mathbf{p} - \mathbf{n} \cdot \mathbf{q}$

$$k = \frac{\mathbf{n} \cdot (\mathbf{p} - \mathbf{q})}{|\mathbf{n}|^2}$$

$$\mathbf{x} = \mathbf{q} + \frac{\mathbf{n} \cdot (\mathbf{p} - \mathbf{q})}{|\mathbf{n}|^2}\,\mathbf{n}$$

PROJECTION ONTO PLANE

PROJECTION IN ARBITRARY DIRECTION

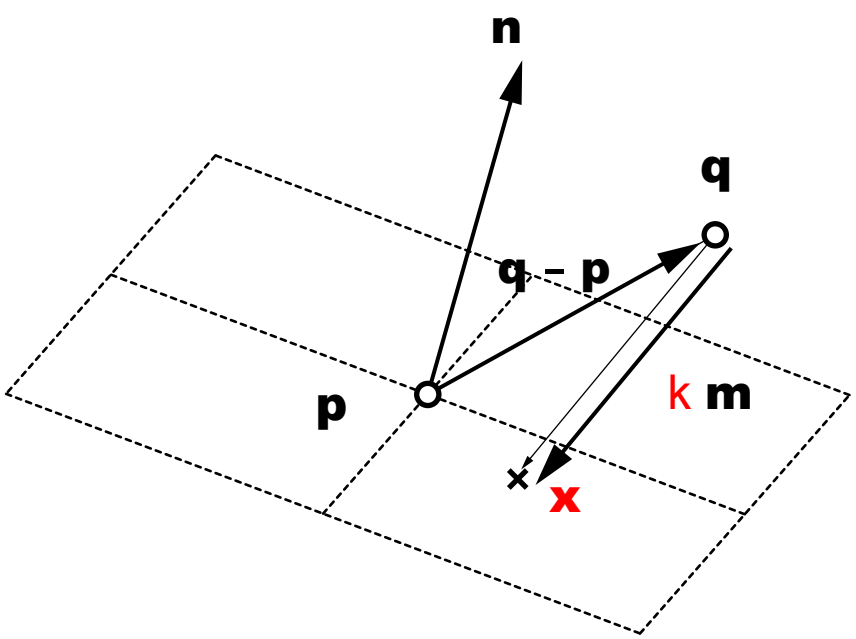


PLANE: $\mathbf{n} \cdot (\mathbf{x} - \mathbf{p}) = 0$

$\mathbf{x} = ?$

PROJECTION ONTO PLANE

PROJECTION IN ARBITRARY DIRECTION



PLANE: $\mathbf{n} \cdot (\mathbf{x} - \mathbf{p}) = 0$

$\mathbf{x} = ?$

$\mathbf{x} = \mathbf{q} + k\,\mathbf{m}$

$\mathbf{n} \cdot ((\mathbf{q} + k\,\mathbf{m}) - \mathbf{p}) = 0$

$\mathbf{n} \cdot \mathbf{q} + k\,\mathbf{n} \cdot \mathbf{m} - \mathbf{n} \cdot \mathbf{p} = 0$

$k\,\mathbf{n} \cdot \mathbf{m} = \mathbf{n} \cdot \mathbf{p} - \mathbf{n} \cdot \mathbf{q}$

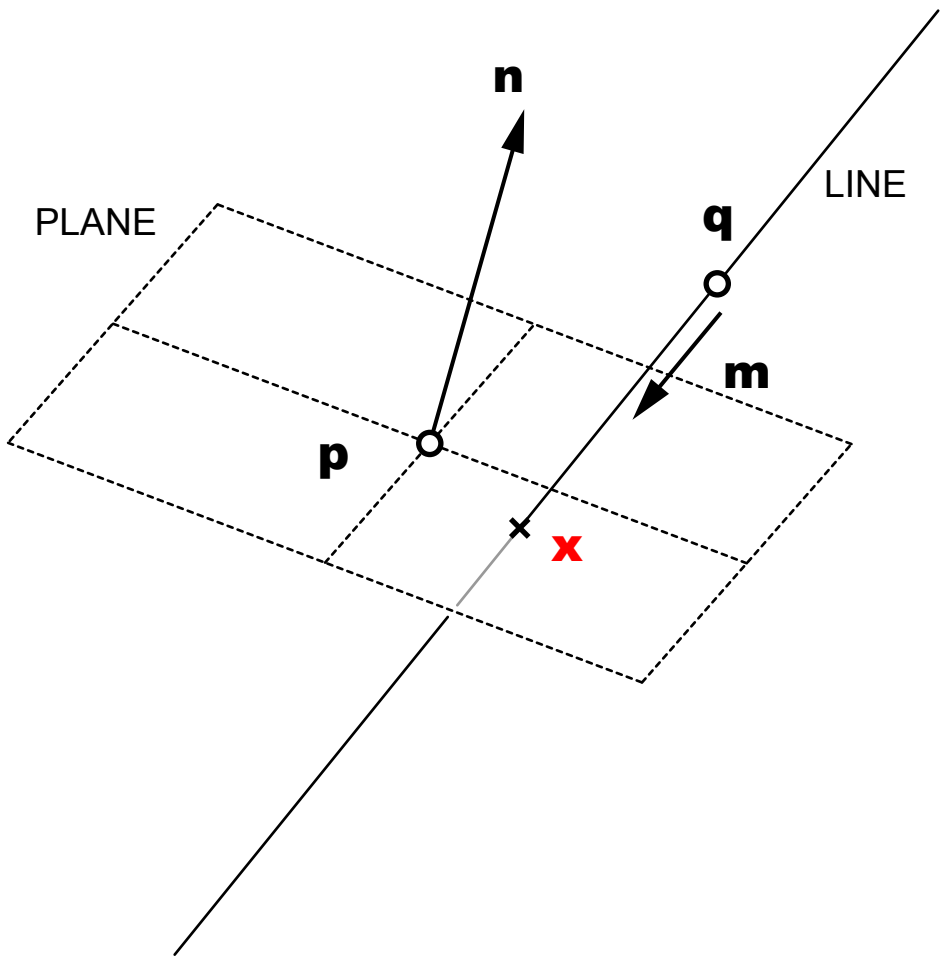
$k\,(\mathbf{n} \cdot \mathbf{m}) = \mathbf{n} \cdot \mathbf{p} - \mathbf{n} \cdot \mathbf{q}$

$$k = \frac{\mathbf{n} \cdot (\mathbf{p} - \mathbf{q})}{\mathbf{n} \cdot \mathbf{m}}$$

$$\mathbf{x} = \mathbf{q} + \frac{\mathbf{n} \cdot (\mathbf{p} - \mathbf{q})}{\mathbf{n} \cdot \mathbf{m}}\,\mathbf{n}$$

INTERSECTION OF PLANE AND LINE

INTERSECTION



PLANE: $\mathbf{n} \cdot (\mathbf{x} - \mathbf{p}) = 0$

$\mathbf{x} = ?$

LINE: $\mathbf{x} = \mathbf{q} + k\,\mathbf{m}$

$$\mathbf{n} \cdot ((\mathbf{q} + k\,\mathbf{m}) - \mathbf{p}) = 0$$

$$\mathbf{n} \cdot \mathbf{q} + k\,\mathbf{n} \cdot \mathbf{m} - \mathbf{n} \cdot \mathbf{p} = 0$$

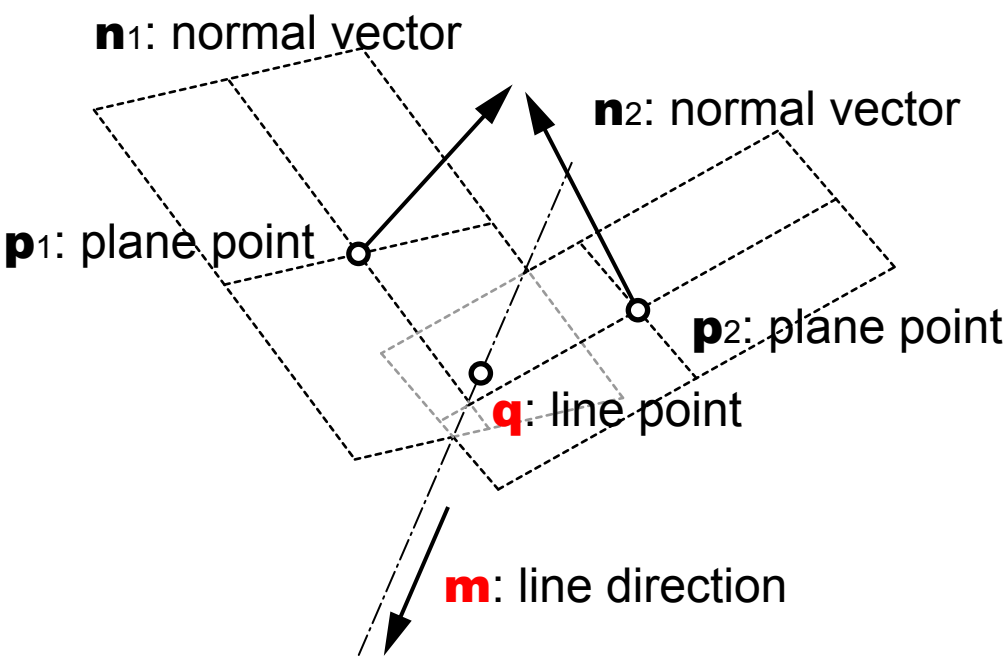
$$k\,\mathbf{n} \cdot \mathbf{m} = \mathbf{n} \cdot \mathbf{p} - \mathbf{n} \cdot \mathbf{q}$$

$$k\,(\mathbf{n} \cdot \mathbf{m}) = \mathbf{n} \cdot \mathbf{p} - \mathbf{n} \cdot \mathbf{q}$$

$$k = \frac{\mathbf{n} \cdot (\mathbf{p} - \mathbf{q})}{\mathbf{n} \cdot \mathbf{m}}$$

$$\mathbf{x} = \mathbf{q} + \frac{\mathbf{n} \cdot (\mathbf{p} - \mathbf{q})}{\mathbf{n} \cdot \mathbf{m}}\,\mathbf{n}$$

INTERSECTION OF PLANES



PLANE 1: $\mathbf{n}_1 \cdot (\mathbf{x} - \mathbf{p}_1) = 0$

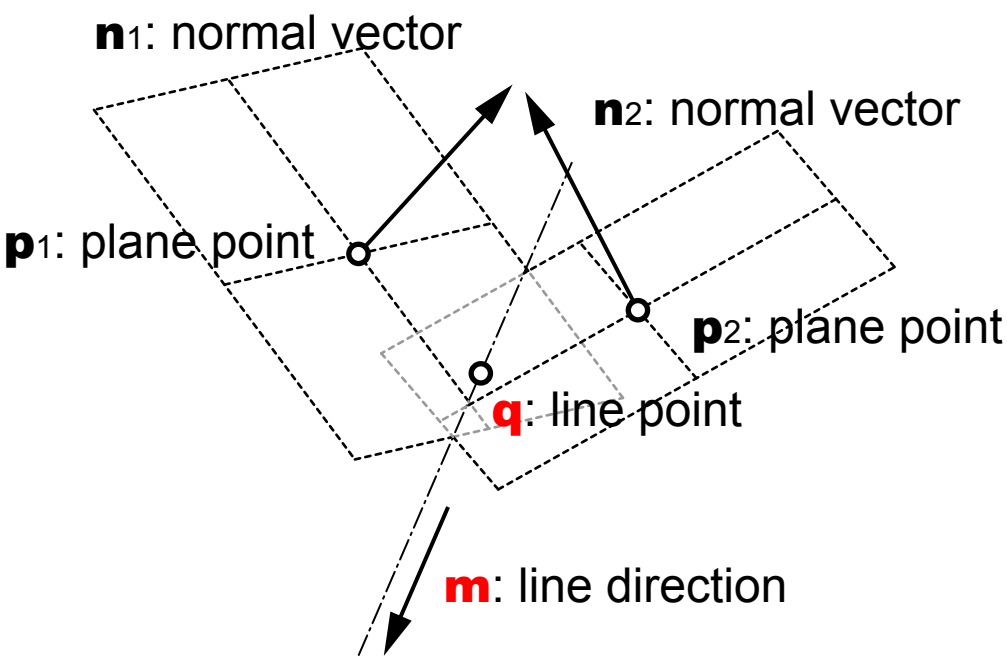
PLANE 2: $\mathbf{n}_2 \cdot (\mathbf{x} - \mathbf{p}_2) = 0$

LINE: $\mathbf{x} = \mathbf{q} + k\,\mathbf{m}$

$\mathbf{m} = ?$

$\mathbf{q} = ?$

INTERSECTION OF PLANES



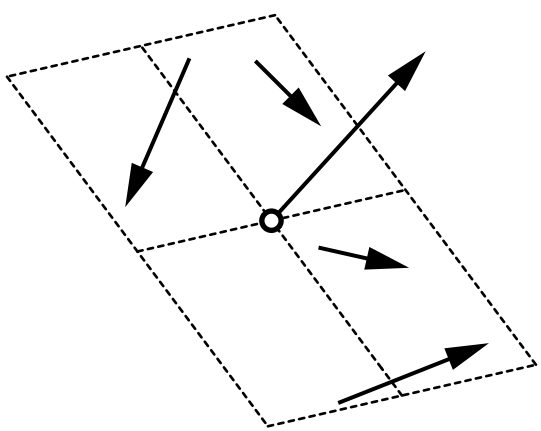
PLANE 1: $\mathbf{n}_1 \cdot (\mathbf{x} - \mathbf{p}_1) = 0$

PLANE 2: $\mathbf{n}_2 \cdot (\mathbf{x} - \mathbf{p}_2) = 0$

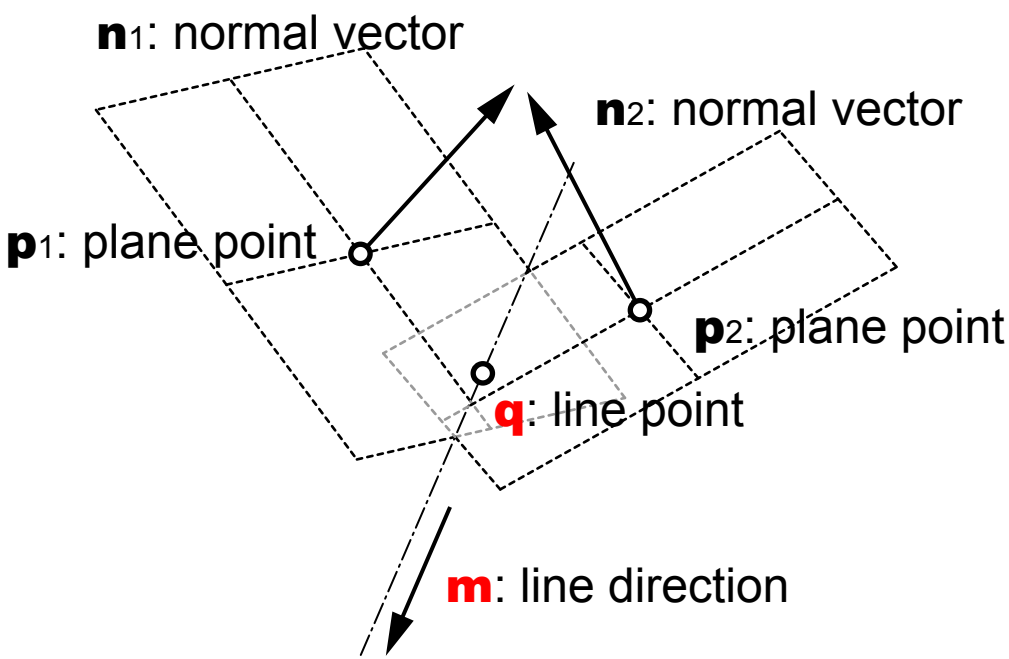
LINE: $\mathbf{x} = \mathbf{q} + k\,\mathbf{m}$

m = ?

q = ?



INTERSECTION OF PLANES



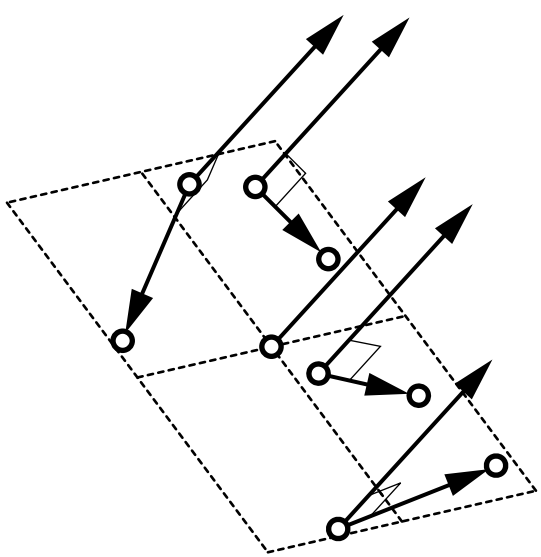
PLANE 1: $\mathbf{n}_1 \cdot (\mathbf{x} - \mathbf{p}_1) = 0$

PLANE 2: $\mathbf{n}_2 \cdot (\mathbf{x} - \mathbf{p}_2) = 0$

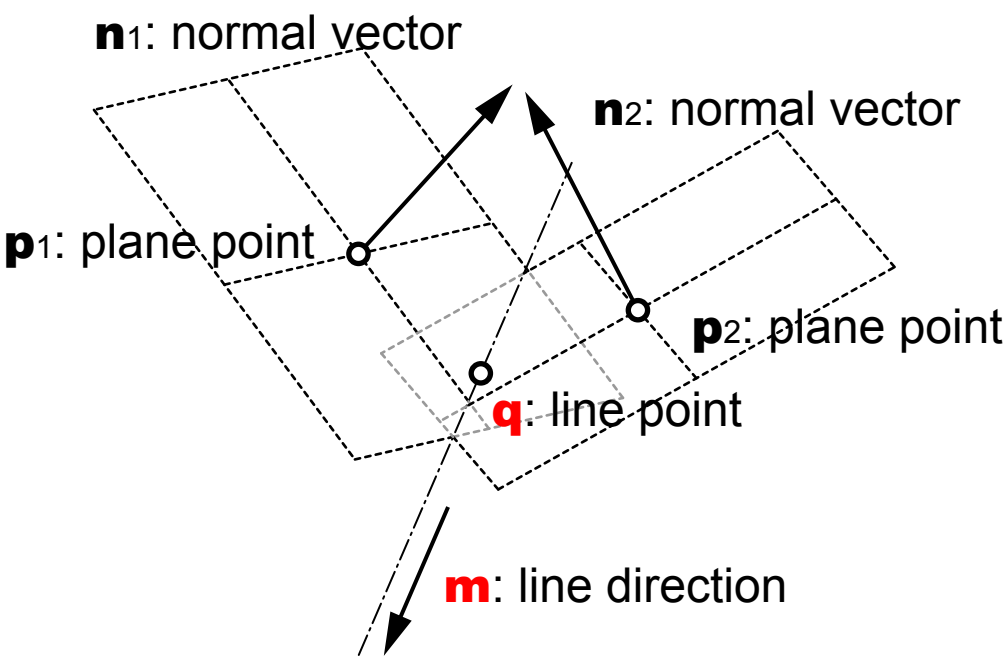
LINE: $\mathbf{x} = \mathbf{q} + k\,\mathbf{m}$

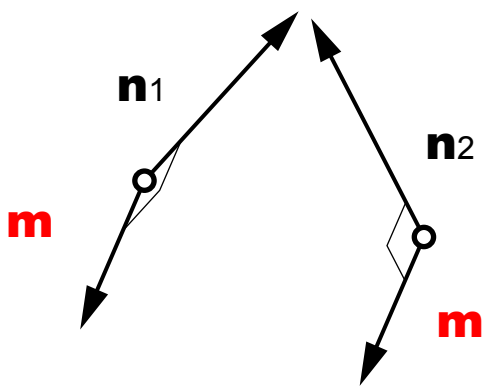
$\mathbf{m} = ?$

$\mathbf{q} = ?$



INTERSECTION OF PLANES





PLANE 1: $\mathbf{n}_1 \cdot (\mathbf{x} - \mathbf{p}_1) = 0$

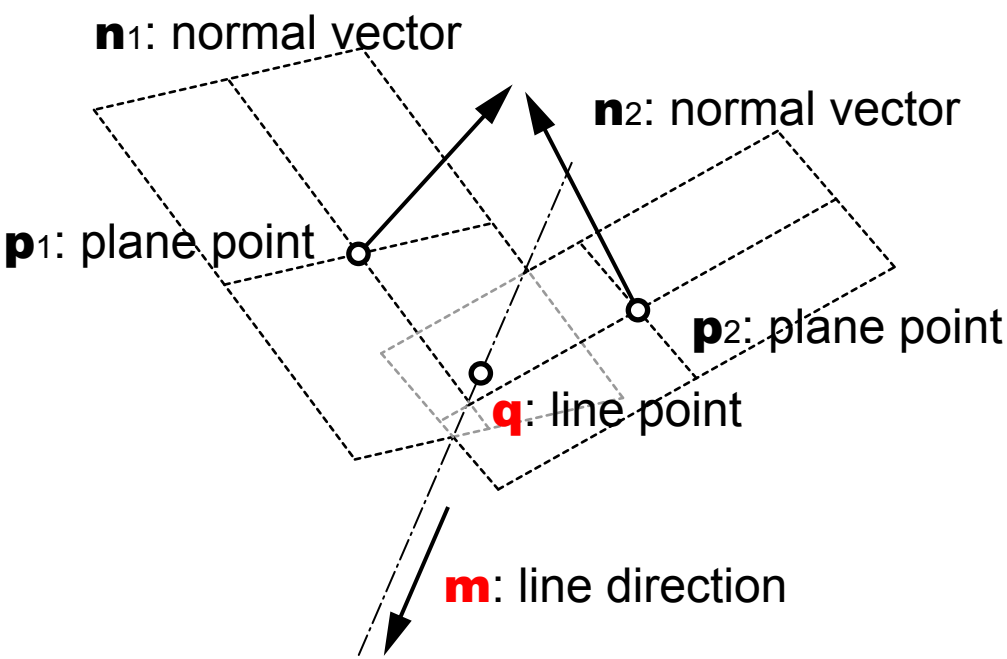
PLANE 2: $\mathbf{n}_2 \cdot (\mathbf{x} - \mathbf{p}_2) = 0$

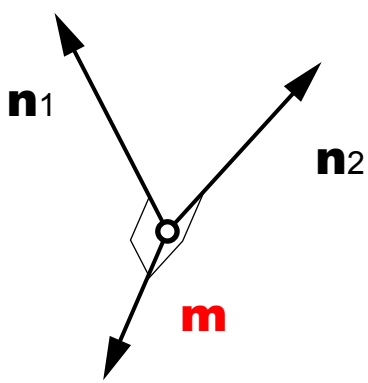
LINE: $\mathbf{x} = \mathbf{q} + k\,\mathbf{m}$

$\mathbf{m} = ?$

$\mathbf{q} = ?$

INTERSECTION OF PLANES





PLANE 1: $\mathbf{n}_1 \cdot (\mathbf{x} - \mathbf{p}_1) = 0$

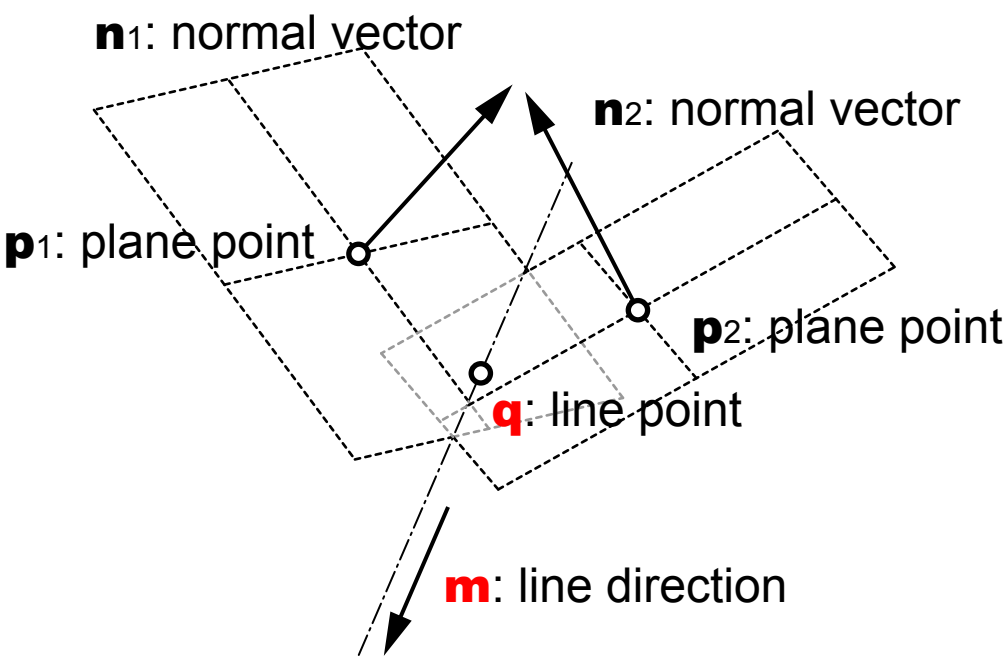
PLANE 2: $\mathbf{n}_2 \cdot (\mathbf{x} - \mathbf{p}_2) = 0$

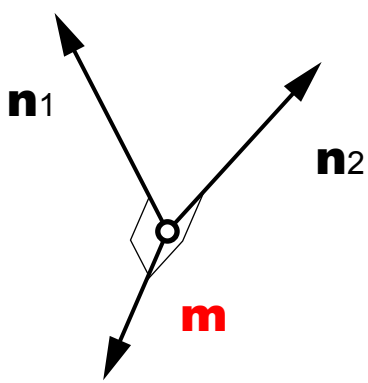
LINE: $\mathbf{x} = \mathbf{q} + k\,\mathbf{m}$

$\mathbf{m} = ?$

$\mathbf{q} = ?$

INTERSECTION OF PLANES





PLANE 1: $\mathbf{n}_1 \cdot (\mathbf{x} - \mathbf{p}_1) = 0$

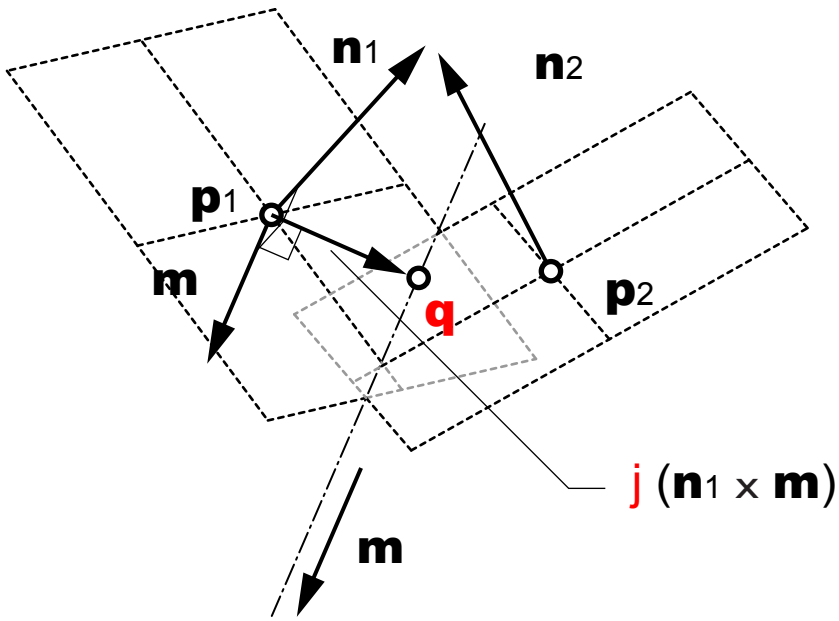
PLANE 2: $\mathbf{n}_2 \cdot (\mathbf{x} - \mathbf{p}_2) = 0$

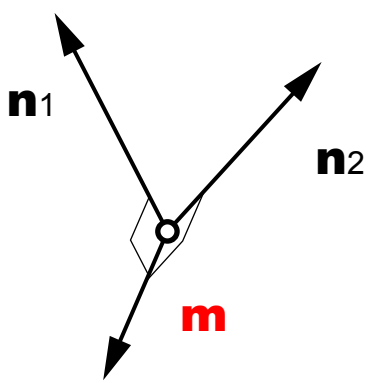
LINE: $\mathbf{x} = \mathbf{q} + k\,\mathbf{m}$

$\mathbf{m} = \mathbf{n}_1 \times \mathbf{n}_2$

$\mathbf{q} = ?$

INTERSECTION OF PLANES





PLANE 1: $\mathbf{n}_1 \cdot (\mathbf{x} - \mathbf{p}_1) = 0$

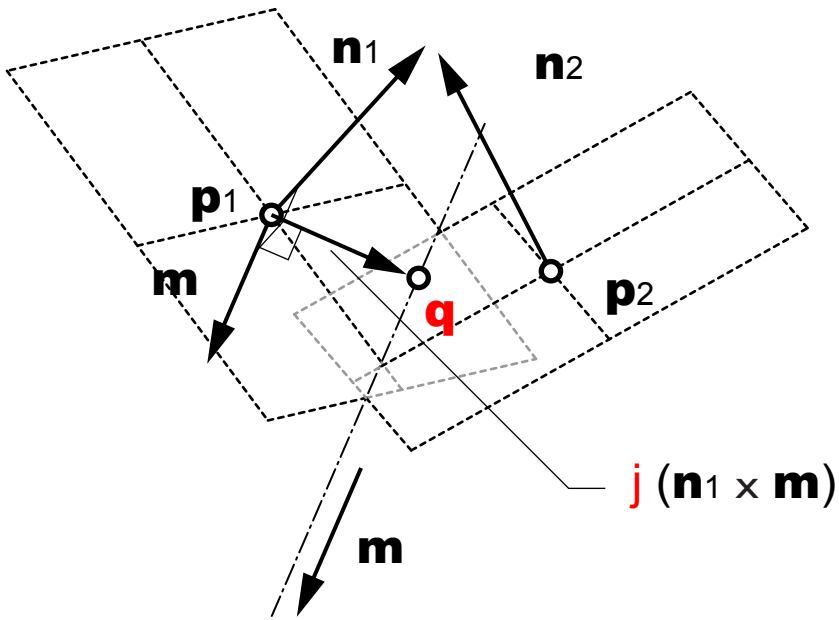
PLANE 2: $\mathbf{n}_2 \cdot (\mathbf{x} - \mathbf{p}_2) = 0$

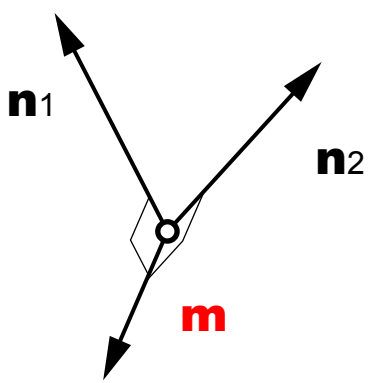
LINE: $\mathbf{x} = \mathbf{q} + k\,\mathbf{m}$

$\mathbf{m} = \mathbf{n}_1 \times \mathbf{n}_2$

$\mathbf{q} = \mathbf{p}_1 + j\,(\mathbf{n}_1 \times \mathbf{m})$

INTERSECTION OF PLANES





PLANE 1: $\mathbf{n}_1 \cdot (\mathbf{x} - \mathbf{p}_1) = 0$

PLANE 2: $\mathbf{n}_2 \cdot (\mathbf{x} - \mathbf{p}_2) = 0$

LINE: $\mathbf{x} = \mathbf{q} + k\,\mathbf{m}$

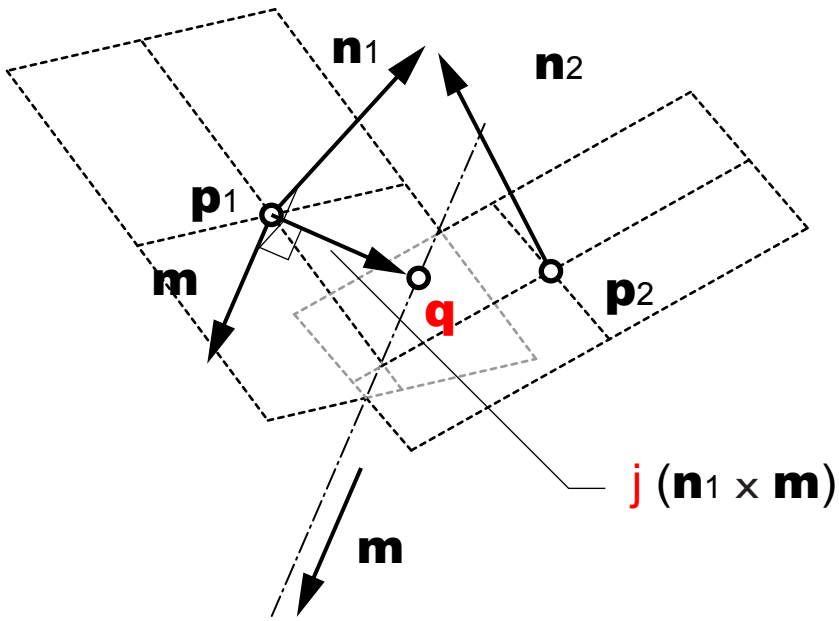
$\mathbf{m} = \mathbf{n}_1 \times \mathbf{n}_2$

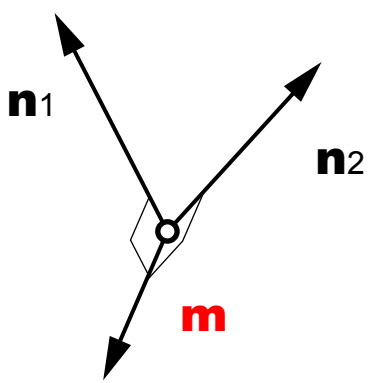
$\mathbf{q} = \mathbf{p}_1 + j\,(\mathbf{n}_1 \times \mathbf{m})$

$\mathbf{n}_2 \cdot (\mathbf{q} - \mathbf{p}_2) = 0$

$\mathbf{n}_2 \cdot (\mathbf{p}_1 + j\,(\mathbf{n}_1 \times \mathbf{m}) - \mathbf{p}_2) = 0$

INTERSECTION OF PLANES





PLANE 1: $\mathbf{n}_1 \cdot (\mathbf{x} - \mathbf{p}_1) = 0$

PLANE 2: $\mathbf{n}_2 \cdot (\mathbf{x} - \mathbf{p}_2) = 0$

LINE: $\mathbf{x} = \mathbf{q} + k\,\mathbf{m}$

$$\mathbf{m} = \mathbf{n}_1 \times \mathbf{n}_2$$

$$\mathbf{q} = \mathbf{p}_1 + j\,(\mathbf{n}_1 \times \mathbf{m})$$

$$\mathbf{n}_2 \cdot (\mathbf{q} - \mathbf{p}_2) = 0$$

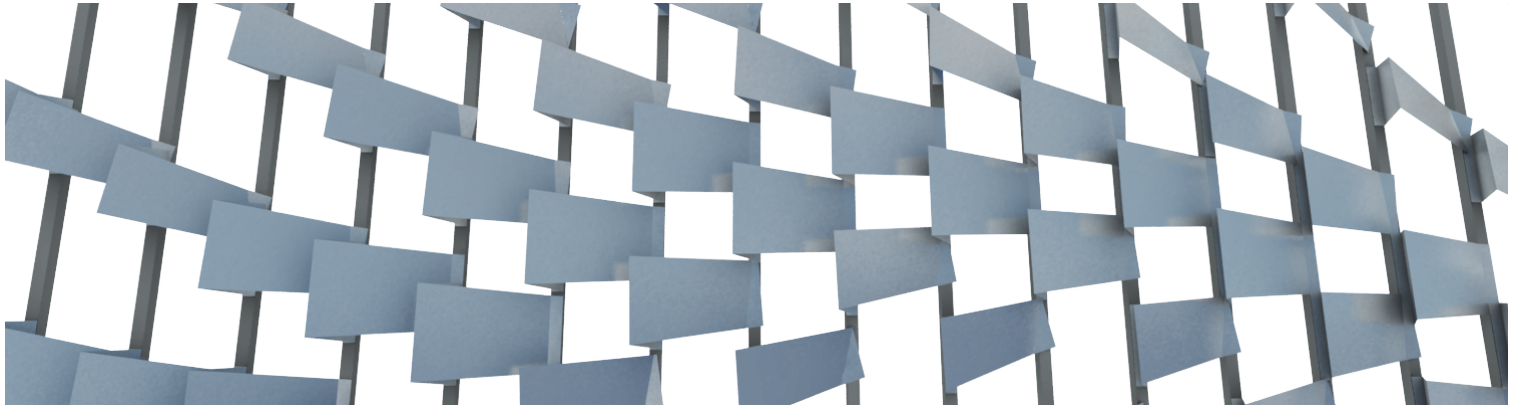
$$\mathbf{n}_2 \cdot (\mathbf{p}_1 + j\,(\mathbf{n}_1 \times \mathbf{m}) - \mathbf{p}_2) = 0$$

$$j\,\mathbf{n}_2 \cdot (\mathbf{n}_1 \times \mathbf{m}) = \mathbf{n}_2 \cdot (\mathbf{p}_2 - \mathbf{p}_1)$$

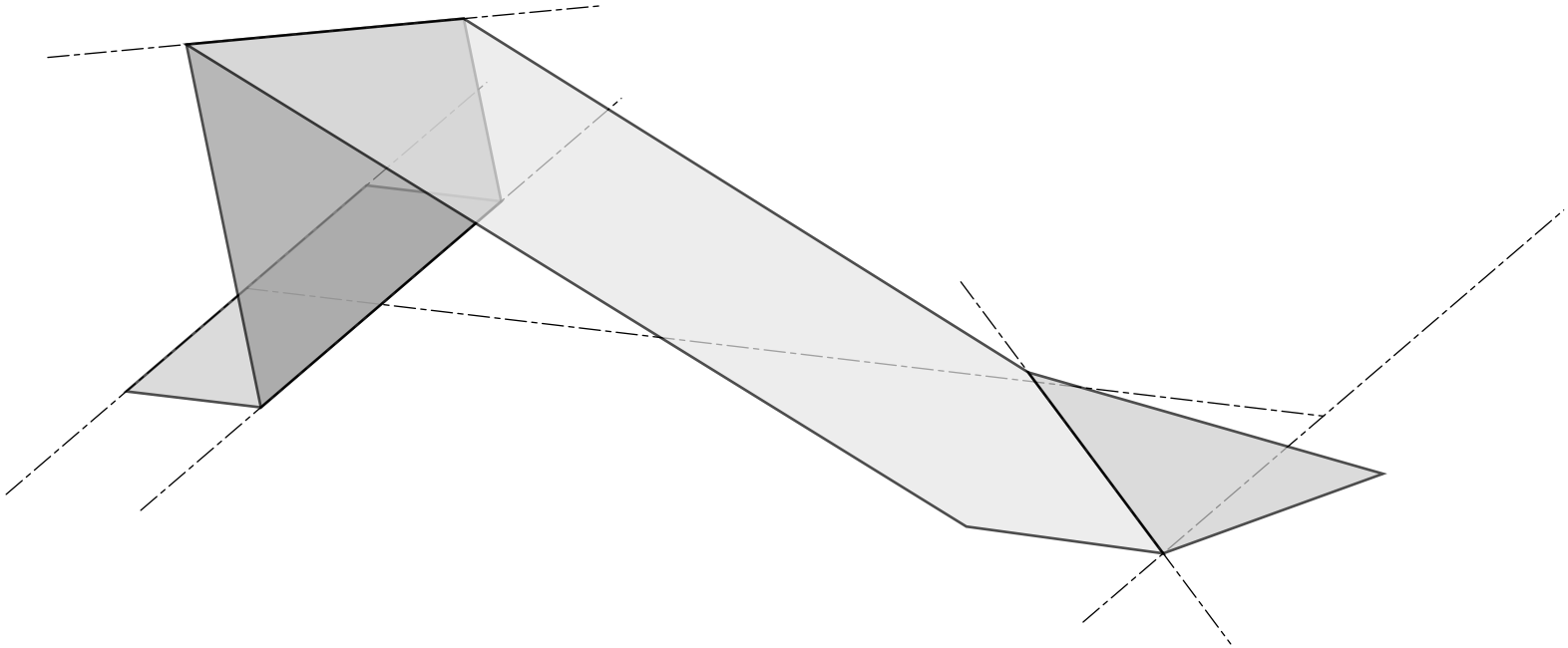
$$j = \frac{\mathbf{n}_2 \cdot (\mathbf{p}_2 - \mathbf{p}_1)}{\mathbf{n}_2 \cdot (\mathbf{n}_1 \times \mathbf{m})}$$

$$\mathbf{q} = \mathbf{p}_1 + \frac{\mathbf{n}_2 \cdot (\mathbf{p}_2 - \mathbf{p}_1)}{\mathbf{n}_2 \cdot (\mathbf{n}_1 \times \mathbf{m})}\,(\mathbf{n}_1 \times \mathbf{m})$$

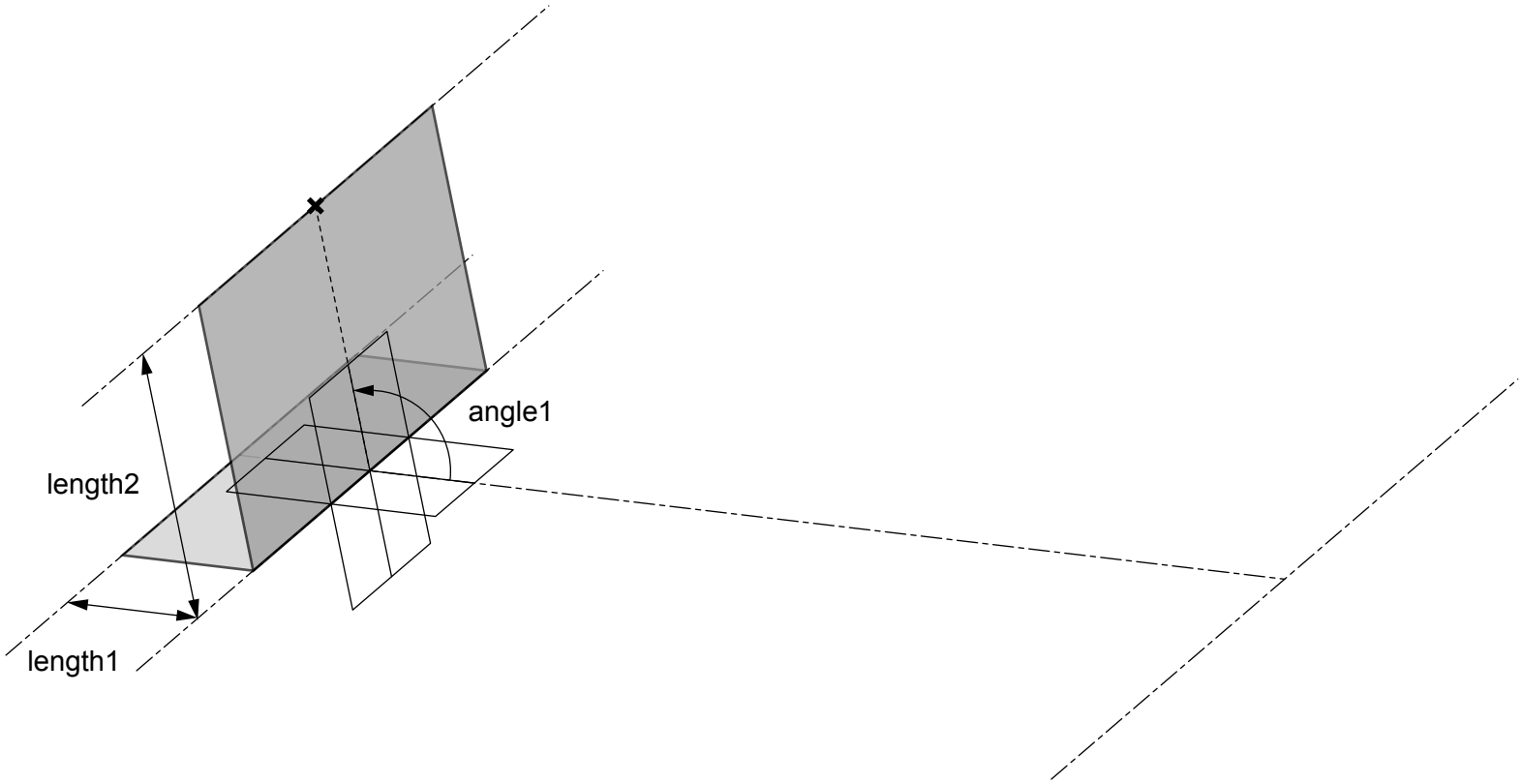
PLANE EXAMPLE : EMERSON COLLEGE SCRIM



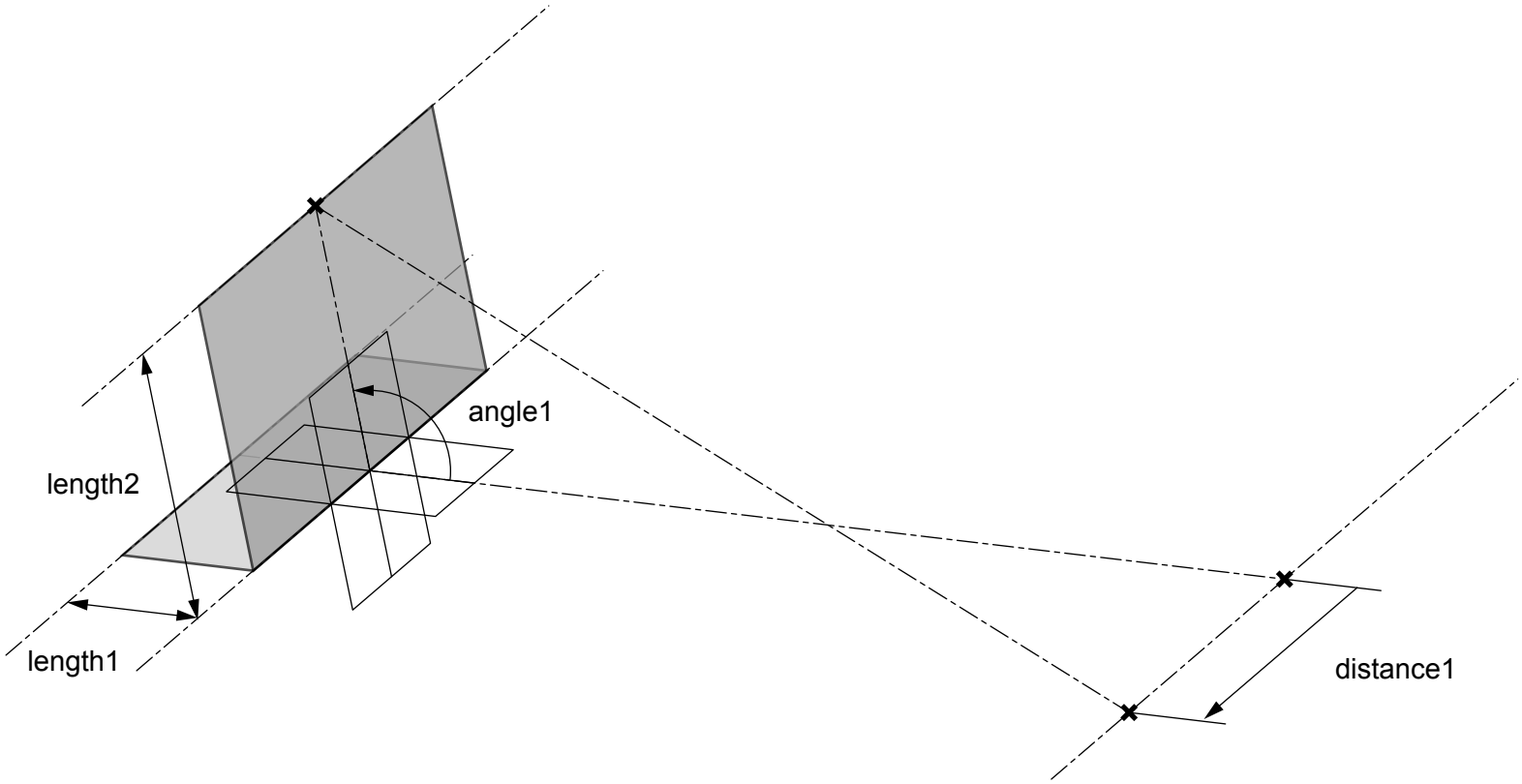
PLANE EXAMPLE : EMERSON COLLEGE SCRIM1



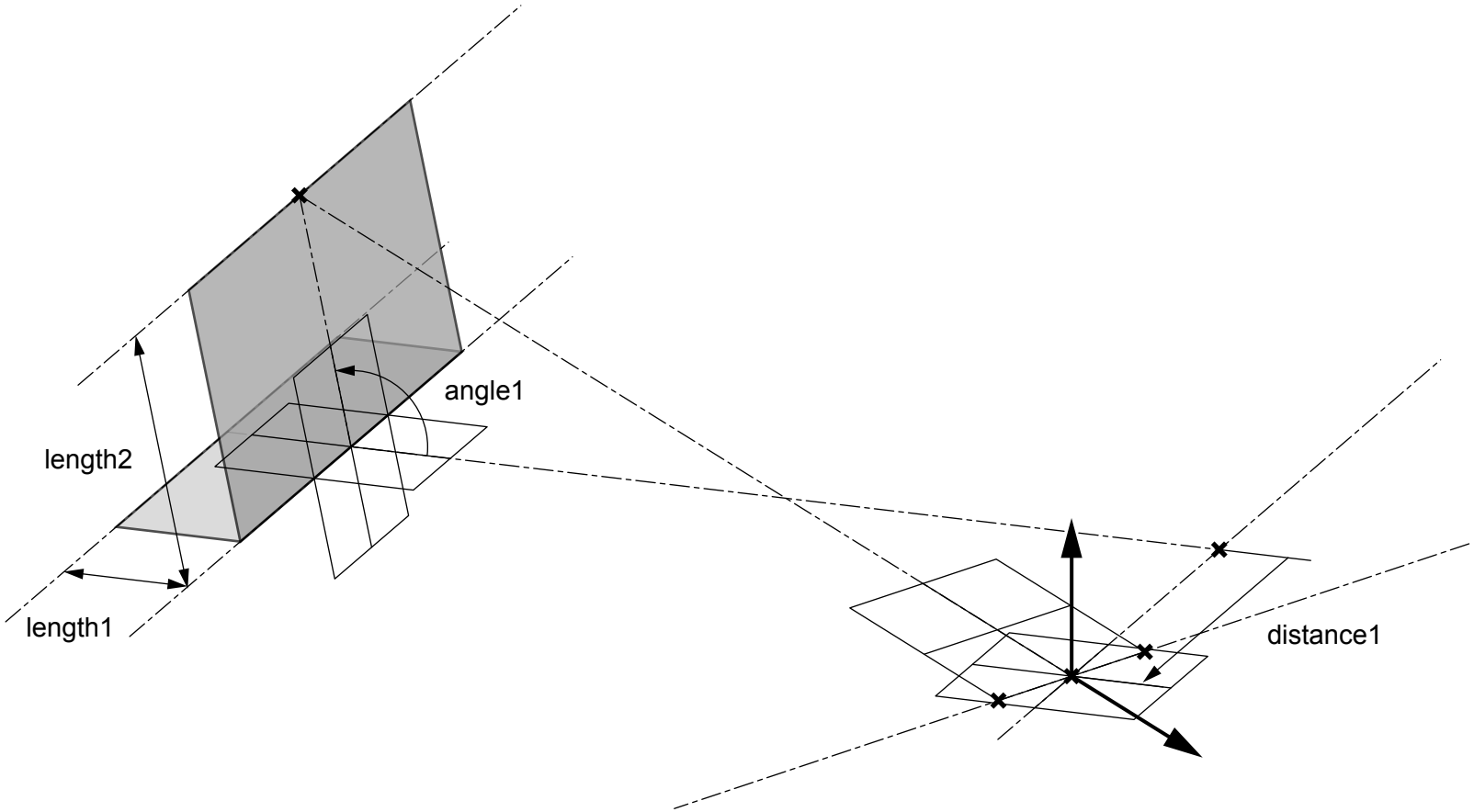
PLANE EXAMPLE : EMERSON COLLEGE SCRIM1



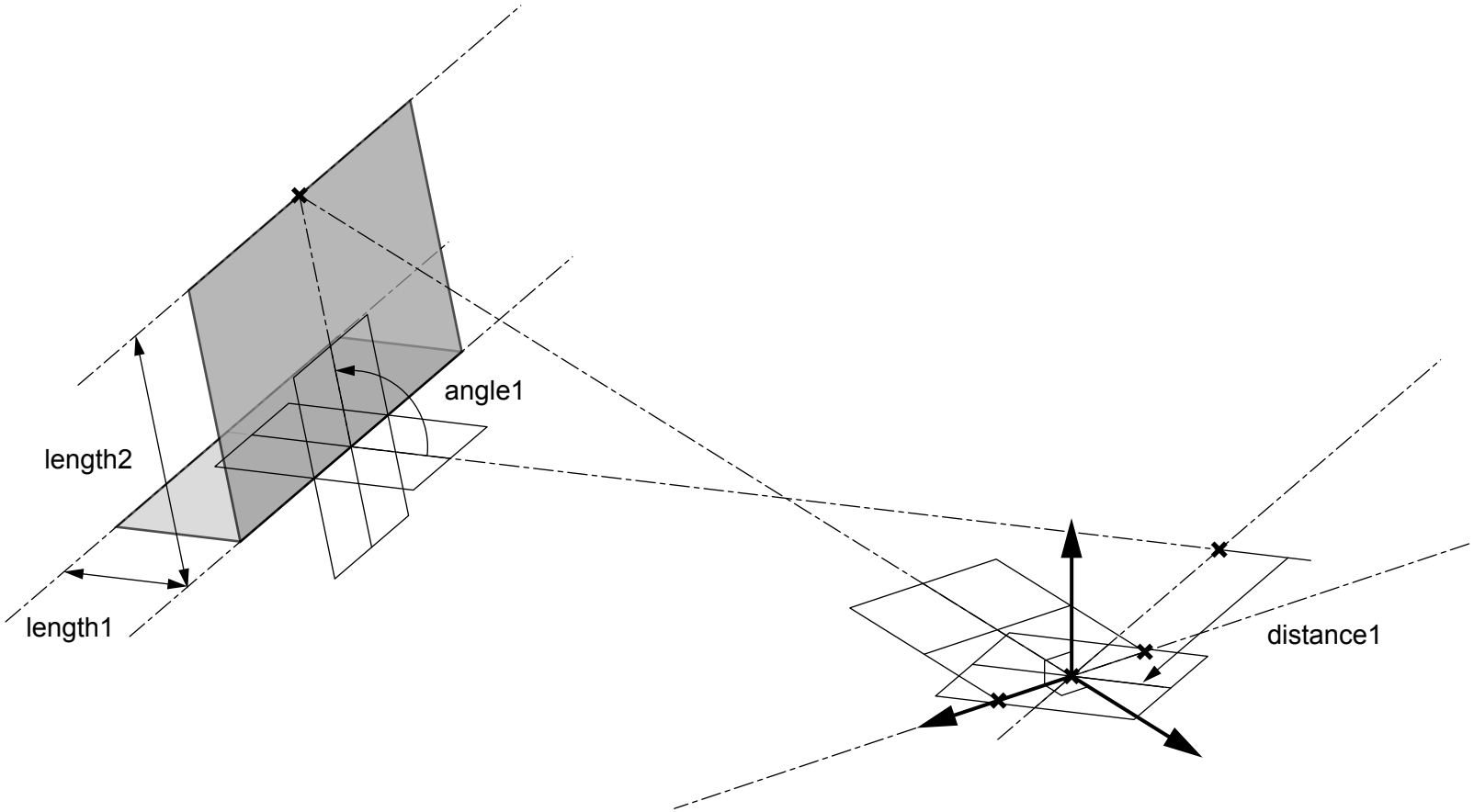
PLANE EXAMPLE : EMERSON COLLEGE SCRIM1



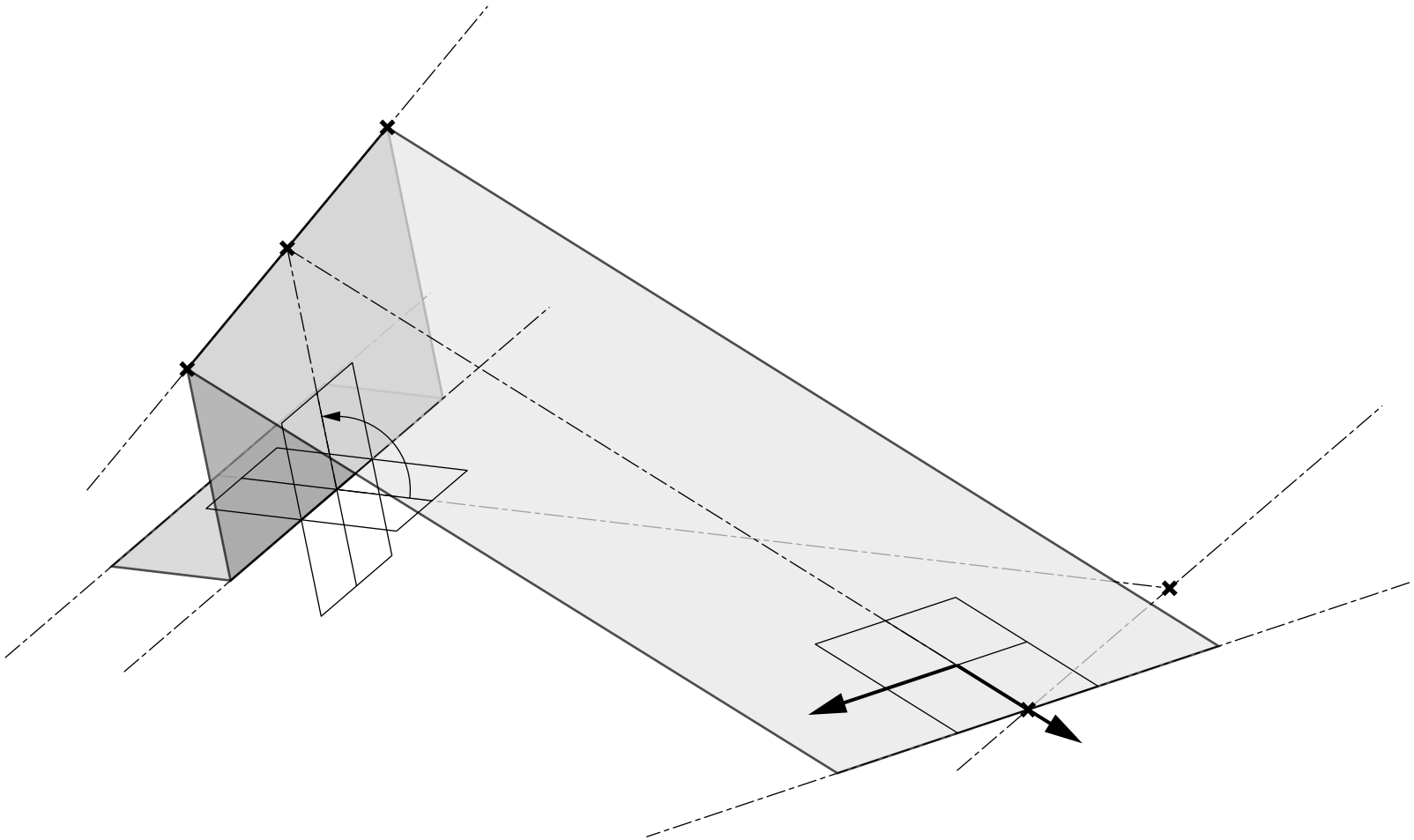
PLANE EXAMPLE : EMERSON COLLEGE SCRIM1



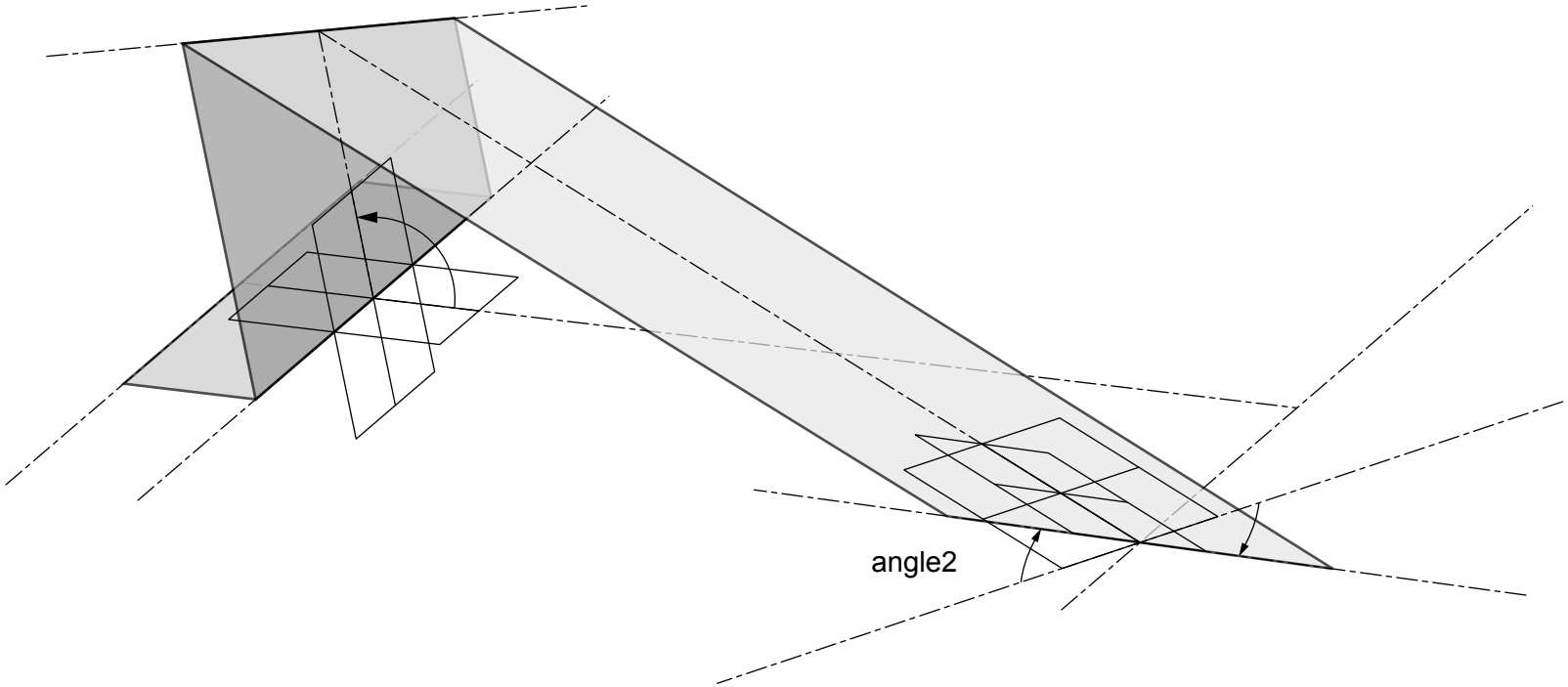
PLANE EXAMPLE : EMERSON COLLEGE SCRIM1



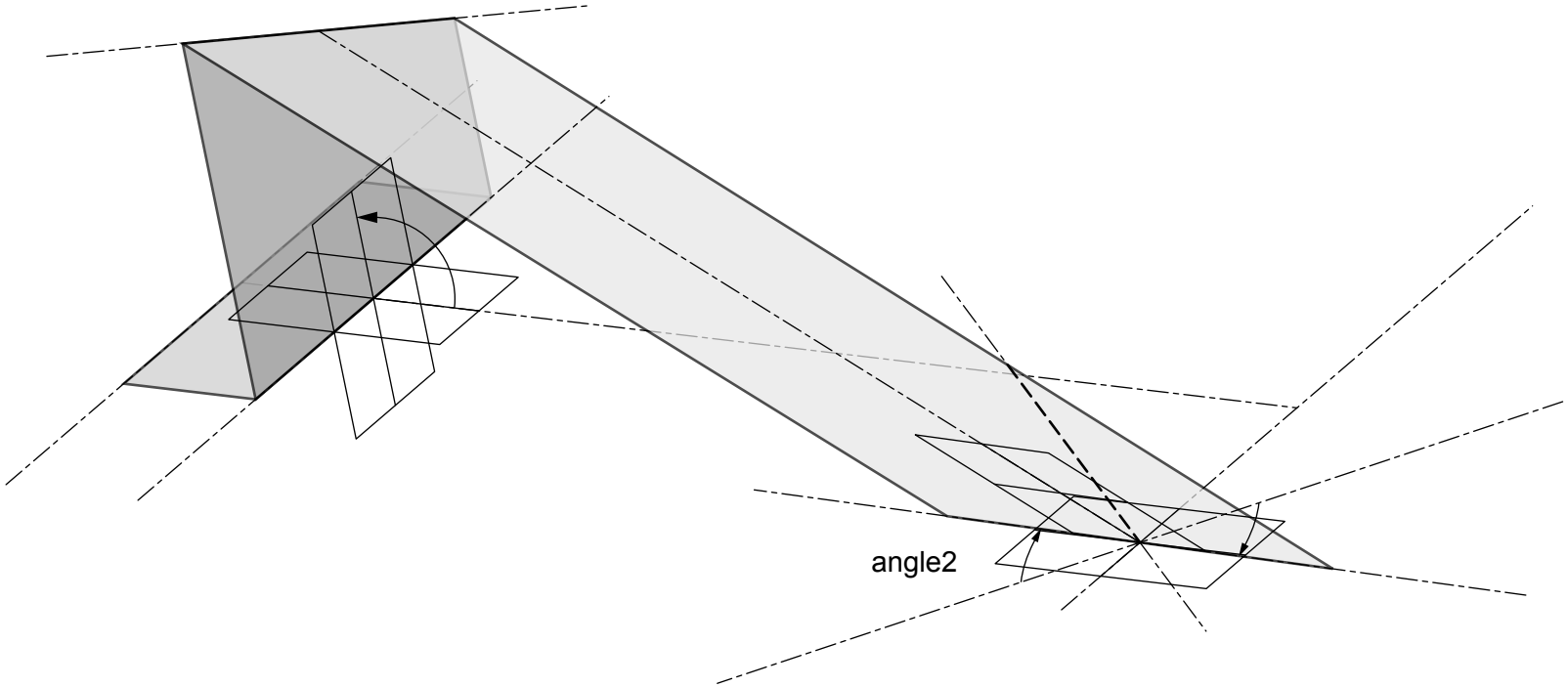
PLANE EXAMPLE : EMERSON COLLEGE SCRIM1



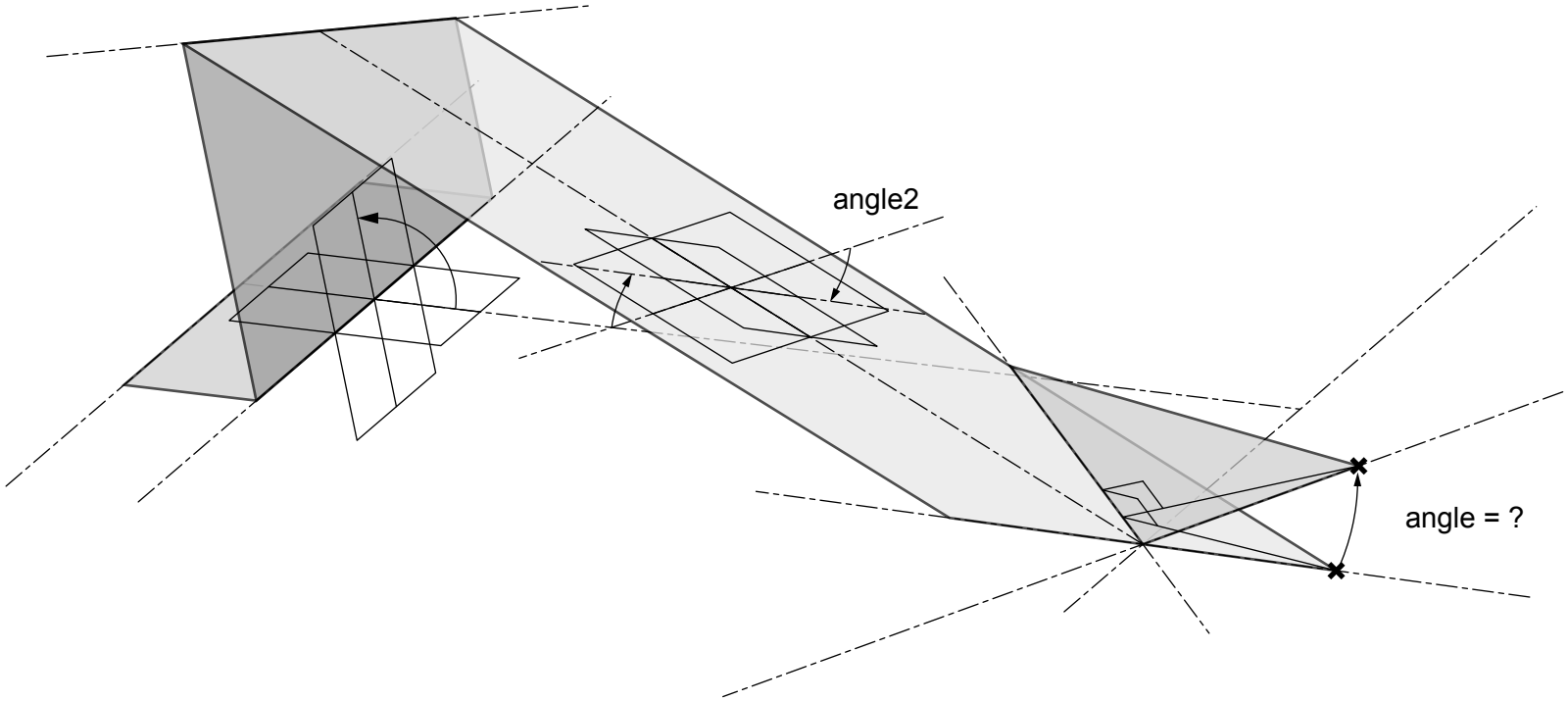
PLANE EXAMPLE : EMERSON COLLEGE SCRIM1



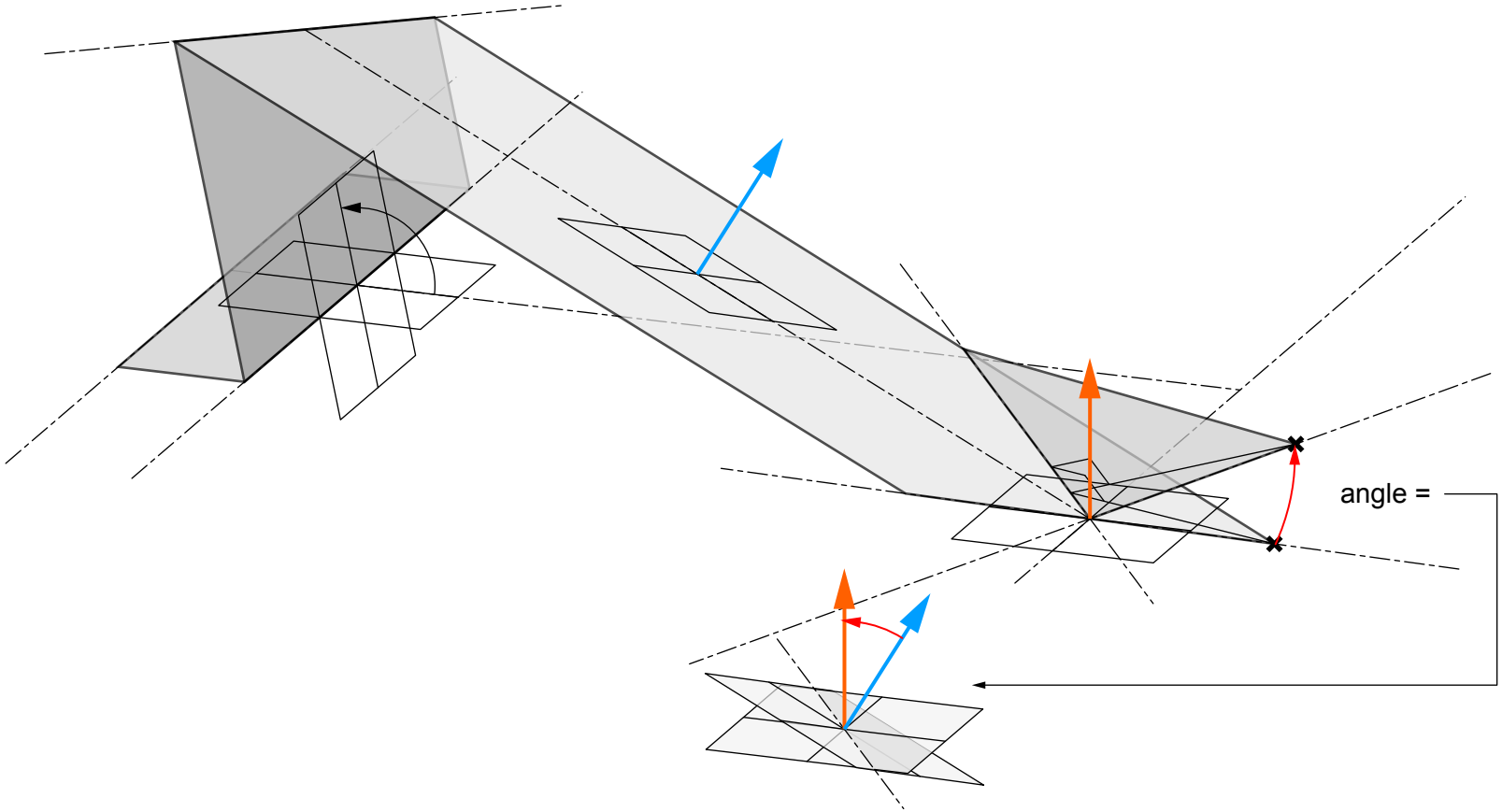
PLANE EXAMPLE : EMERSON COLLEGE SCRIM1



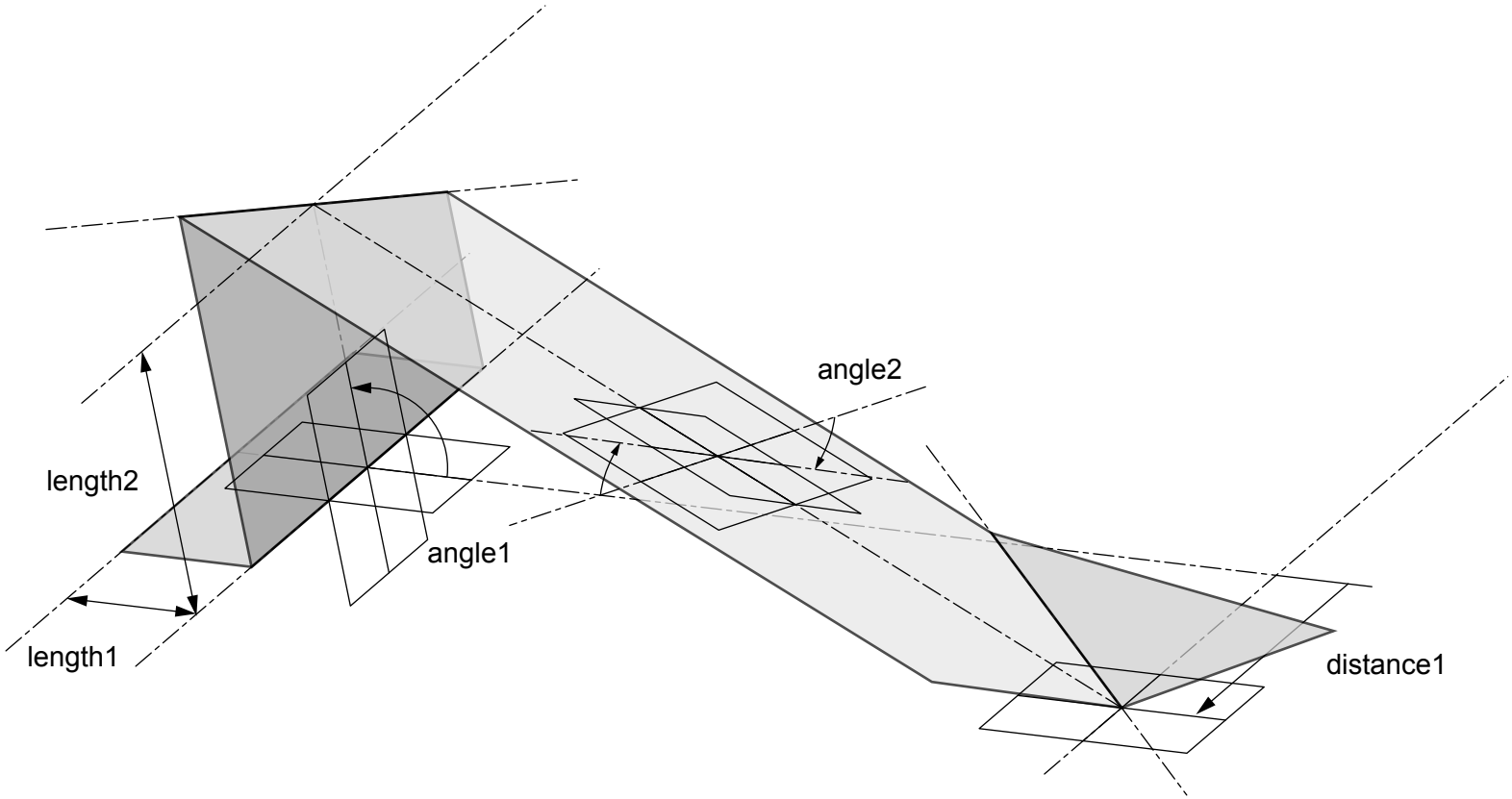
PLANE EXAMPLE : EMERSON COLLEGE SCRIM1



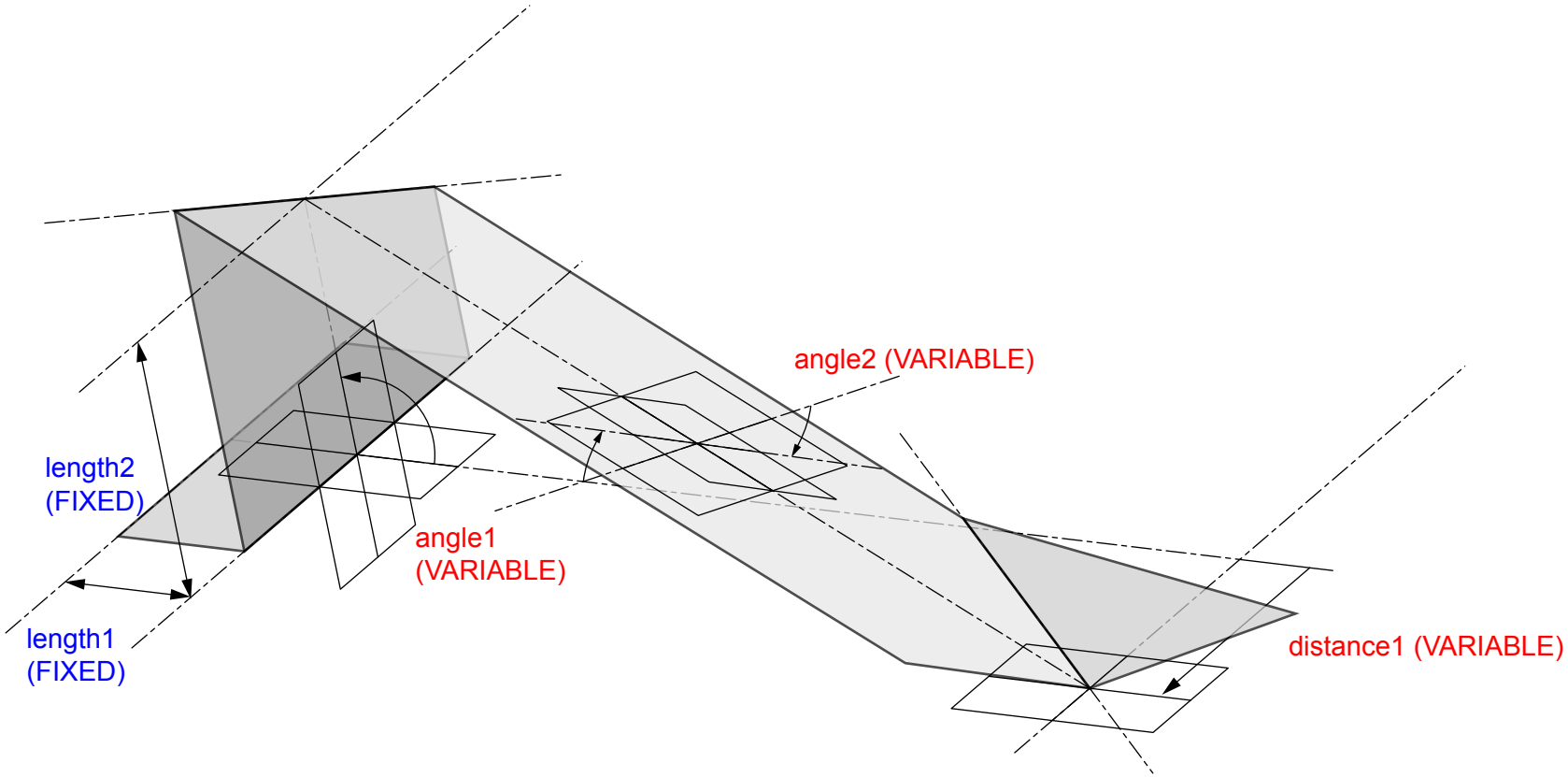
PLANE EXAMPLE : EMERSON COLLEGE SCRIM1



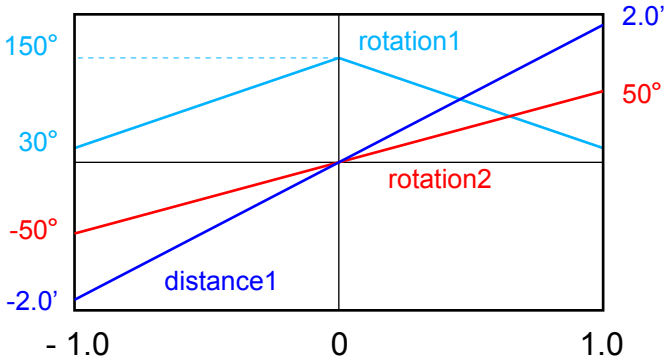
PLANE EXAMPLE : EMERSON COLLEGE SCRIM1

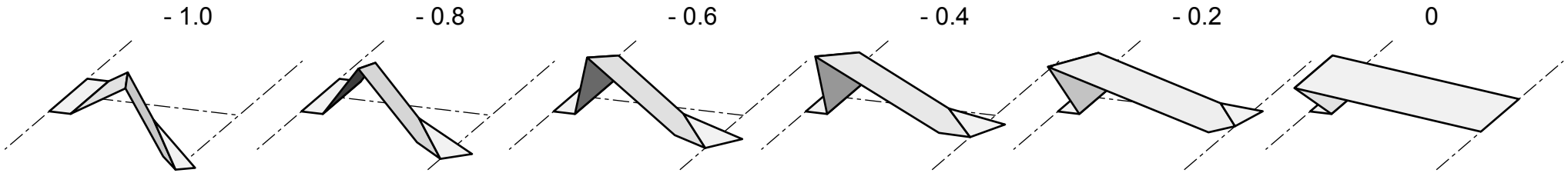


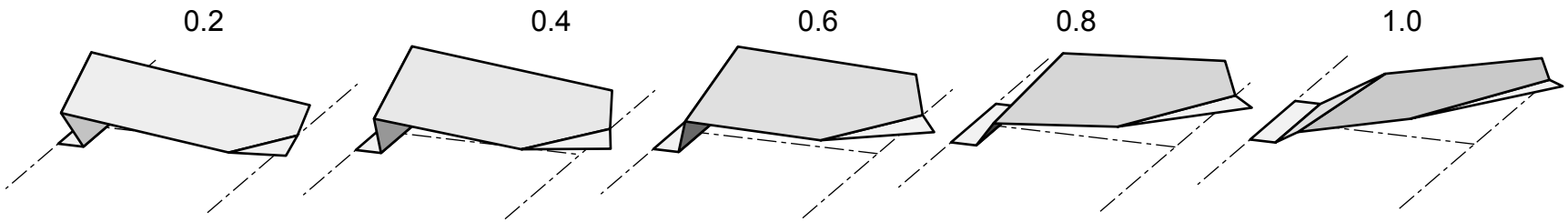
PLANE EXAMPLE : EMERSON COLLEGE SCRIM1



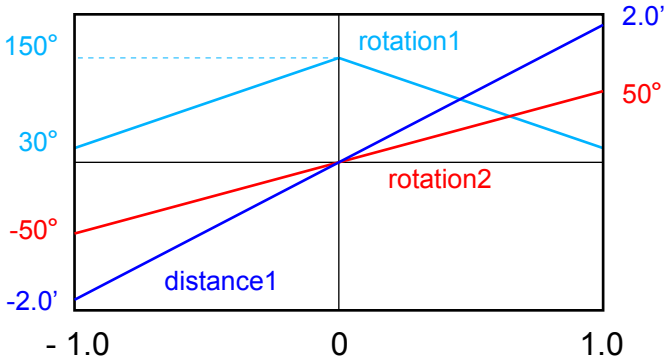
PLANE EXAMPLE : EMERSON COLLEGE SCRIM1

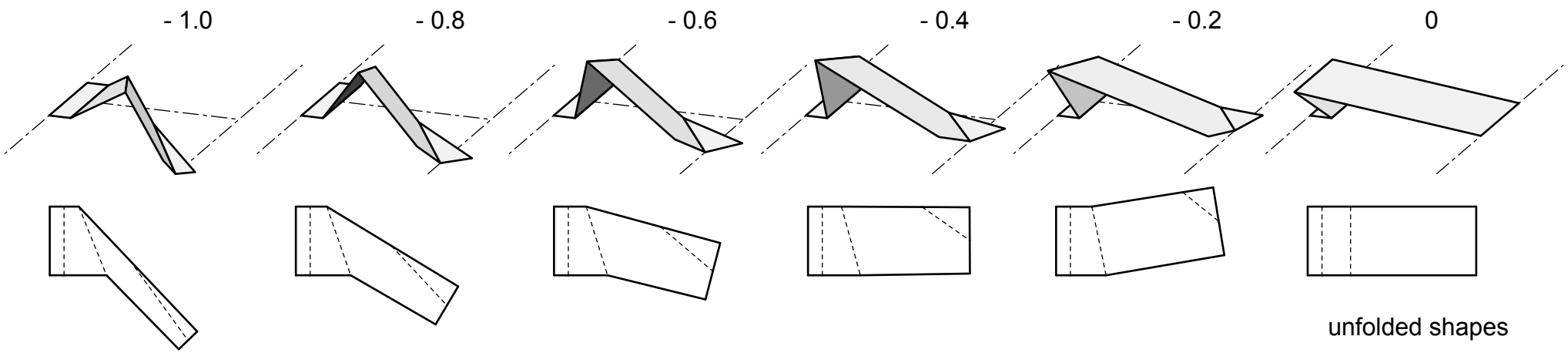


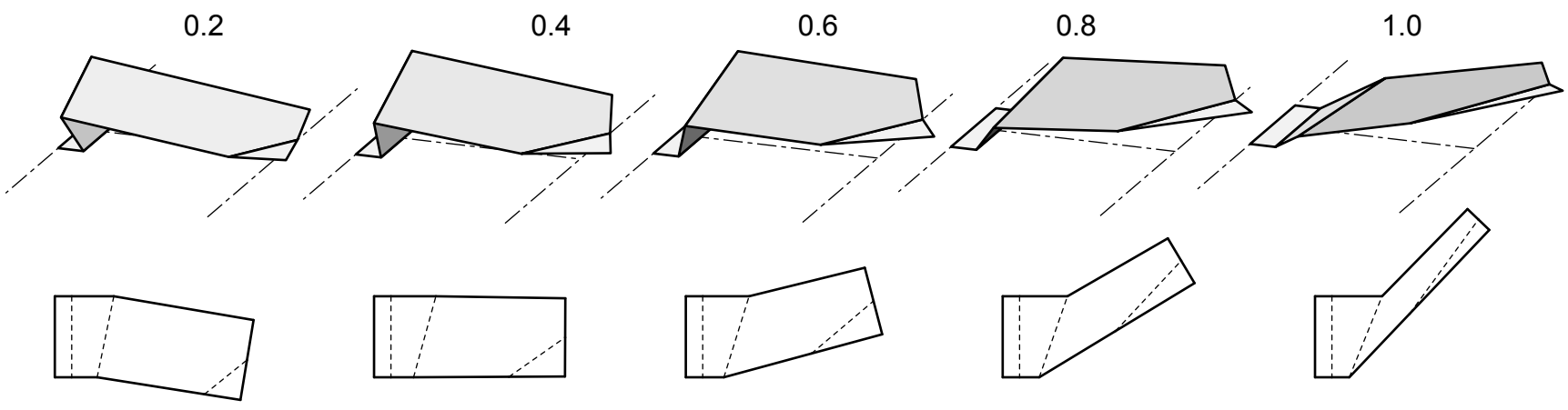




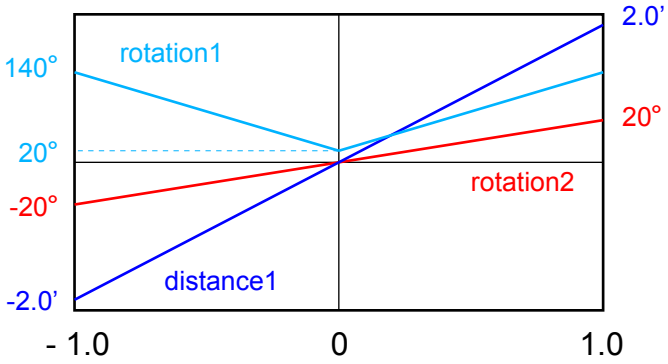
PLANE EXAMPLE : EMERSON COLLEGE SCRIM1

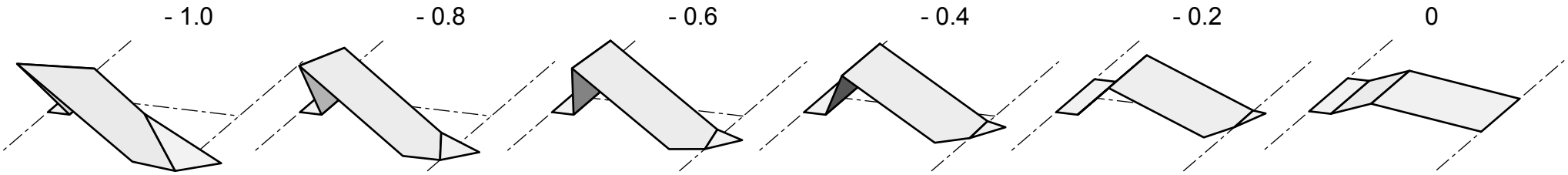


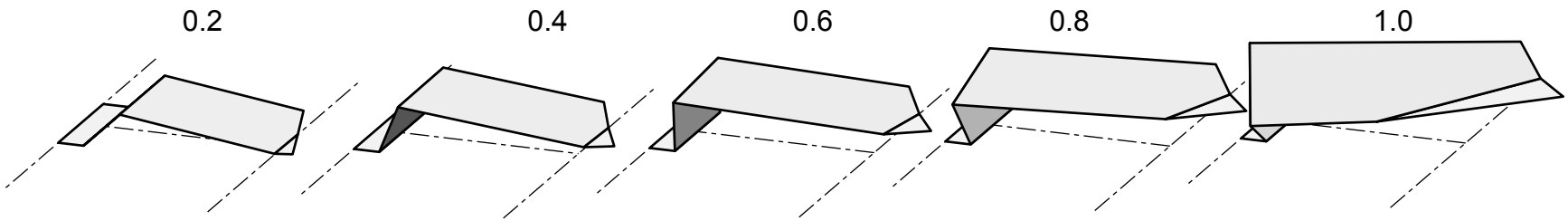




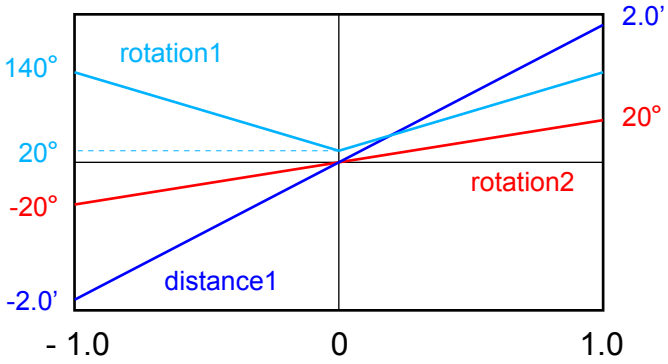
PLANE EXAMPLE : EMERSON COLLEGE SCRIM1

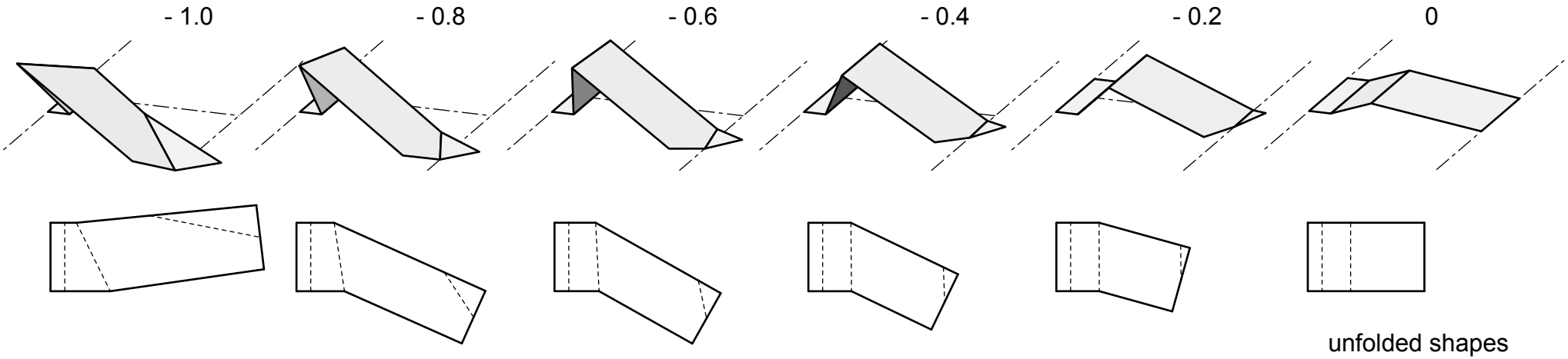


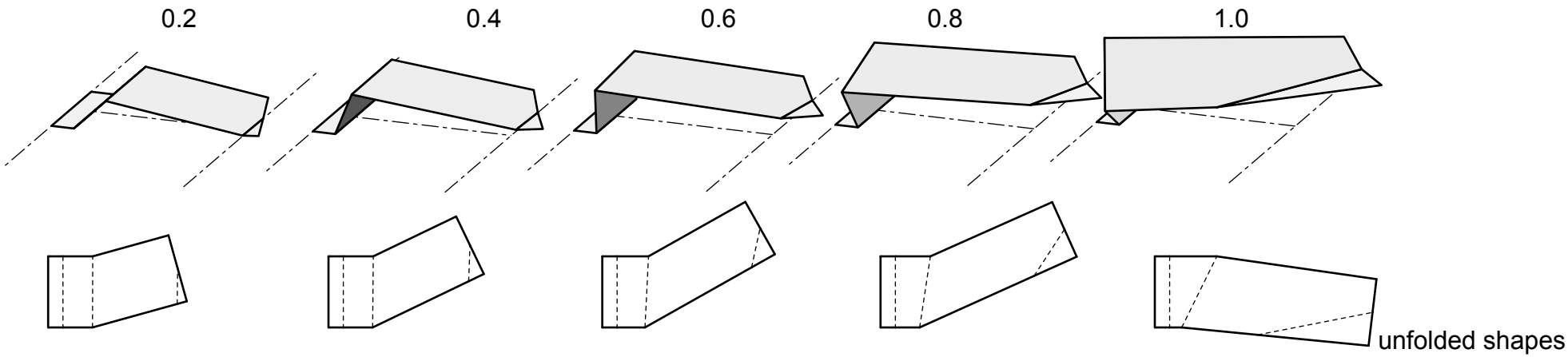




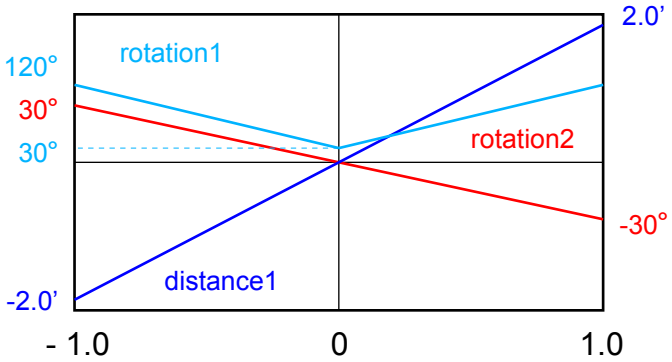
PLANE EXAMPLE : EMERSON COLLEGE SCRIM1

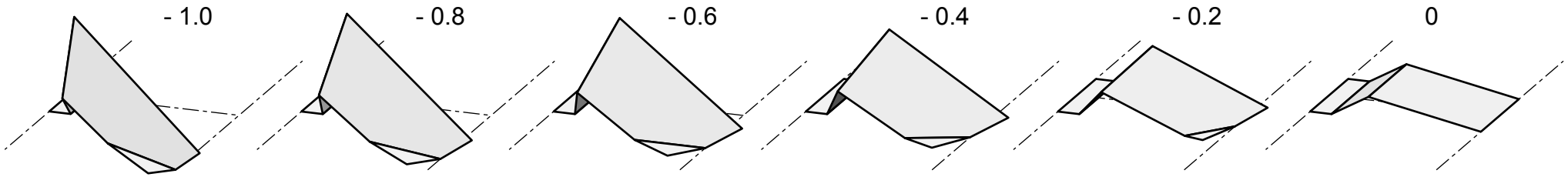


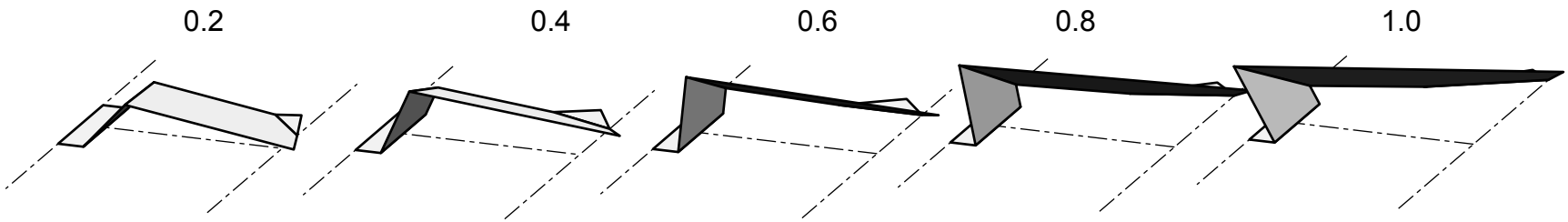




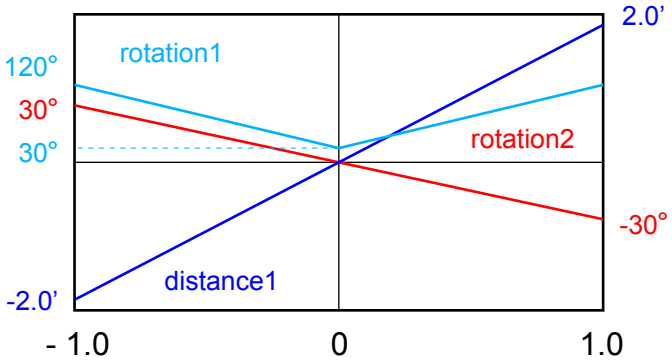
PLANE EXAMPLE : EMERSON COLLEGE SCRIM1

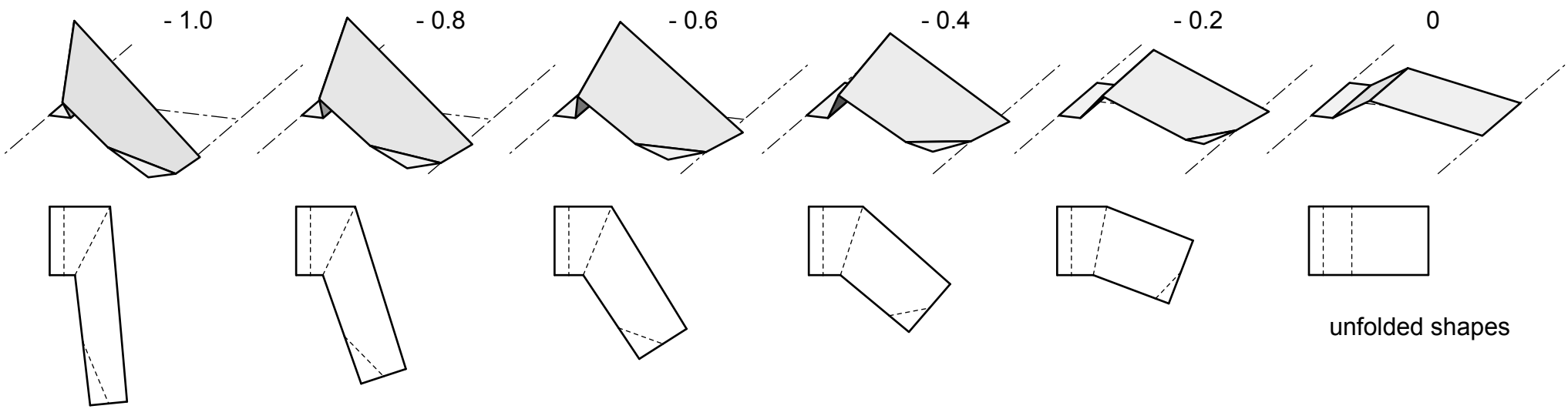


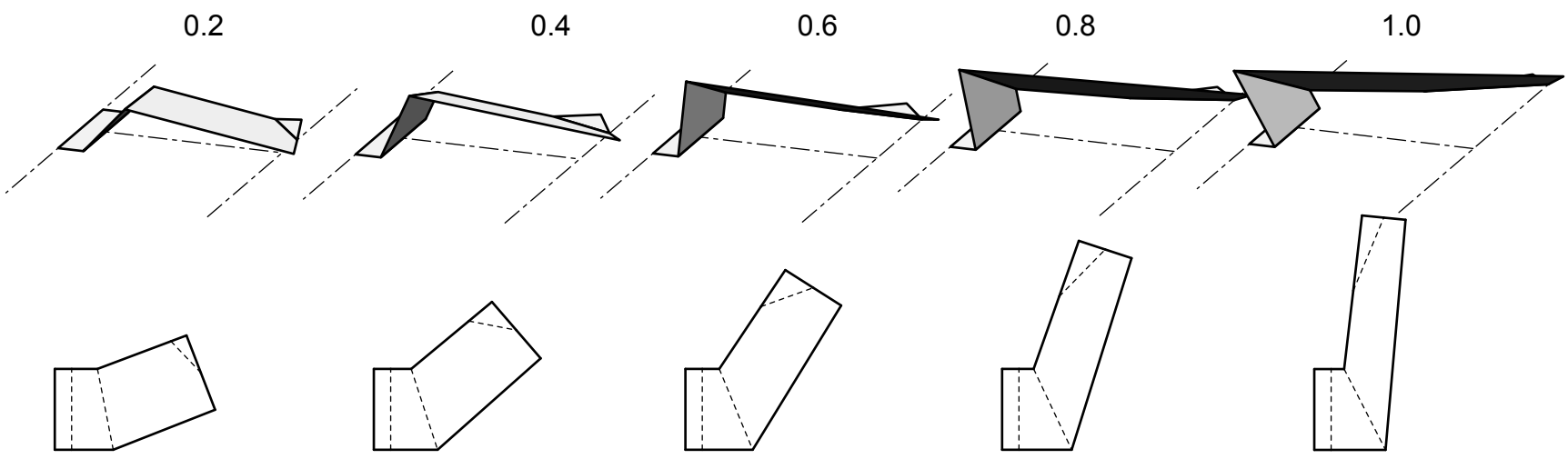




PLANE EXAMPLE : EMERSON COLLEGE SCRIM1

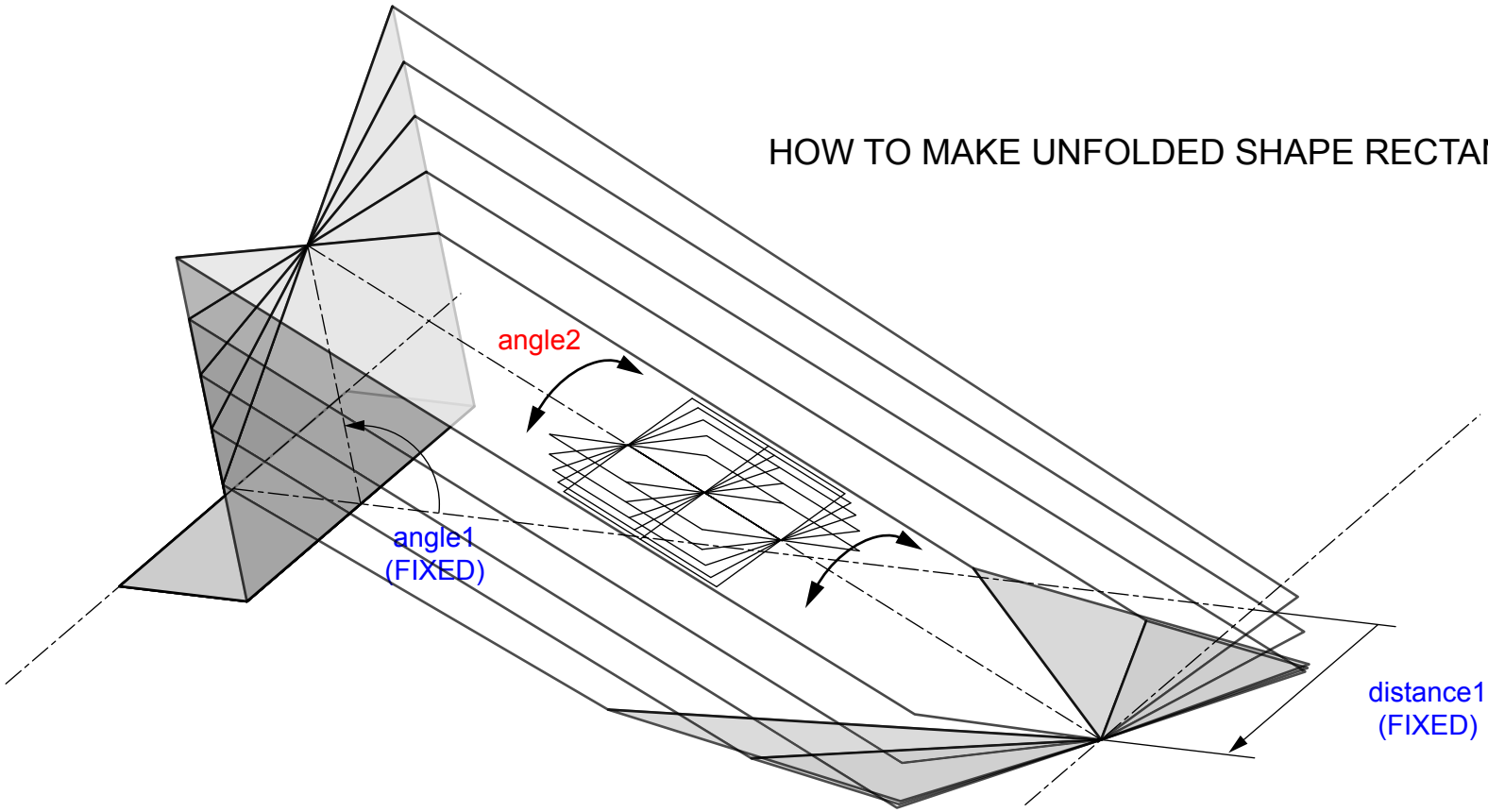


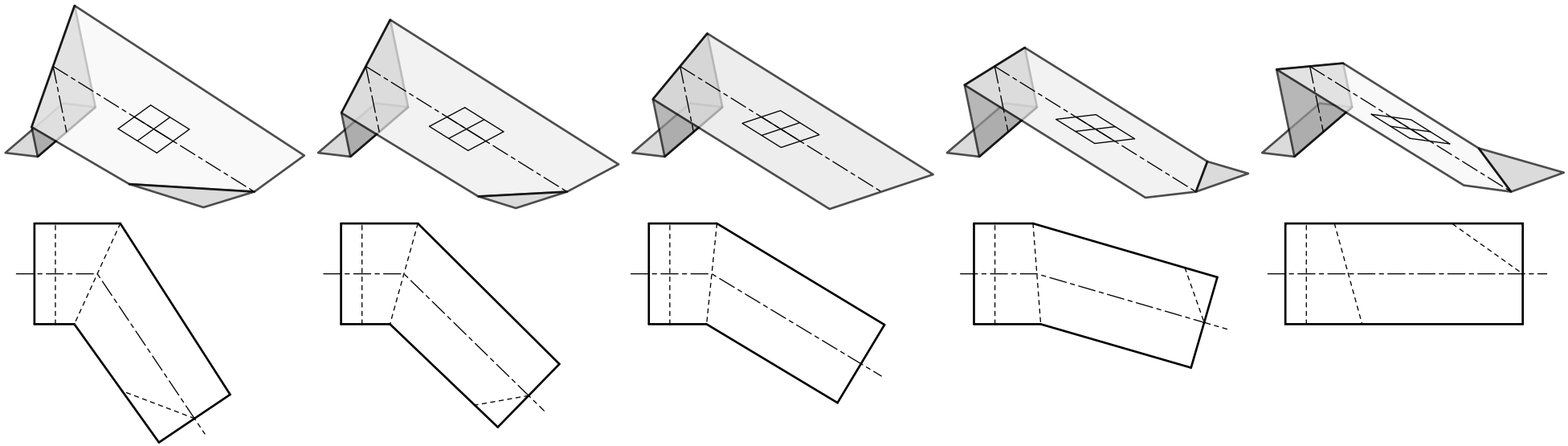




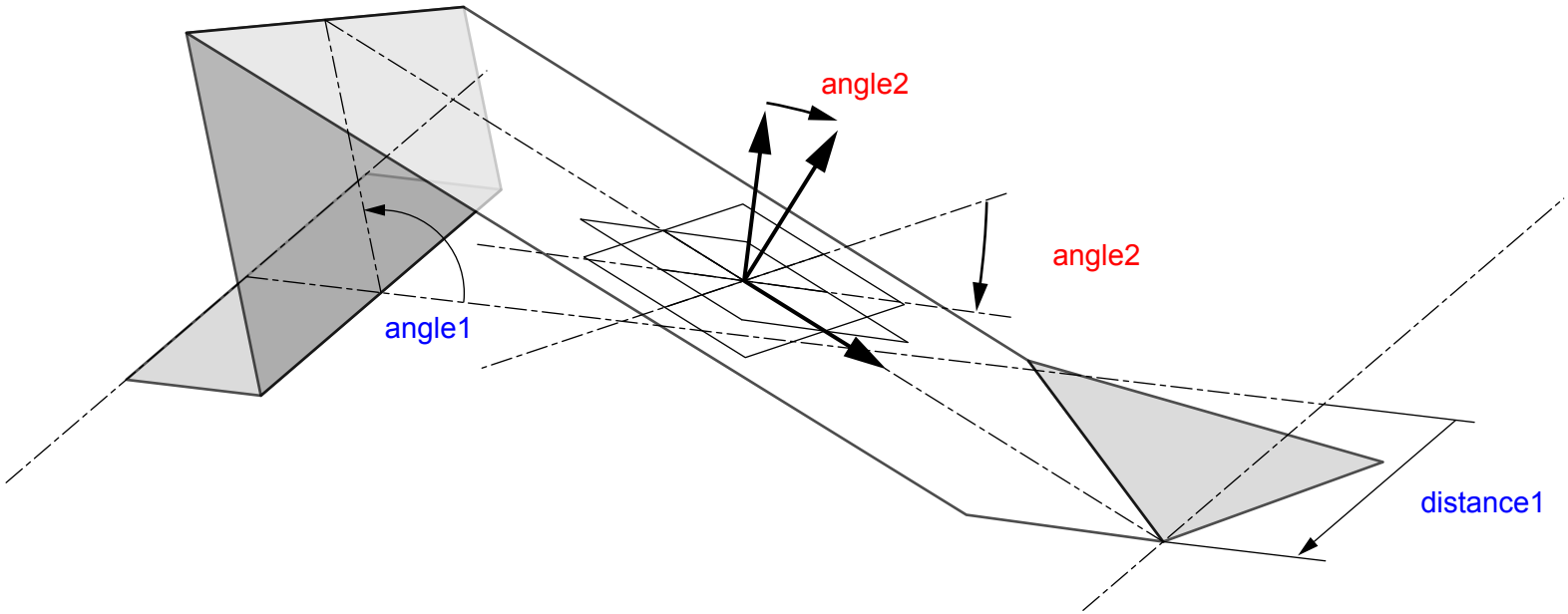
PLANE EXAMPLE : EMERSON COLLEGE SCRIM2

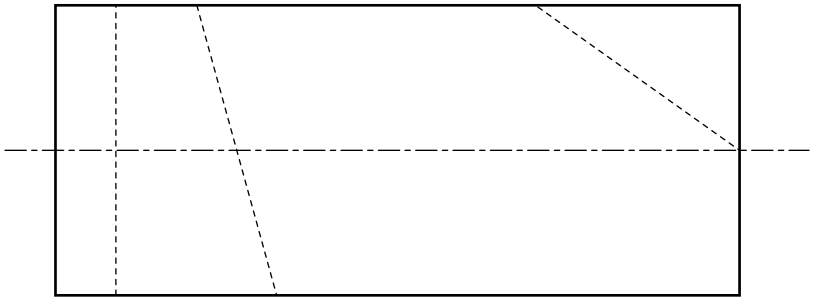
HOW TO MAKE UNFOLDED SHAPE RECTANGULAR?





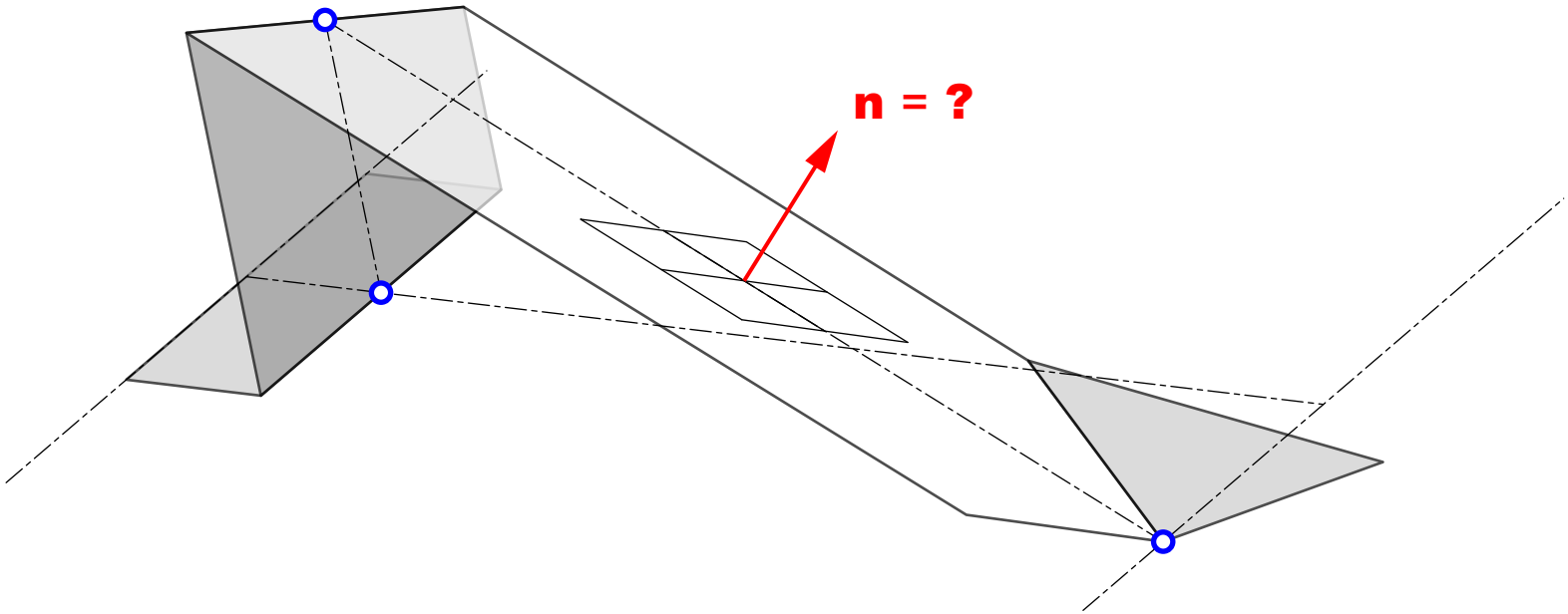
PLANE EXAMPLE : EMERSON COLLEGE SCRIM2

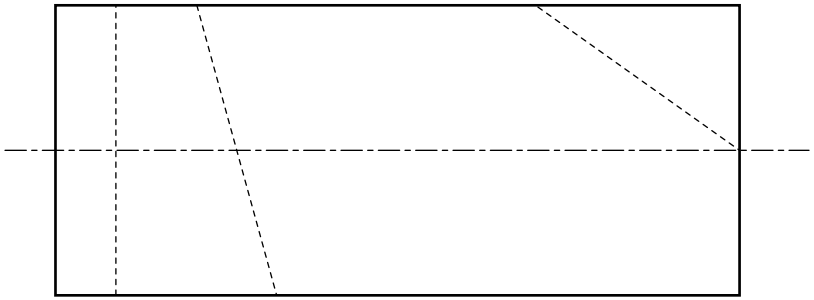




unfolded shape

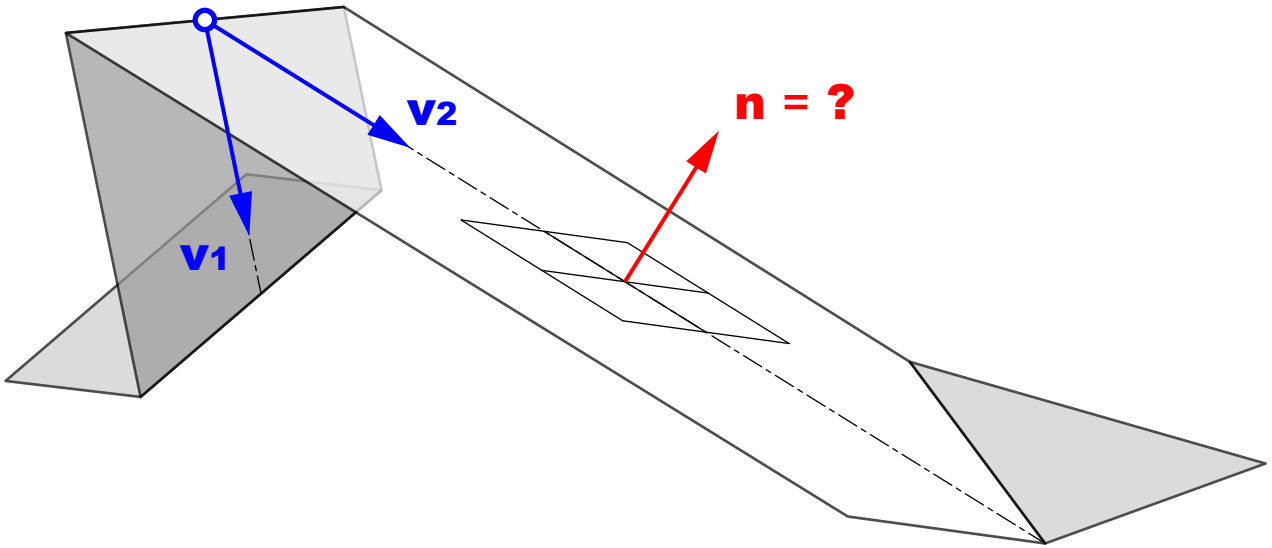
PLANE EXAMPLE : EMERSON COLLEGE SCRIM2

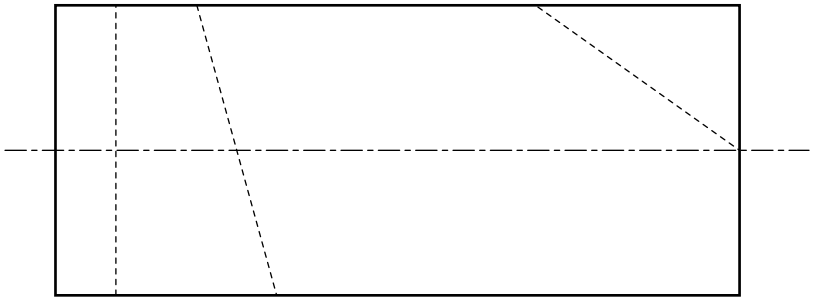




unfolded shape

PLANE EXAMPLE : EMERSON COLLEGE SCRIM2

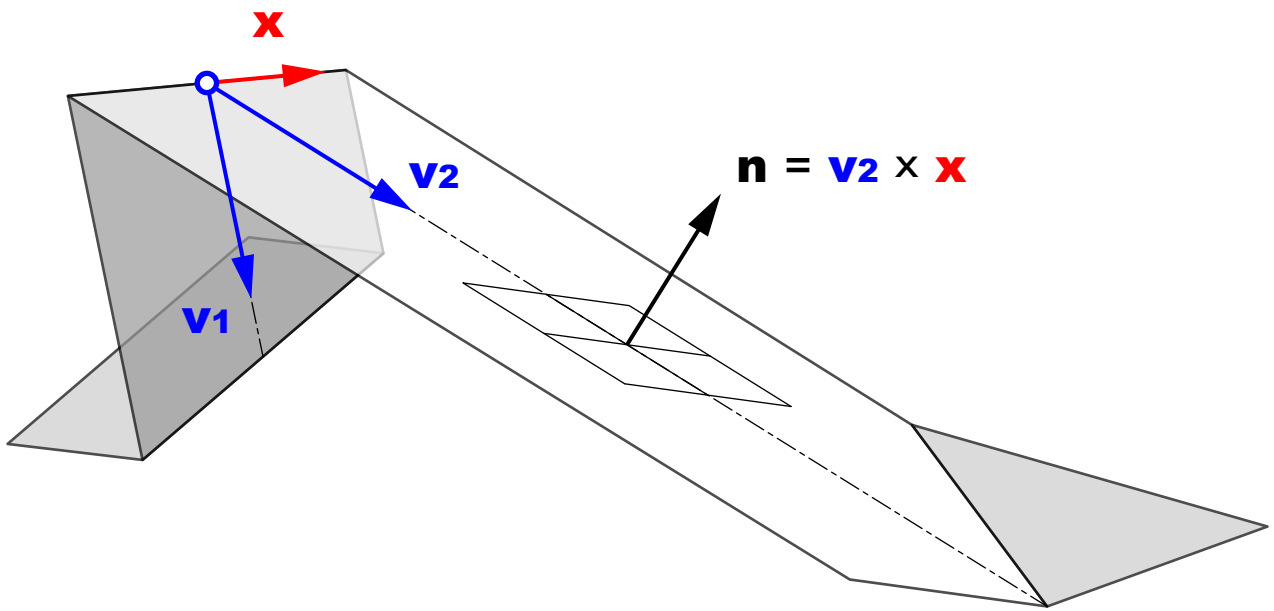


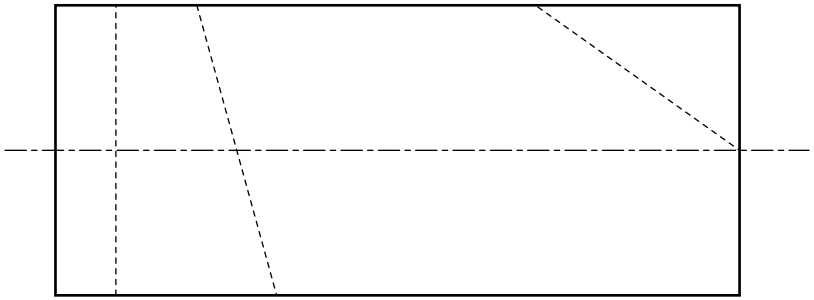


unfolded shape

PLANE EXAMPLE : EMERSON COLLEGE SCRIM2

$|\mathbf{v_1}| = 1, |\mathbf{v_2}| = 1, |\mathbf{x}| = 1$

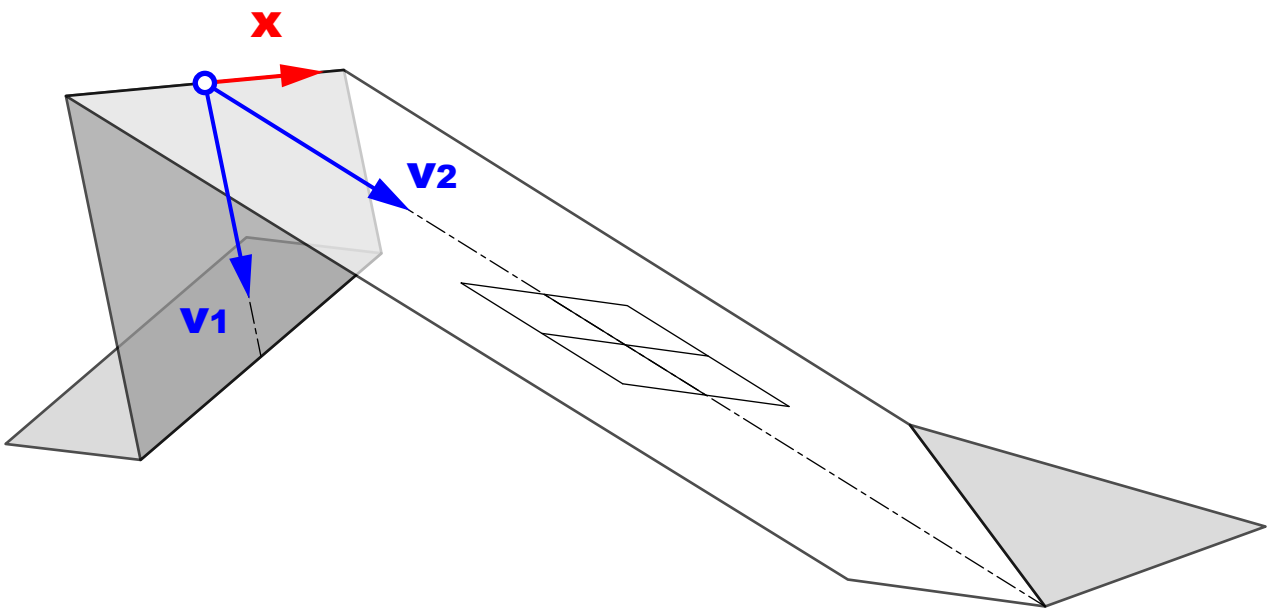


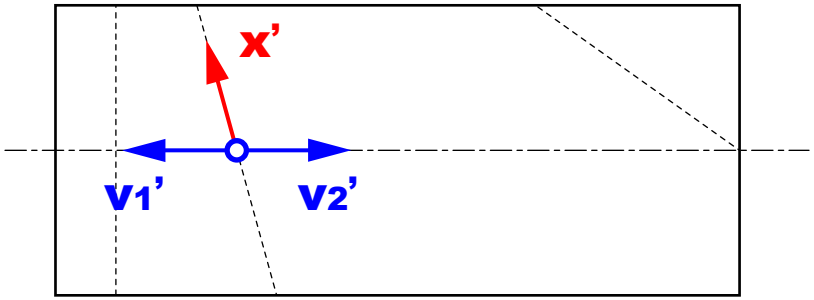


unfolded shape

PLANE EXAMPLE : EMERSON COLLEGE SCRIM2

$|\mathbf{v_1}| = 1, |\mathbf{v_2}| = 1, |\mathbf{x}| = 1$

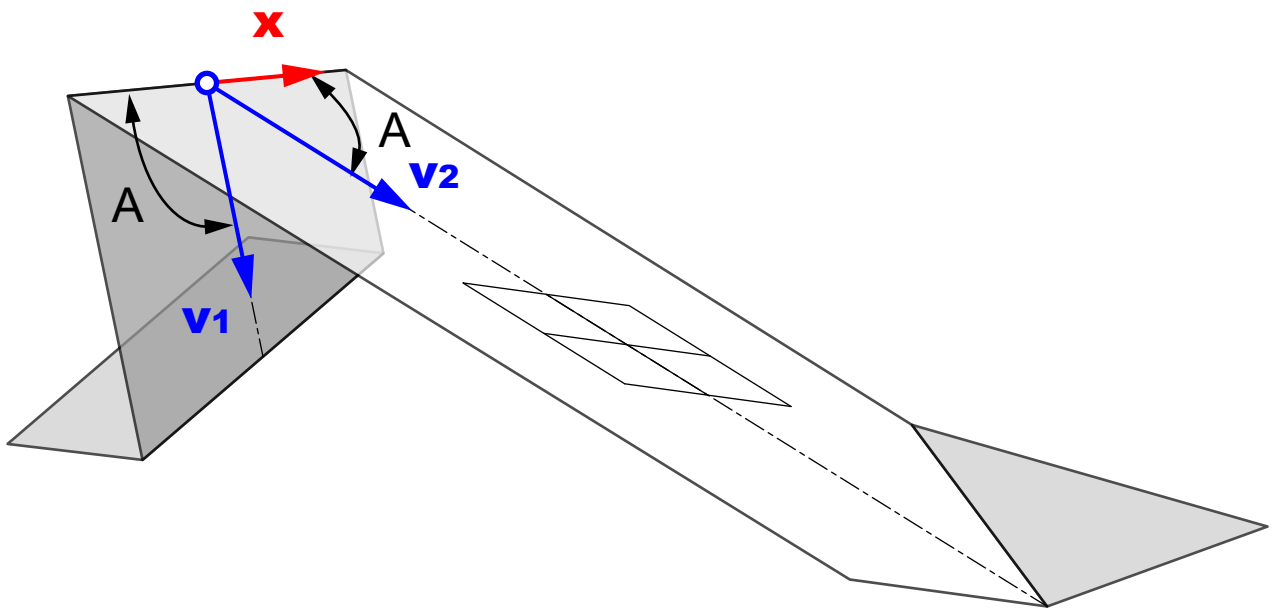


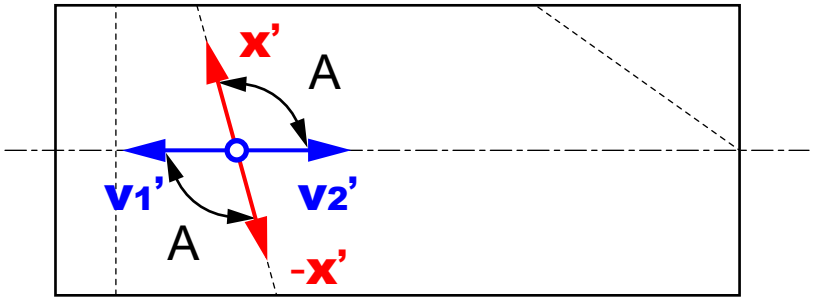


unfolded shape

PLANE EXAMPLE : EMERSON COLLEGE SCRIM2

$|\mathbf{v_1}| = 1, |\mathbf{v_2}| = 1, |\mathbf{x}| = 1$





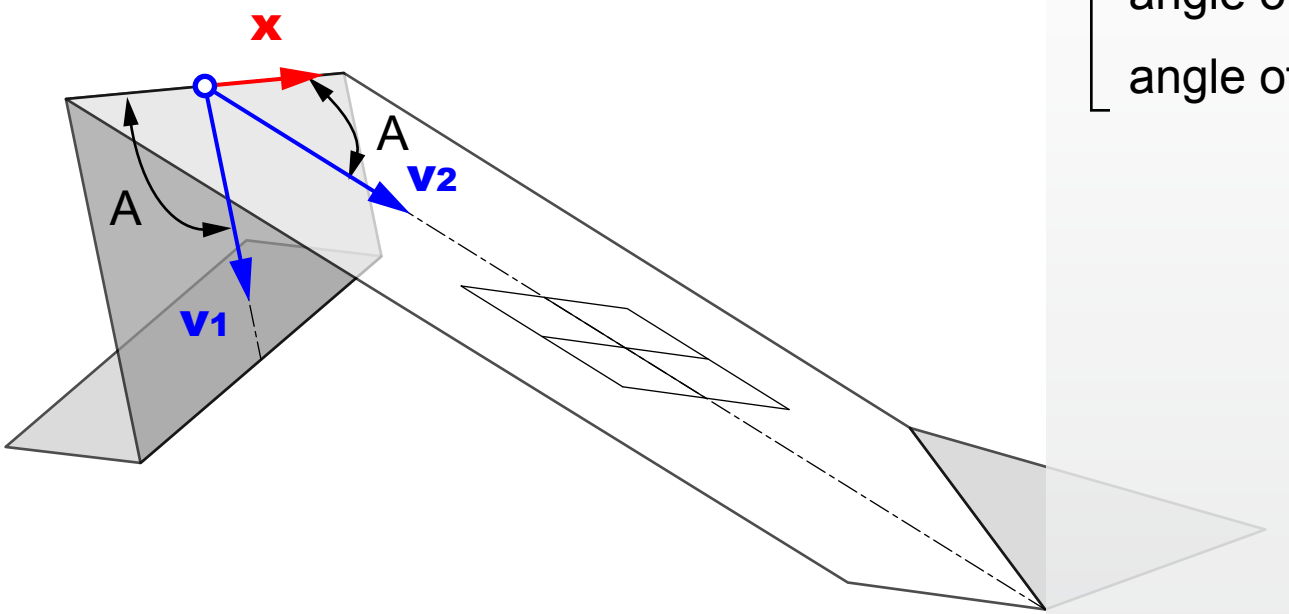
unfolded shape

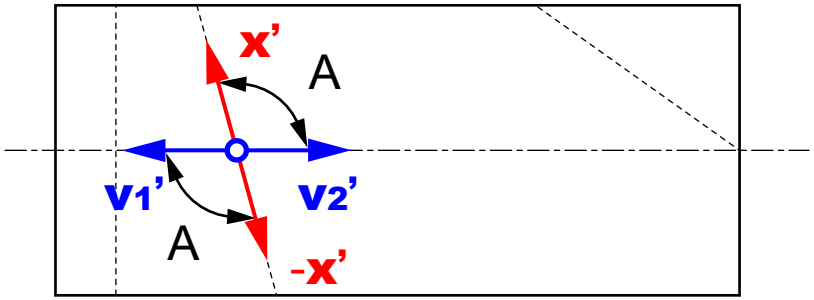
PLANE EXAMPLE : EMERSON COLLEGE SCRIM2

$|\mathbf{v_1}| = 1, |\mathbf{v_2}| = 1, |\mathbf{x}| = 1$

angle of $\mathbf{x}$ and $\mathbf{v_2}$ = A

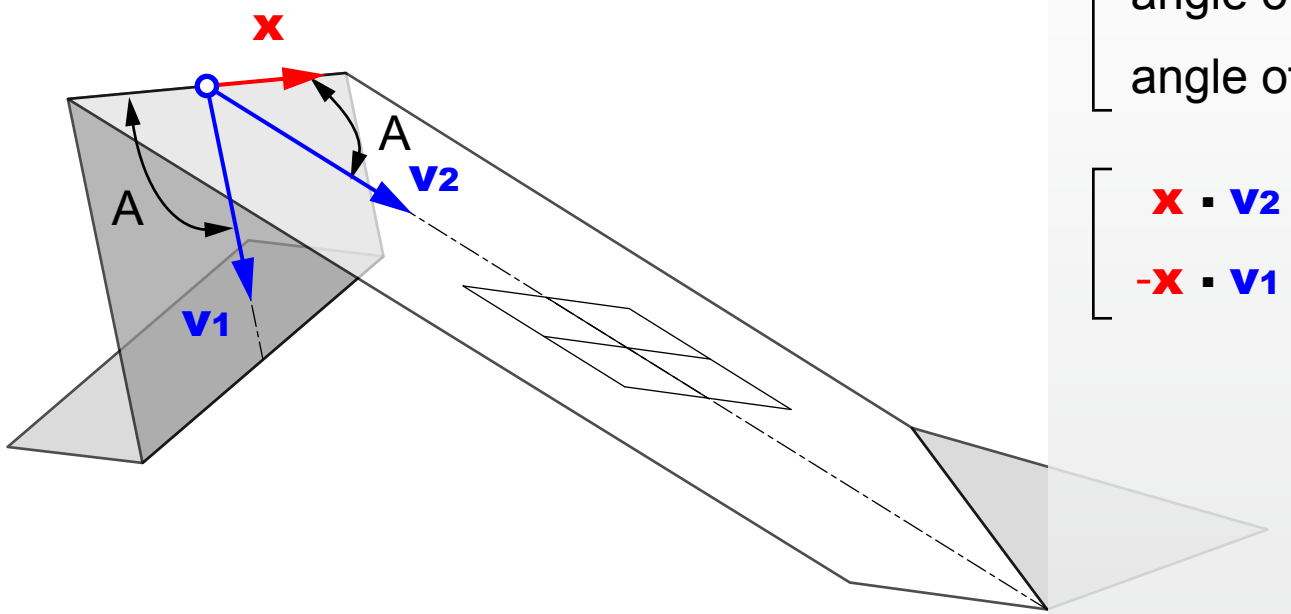
angle of $-\mathbf{x}$ and $\mathbf{v_1}$ = A

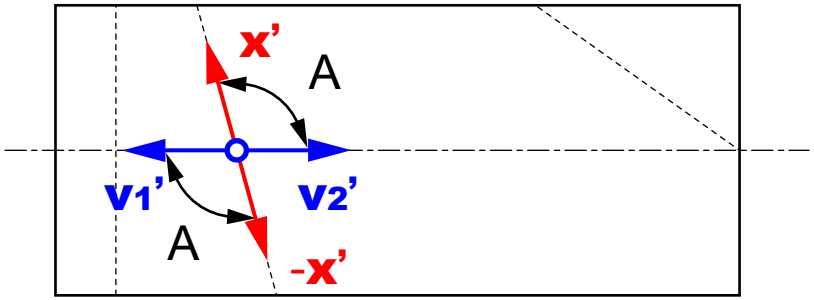




unfolded shape

PLANE EXAMPLE : EMERSON COLLEGE SCRIM2





unfolded shape

$|\mathbf{v_1}| = 1, |\mathbf{v_2}| = 1, |\mathbf{x}| = 1$

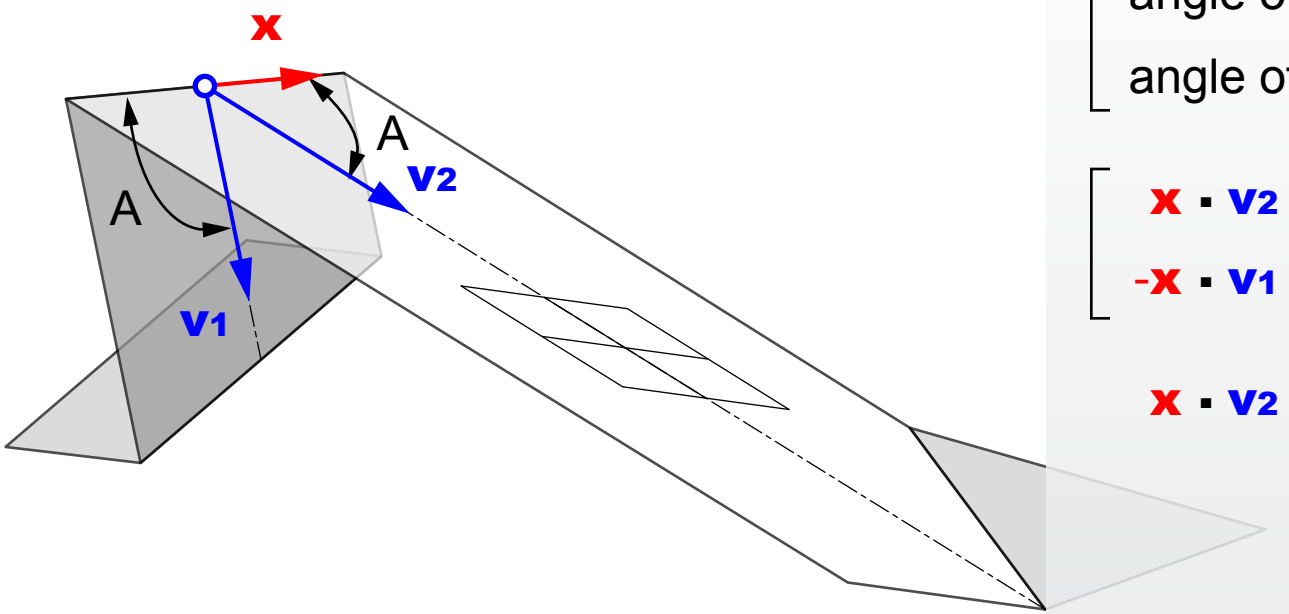
angle of $\mathbf{x}$ and $\mathbf{v_2}$ = A

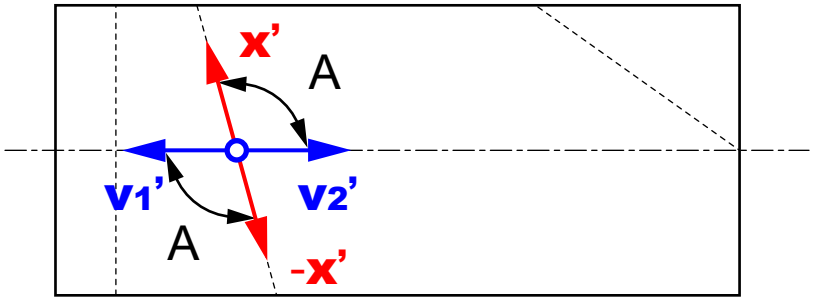
angle of $-\mathbf{x}$ and $\mathbf{v_1}$ = A

$\mathbf{x} \cdot \mathbf{v_2} = \cos A$

$-\mathbf{x} \cdot \mathbf{v_1} = \cos A$

PLANE EXAMPLE : EMERSON COLLEGE SCRIM2





unfolded shape

$|\mathbf{v_1}| = 1, |\mathbf{v_2}| = 1, |\mathbf{x}| = 1$

angle of $\mathbf{x}$ and $\mathbf{v_2}$ = A

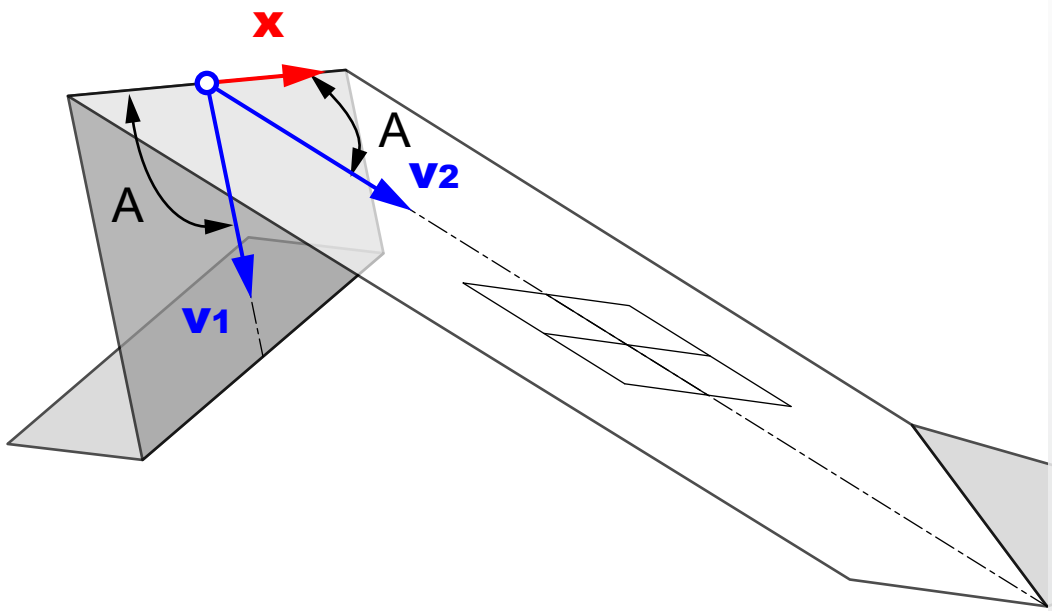
angle of $-\mathbf{x}$ and $\mathbf{v_1}$ = A

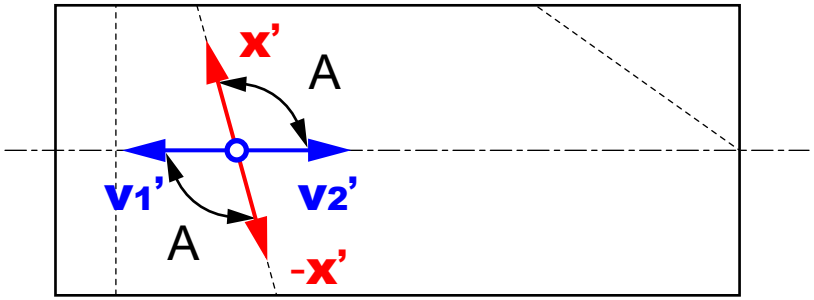
$$\mathbf{x} \cdot \mathbf{v_2} = \cos A$$

$$-\mathbf{x} \cdot \mathbf{v_1} = \cos A$$

$$\mathbf{x} \cdot \mathbf{v_2} = -\mathbf{x} \cdot \mathbf{v_1}$$

PLANE EXAMPLE : EMERSON COLLEGE SCRIM2





unfolded shape

$|\mathbf{v_1}| = 1, |\mathbf{v_2}| = 1, |\mathbf{x}| = 1$

angle of $\mathbf{x}$ and $\mathbf{v_2}$ = A

angle of $-\mathbf{x}$ and $\mathbf{v_1}$ = A

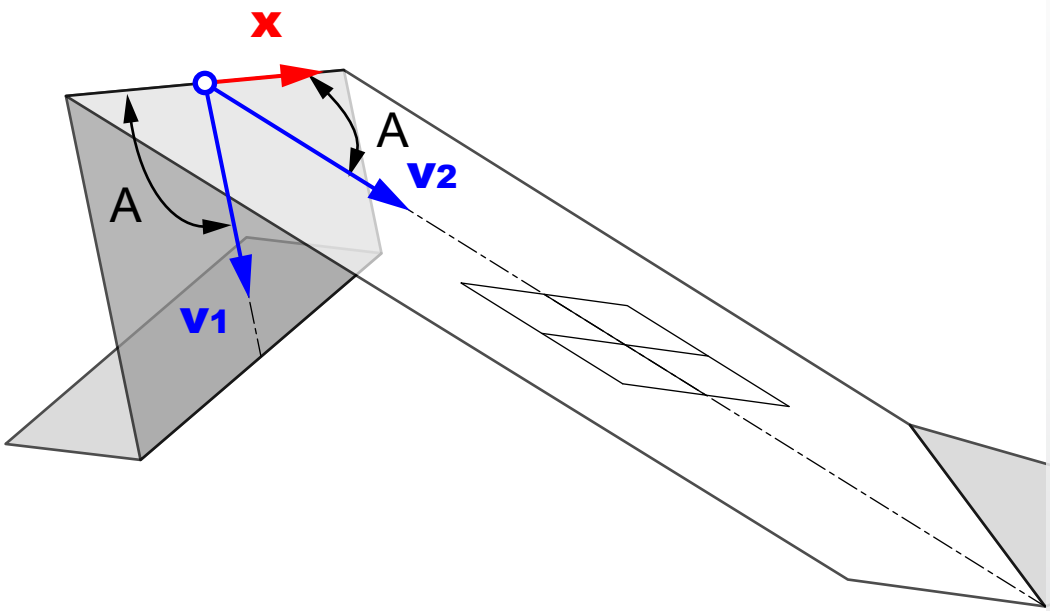
$$\mathbf{x} \cdot \mathbf{v_2} = \cos A$$

$$-\mathbf{x} \cdot \mathbf{v_1} = \cos A$$

$$\mathbf{x} \cdot \mathbf{v_2} = -\mathbf{x} \cdot \mathbf{v_1}$$

$$\mathbf{x} \cdot \mathbf{v_2} + \mathbf{x} \cdot \mathbf{v_1} = 0$$

PLANE EXAMPLE : EMERSON COLLEGE SCRIM2



$|\mathbf{v_1}| = 1, |\mathbf{v_2}| = 1, |\mathbf{x}| = 1$

angle of $\mathbf{x}$ and $\mathbf{v_2}$ = A

angle of $-\mathbf{x}$ and $\mathbf{v_1}$ = A

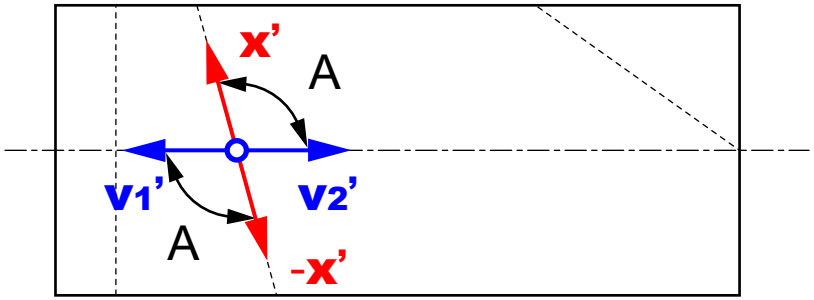
$$\mathbf{x} \cdot \mathbf{v_2} = \cos A$$

$$-\mathbf{x} \cdot \mathbf{v_1} = \cos A$$

$$\mathbf{x} \cdot \mathbf{v_2} = -\mathbf{x} \cdot \mathbf{v_1}$$

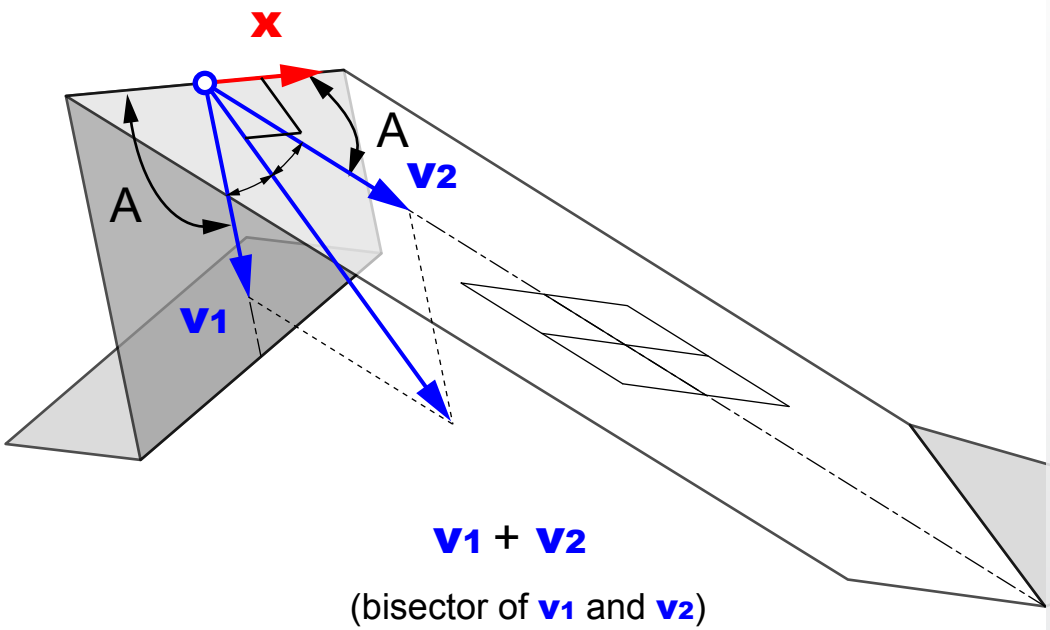
$$\mathbf{x} \cdot \mathbf{v_2} + \mathbf{x} \cdot \mathbf{v_1} = 0$$

$$\mathbf{x} \cdot (\mathbf{v_2} + \mathbf{v_1}) = 0$$



unfolded shape

PLANE EXAMPLE : EMERSON COLLEGE SCRIM2



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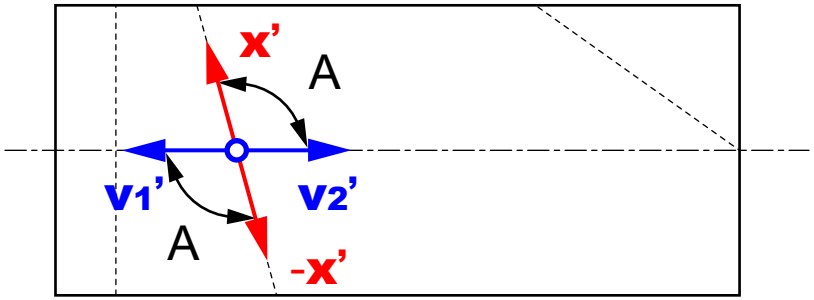
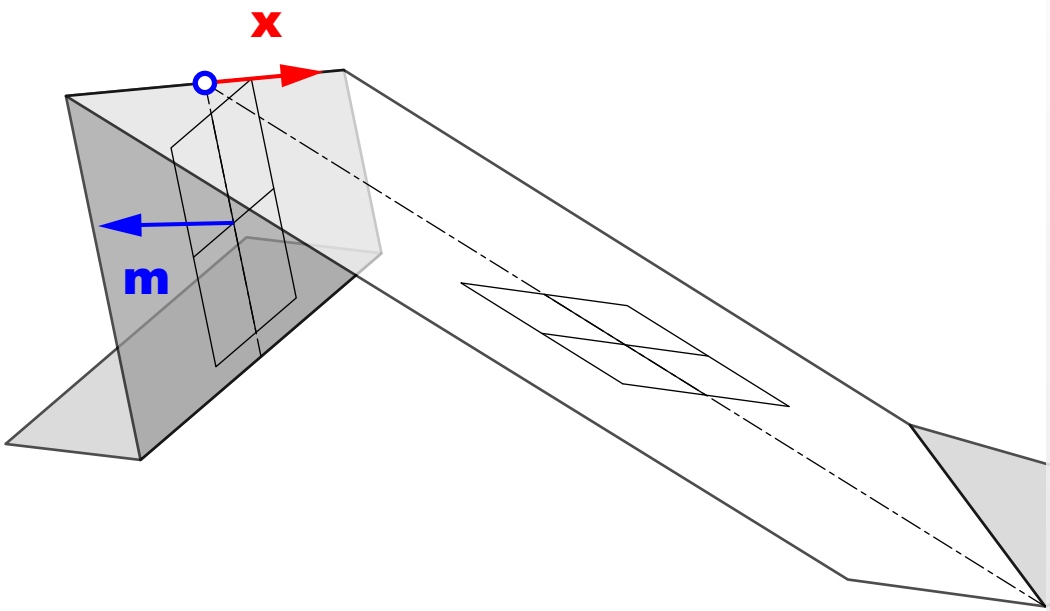
$$\mathbf{x} \cdot (\mathbf{v_2} + \mathbf{v_1}) = 0$$



unfolded shape

PLANE EXAMPLE : EMERSON COLLEGE SCRIM2



$|\mathbf{v_1}| = 1, |\mathbf{v_2}| = 1, |\mathbf{x}| = 1$

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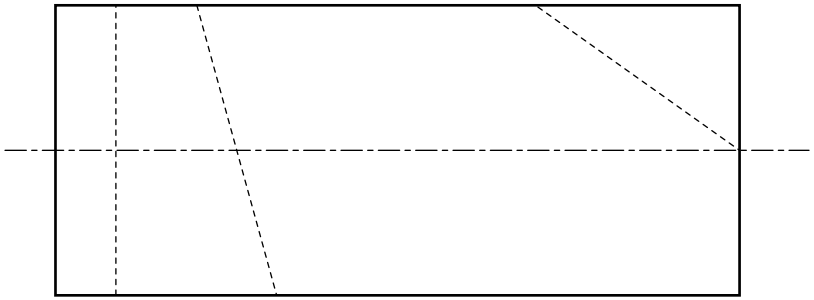
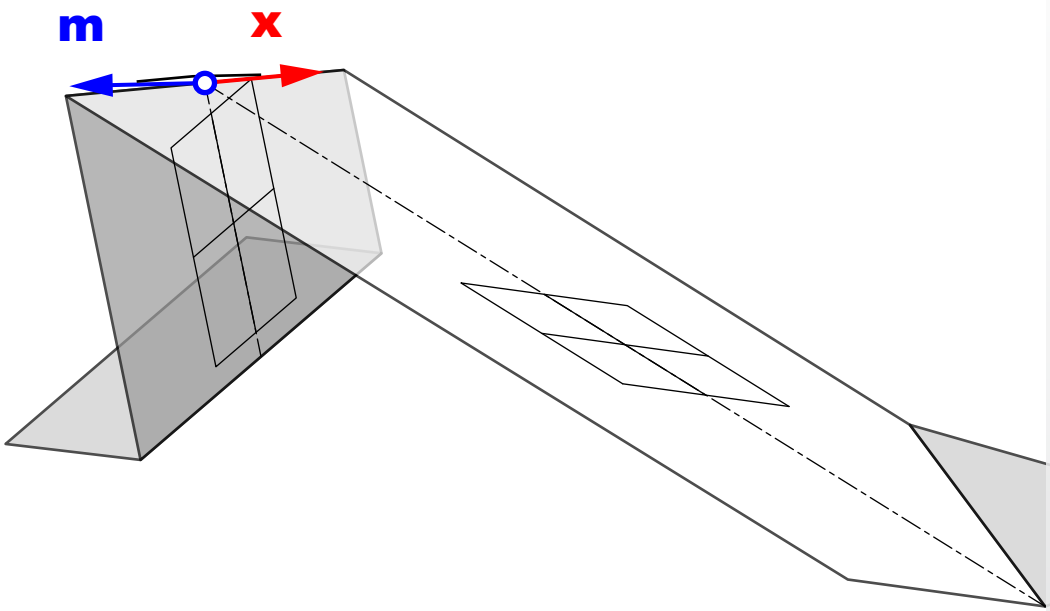
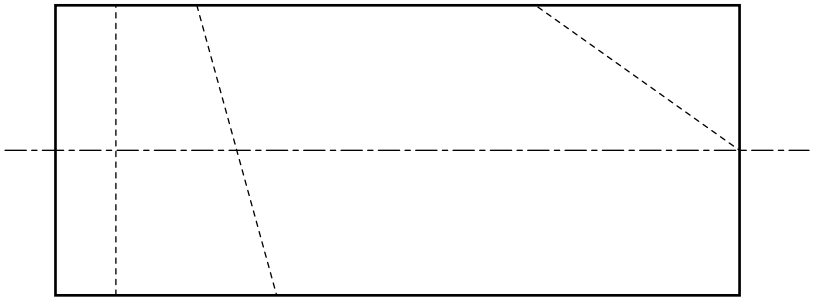
$$\mathbf{x} \cdot (\mathbf{v_2} + \mathbf{v_1}) = 0$$



unfolded shape

PLANE EXAMPLE : EMERSON COLLEGE SCRIM2





unfolded shape

$|\mathbf{v_1}| = 1, |\mathbf{v_2}| = 1, |\mathbf{x}| = 1$

angle of $\mathbf{x}$ and $\mathbf{v_2}$ = A

angle of $-\mathbf{x}$ and $\mathbf{v_1}$ = A

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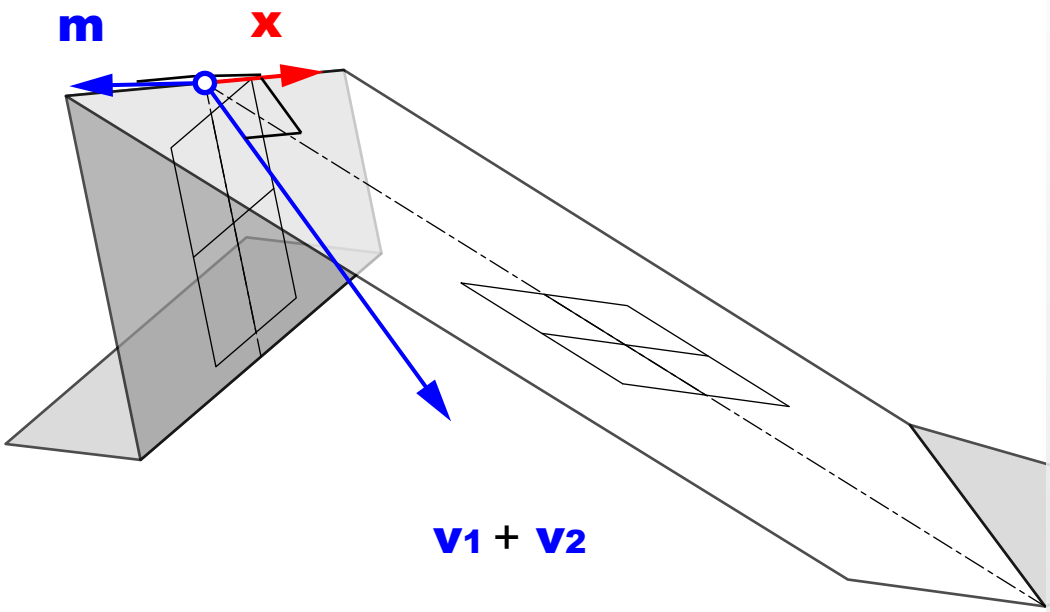
$$\mathbf{x} \cdot (\mathbf{v_2} + \mathbf{v_1}) = 0$$

$$\mathbf{x} \cdot \mathbf{m} = 0$$

PLANE EXAMPLE : EMERSON COLLEGE SCRIM2



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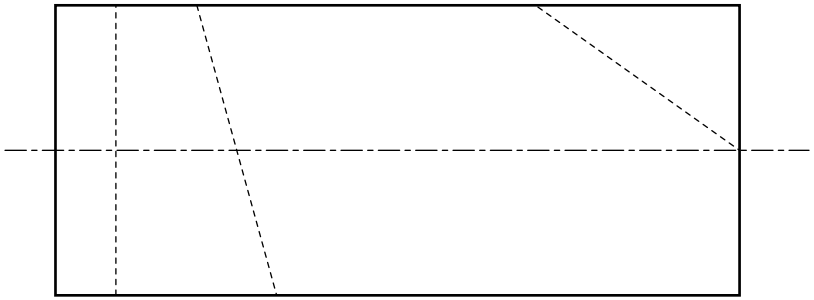
$$\mathbf{x} \cdot \mathbf{v_2} + \mathbf{x} \cdot \mathbf{v_1} = 0$$

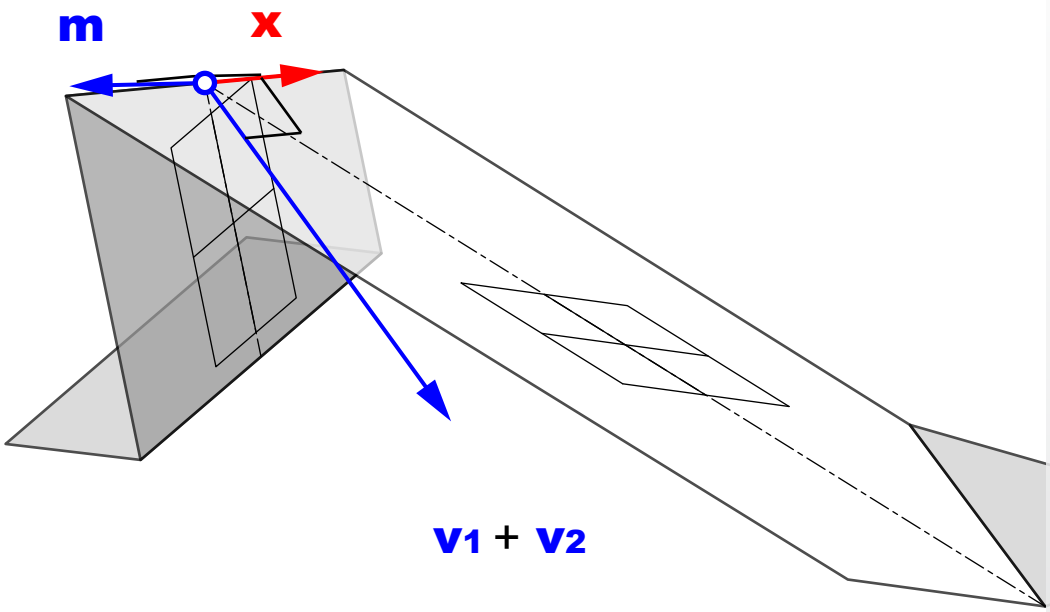
$$\mathbf{x} \cdot (\mathbf{v_2} + \mathbf{v_1}) = 0$$

$$\mathbf{x} \cdot \mathbf{m} = 0$$



unfolded shape

PLANE EXAMPLE : EMERSON COLLEGE SCRIM2



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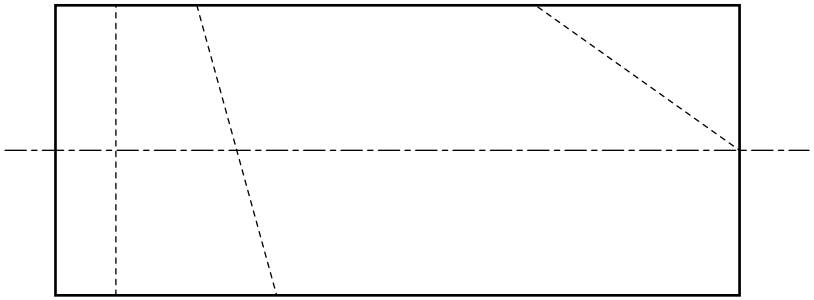
$$\mathbf{x} \cdot (\mathbf{v_2} + \mathbf{v_1}) = 0$$

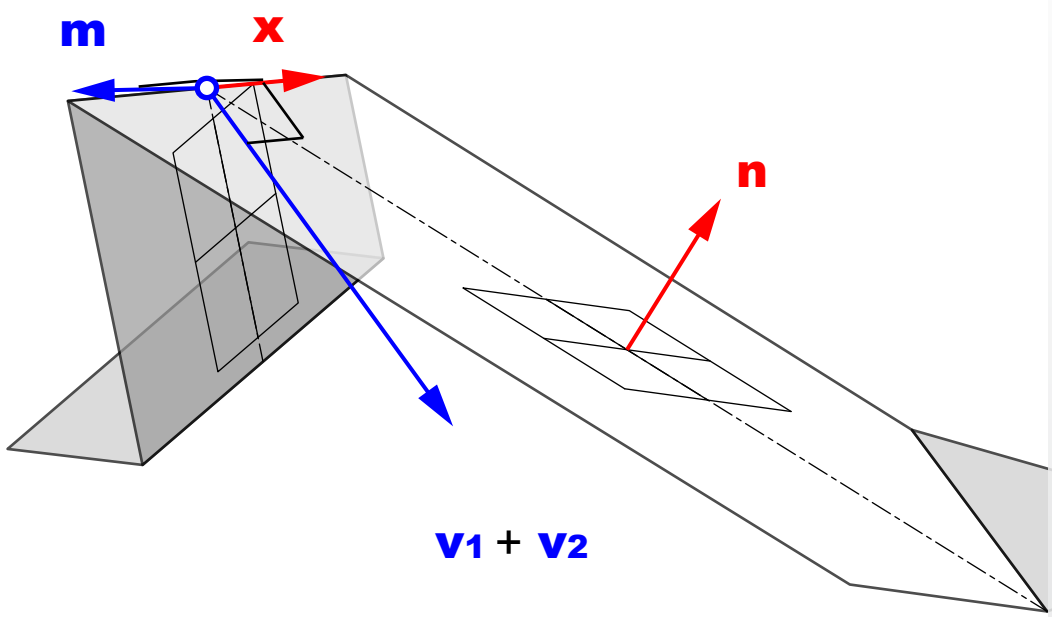
$$\mathbf{x} \cdot \mathbf{m} = 0$$

$$\mathbf{x} = \mathbf{m} \times (\mathbf{v_2} + \mathbf{v_1})$$



unfolded shape

PLANE EXAMPLE : EMERSON COLLEGE SCRIM2



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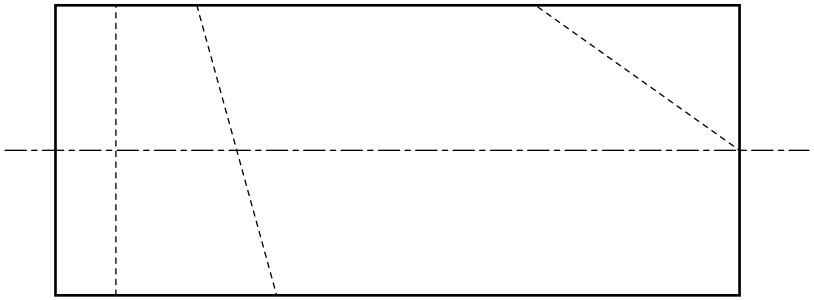
$$\mathbf{x} \cdot \mathbf{m} = 0$$

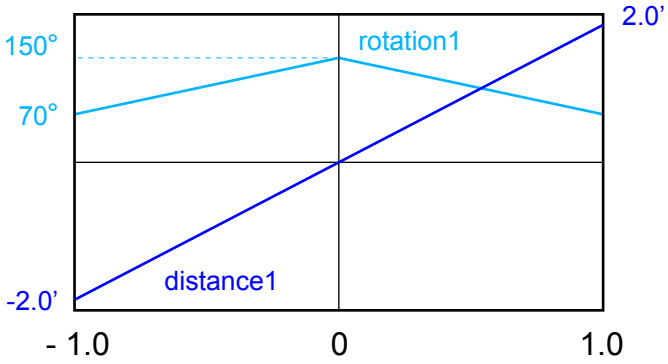
$$\mathbf{x} = \mathbf{m} \times (\mathbf{v_2} + \mathbf{v_1})$$

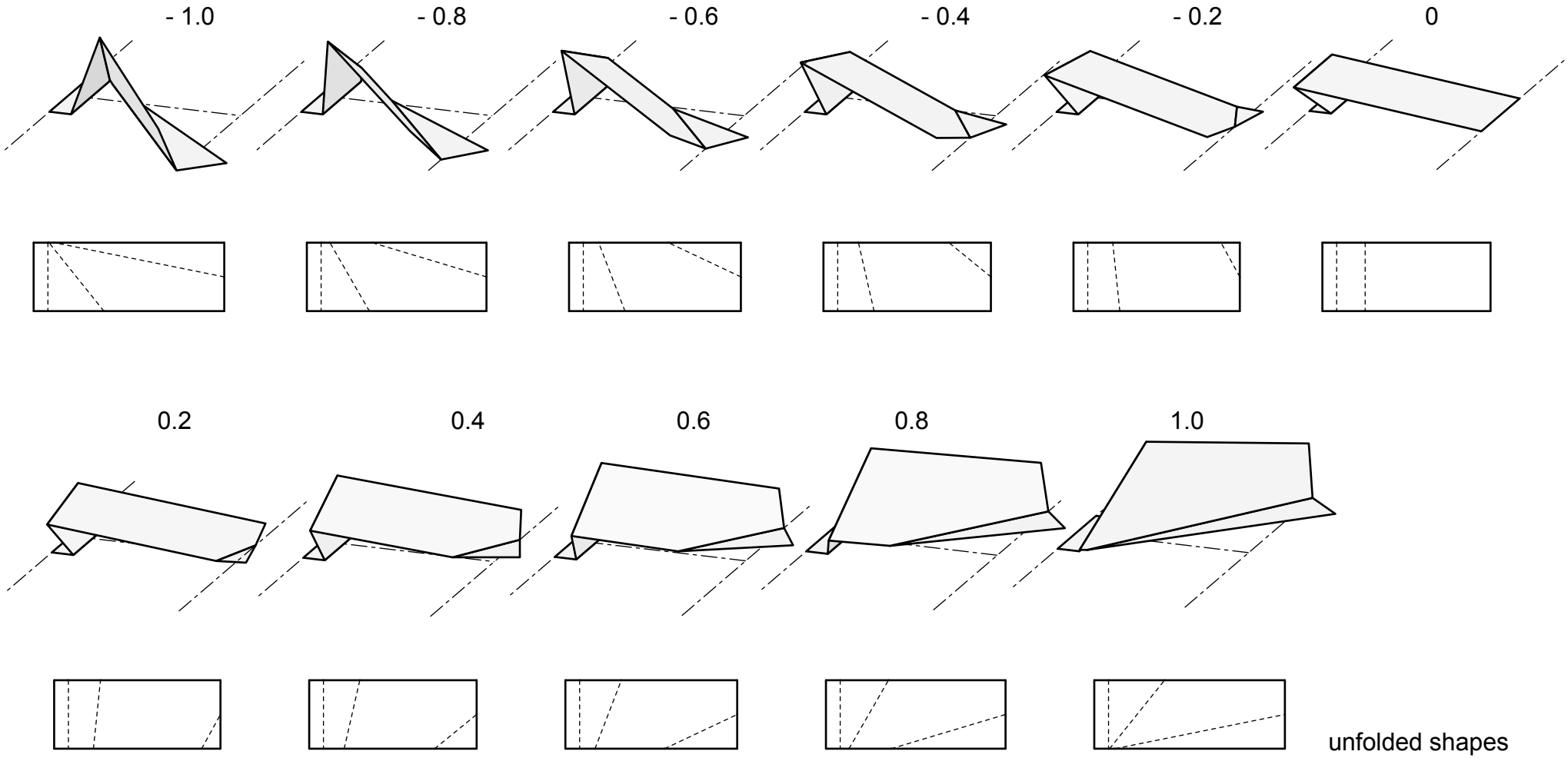
$$\mathbf{n} = \mathbf{v_2} \times (\mathbf{m} \times (\mathbf{v_2} + \mathbf{v_1}))$$



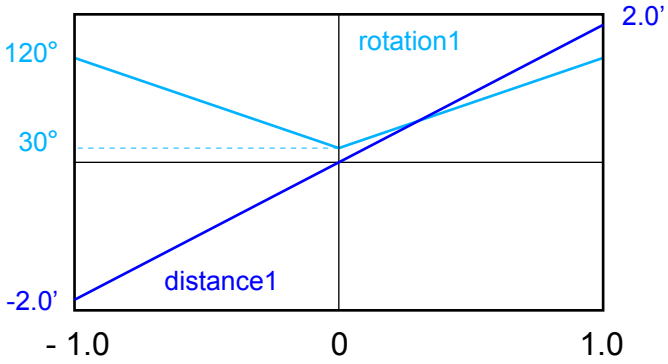
unfolded shape

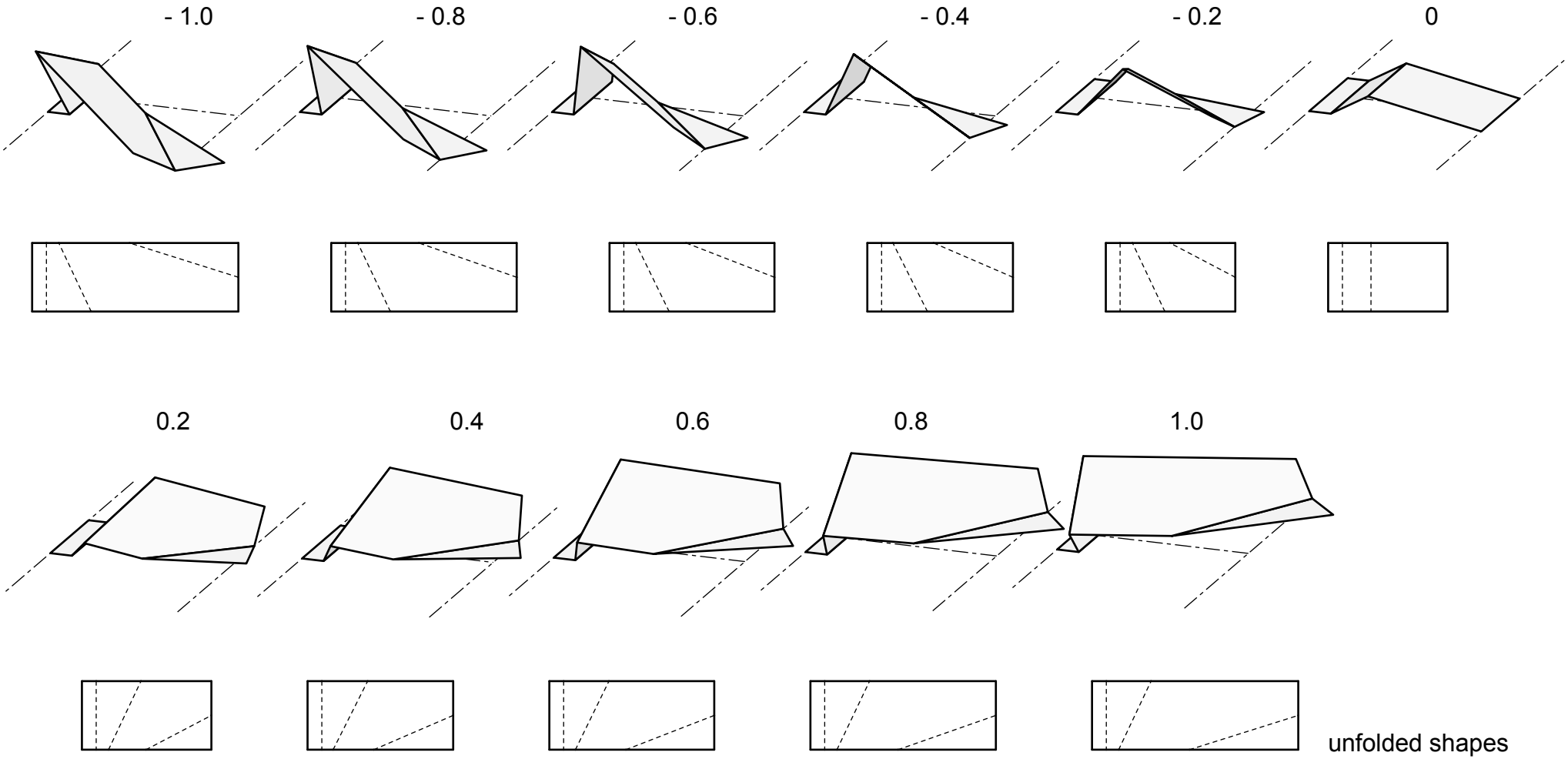
PLANE EXAMPLE : EMERSON COLLEGE SCRIM2



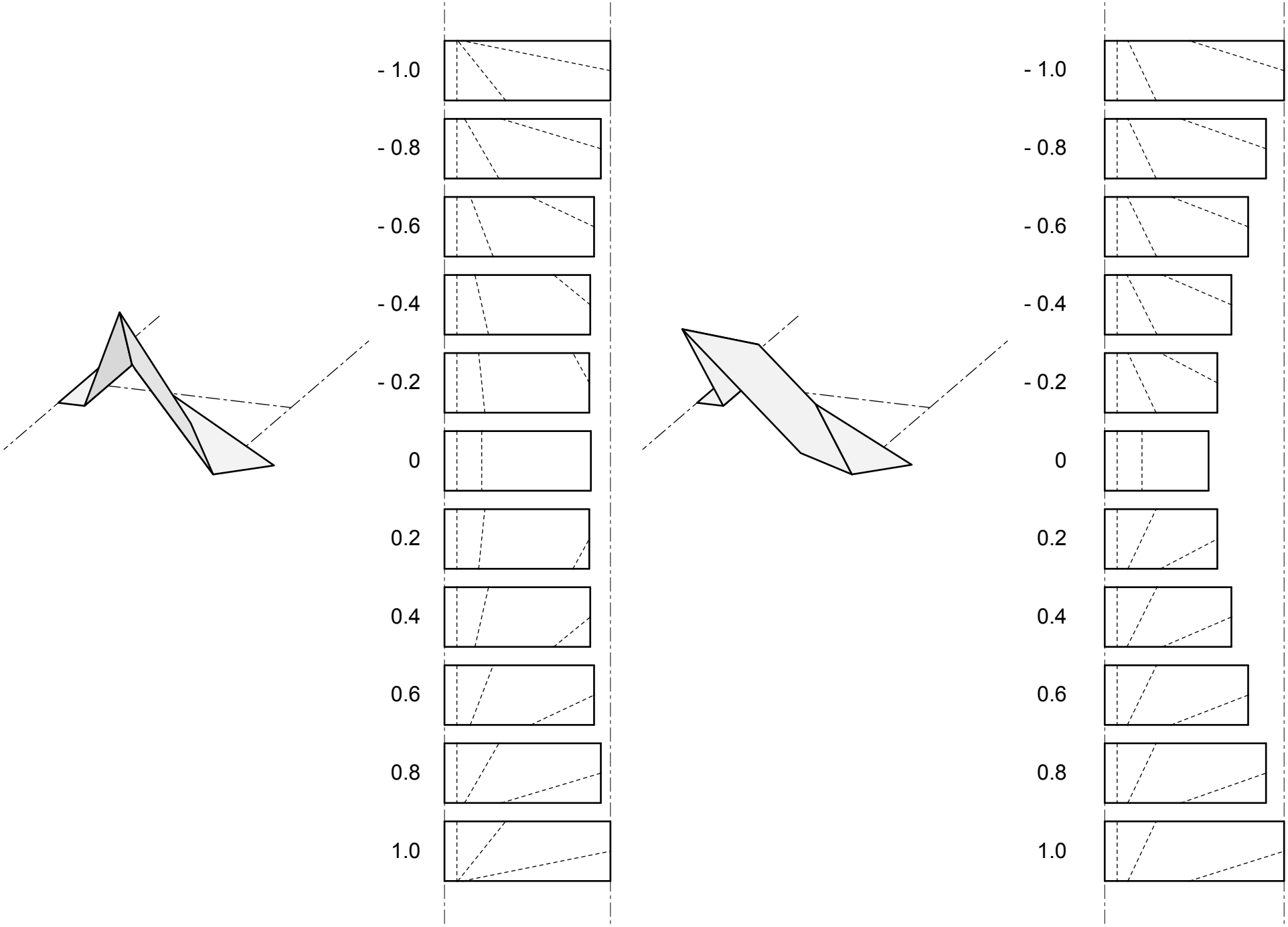


PLANE EXAMPLE : EMERSON COLLEGE SCRIM2

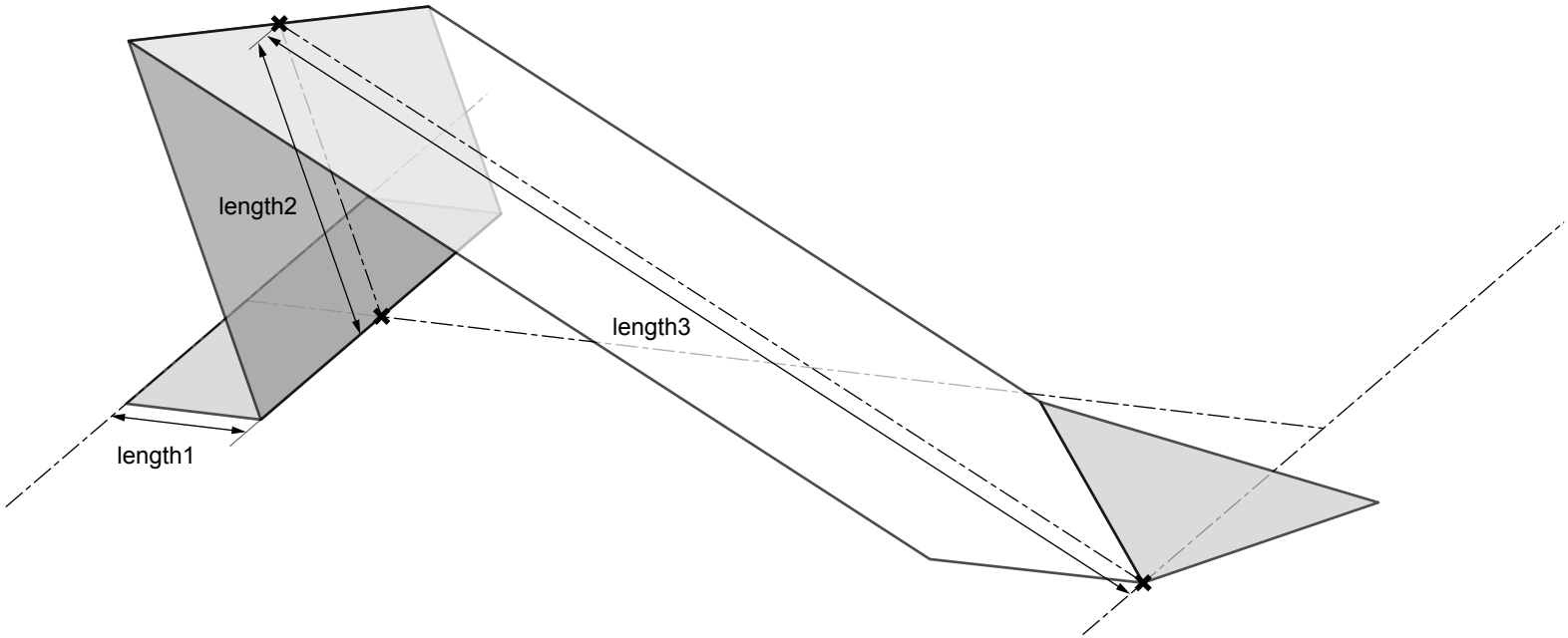


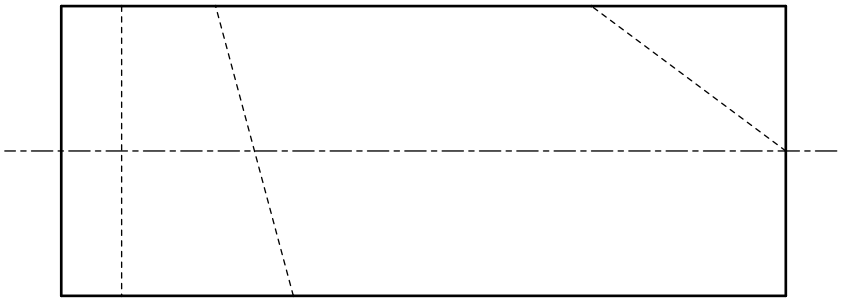


PLANE EXAMPLE : EMERSON COLLEGE SCRIM2



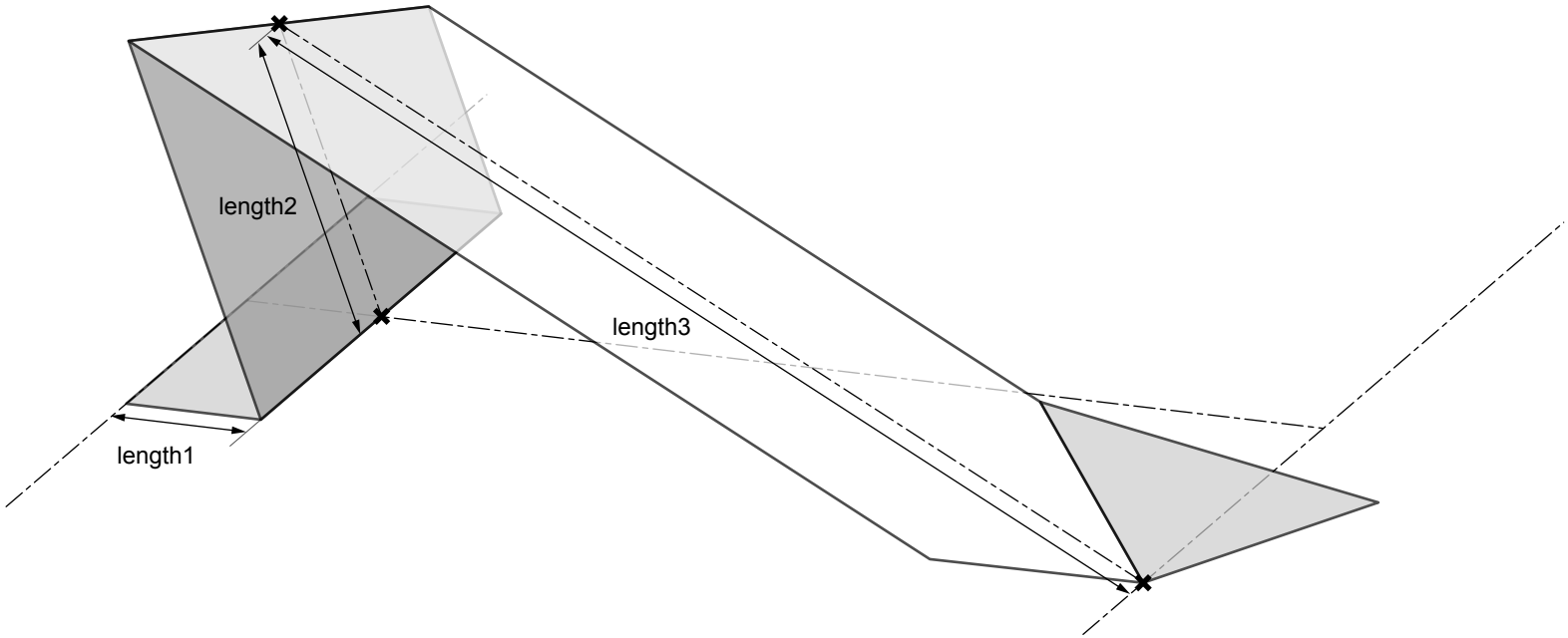
PLANE EXAMPLE : EMERSON COLLEGE SCRIM3

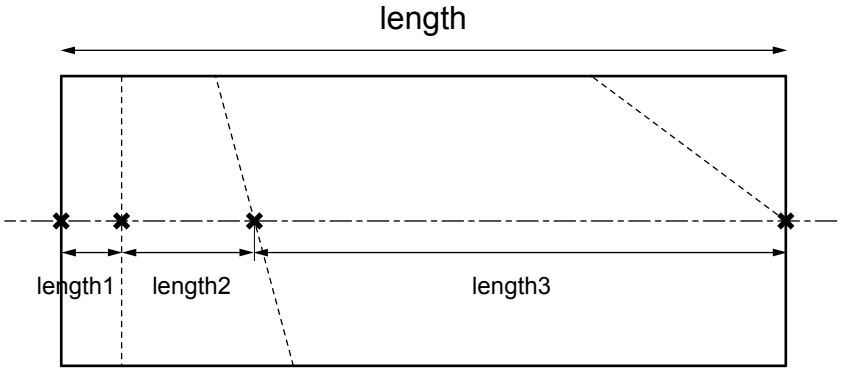




unfolded shape

PLANE EXAMPLE : EMERSON COLLEGE SCRIM3

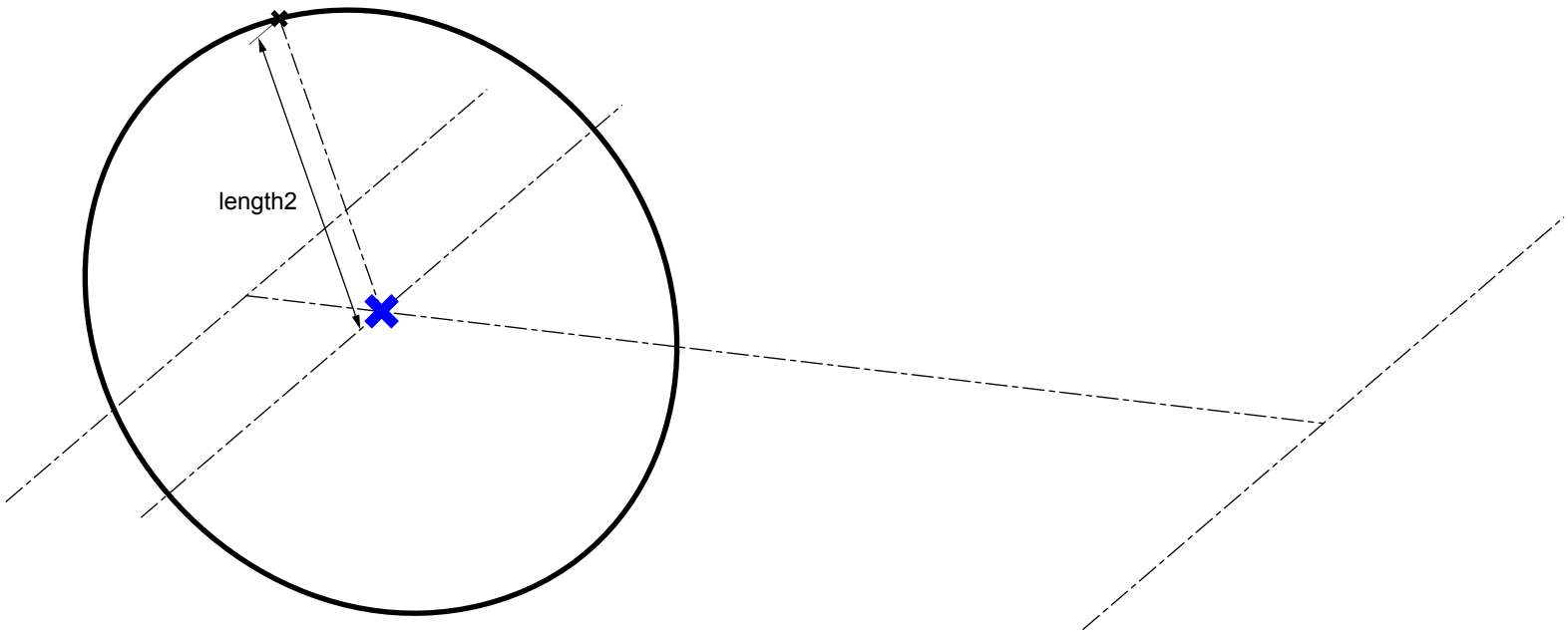


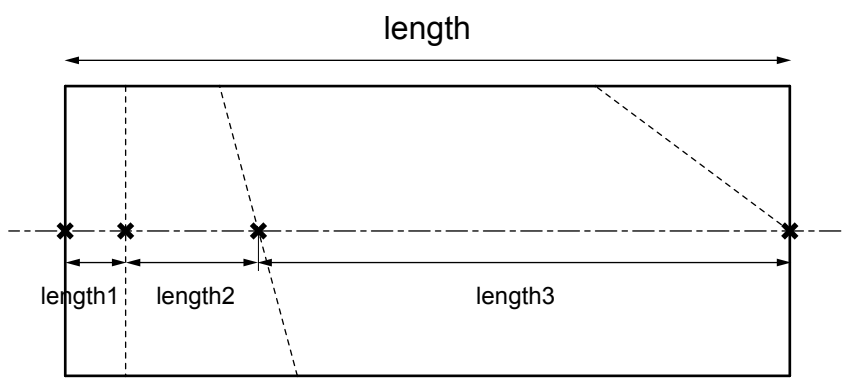


length = length1 + length2 + length3

unfolded shape

PLANE EXAMPLE : EMERSON COLLEGE SCRIM3

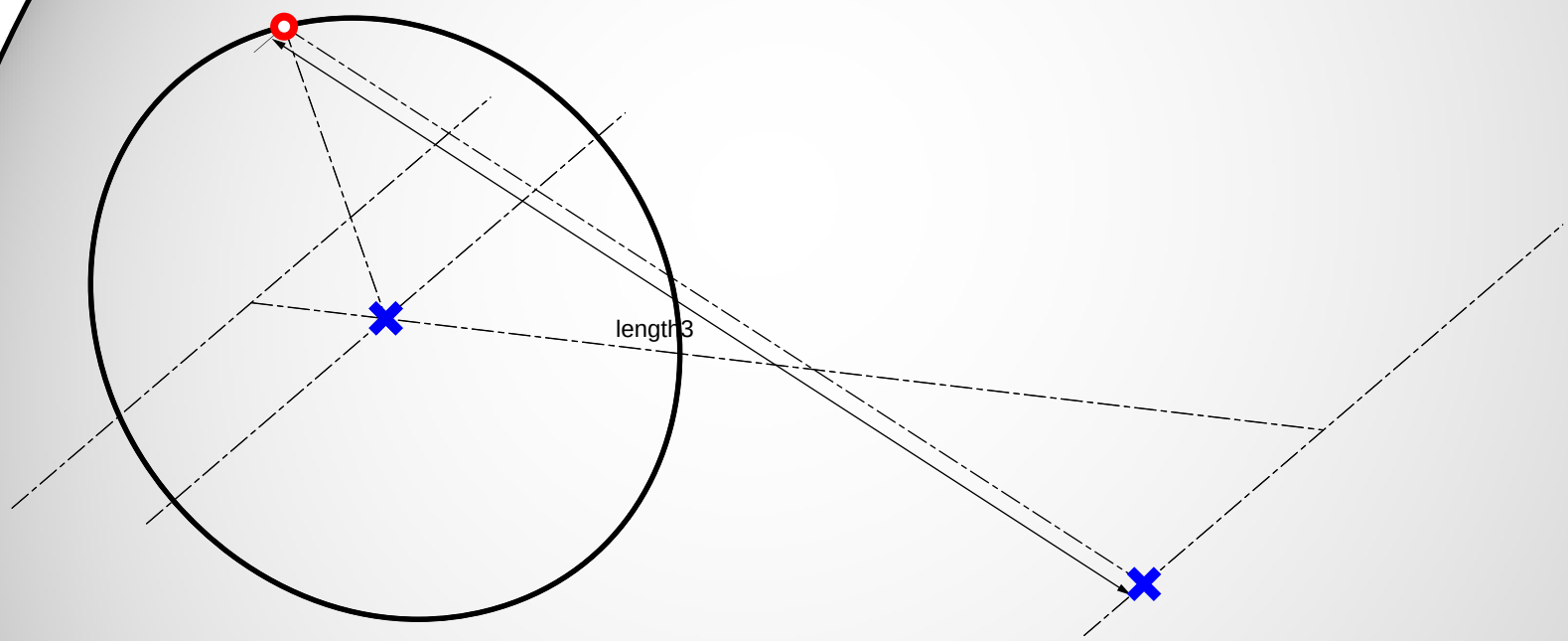


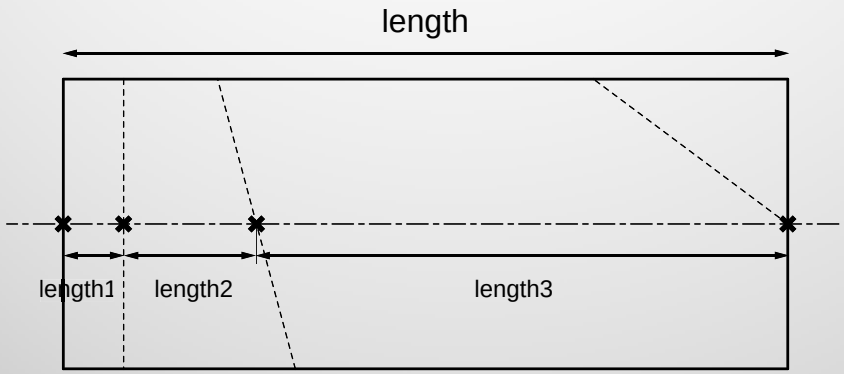


length = length1 + length2 + length3

unfolded shape

PLANE EXAMPLE : EMERSON COLLEGE SCRIM3





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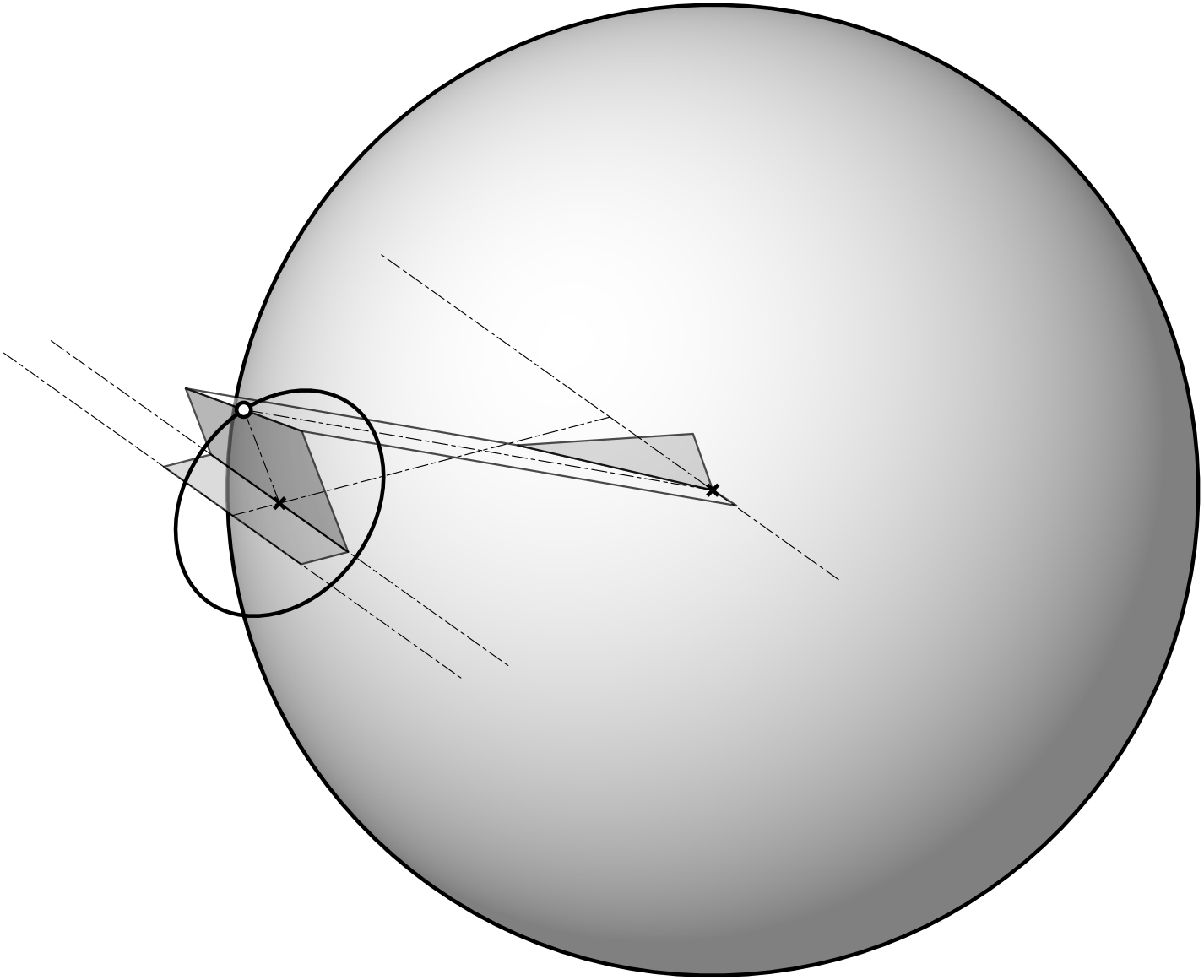
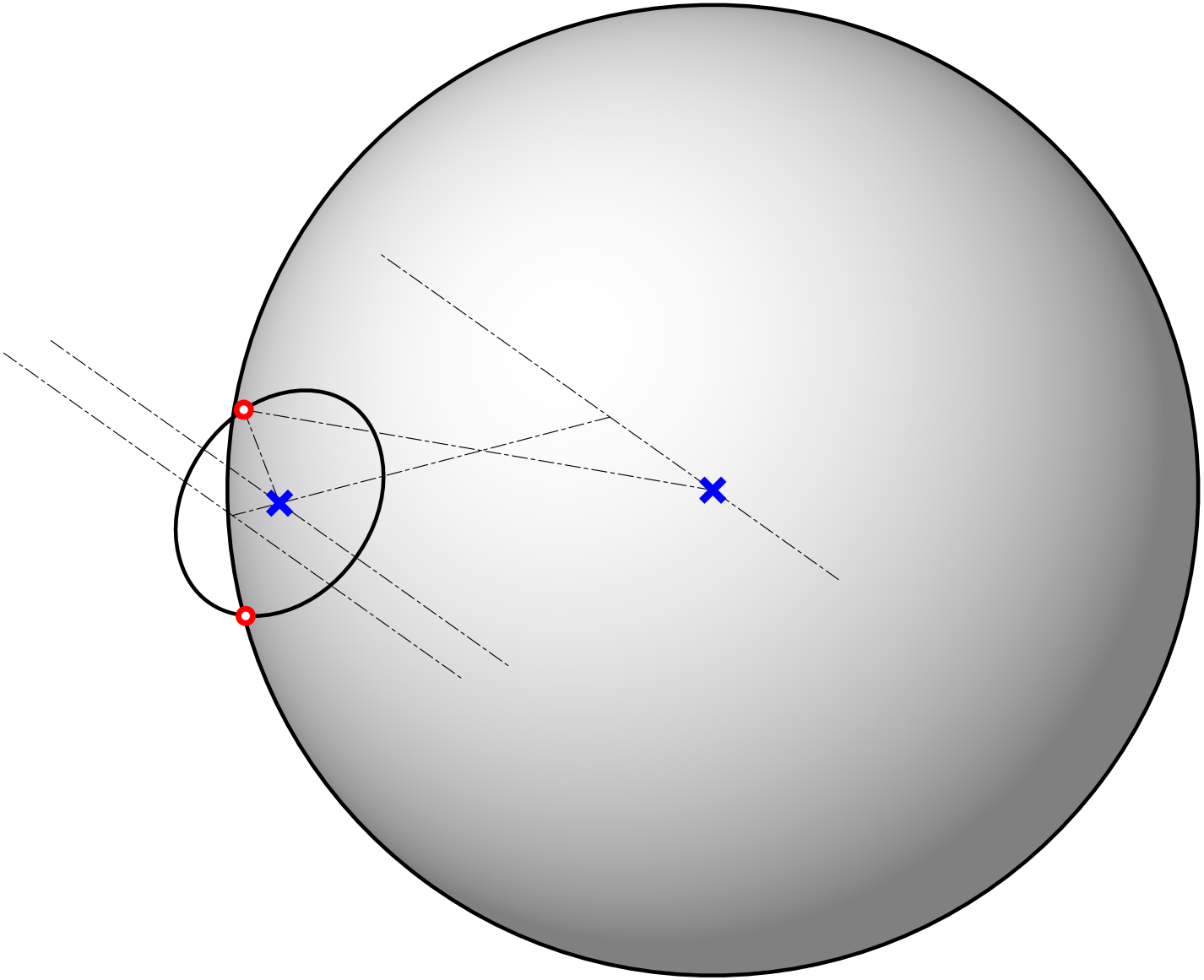
unfolded shape

PLANE EXAMPLE : EMERSON COLLEGE SCRIM3



PLANE EXAMPLE : EMERSON COLLEGE SCRIM3



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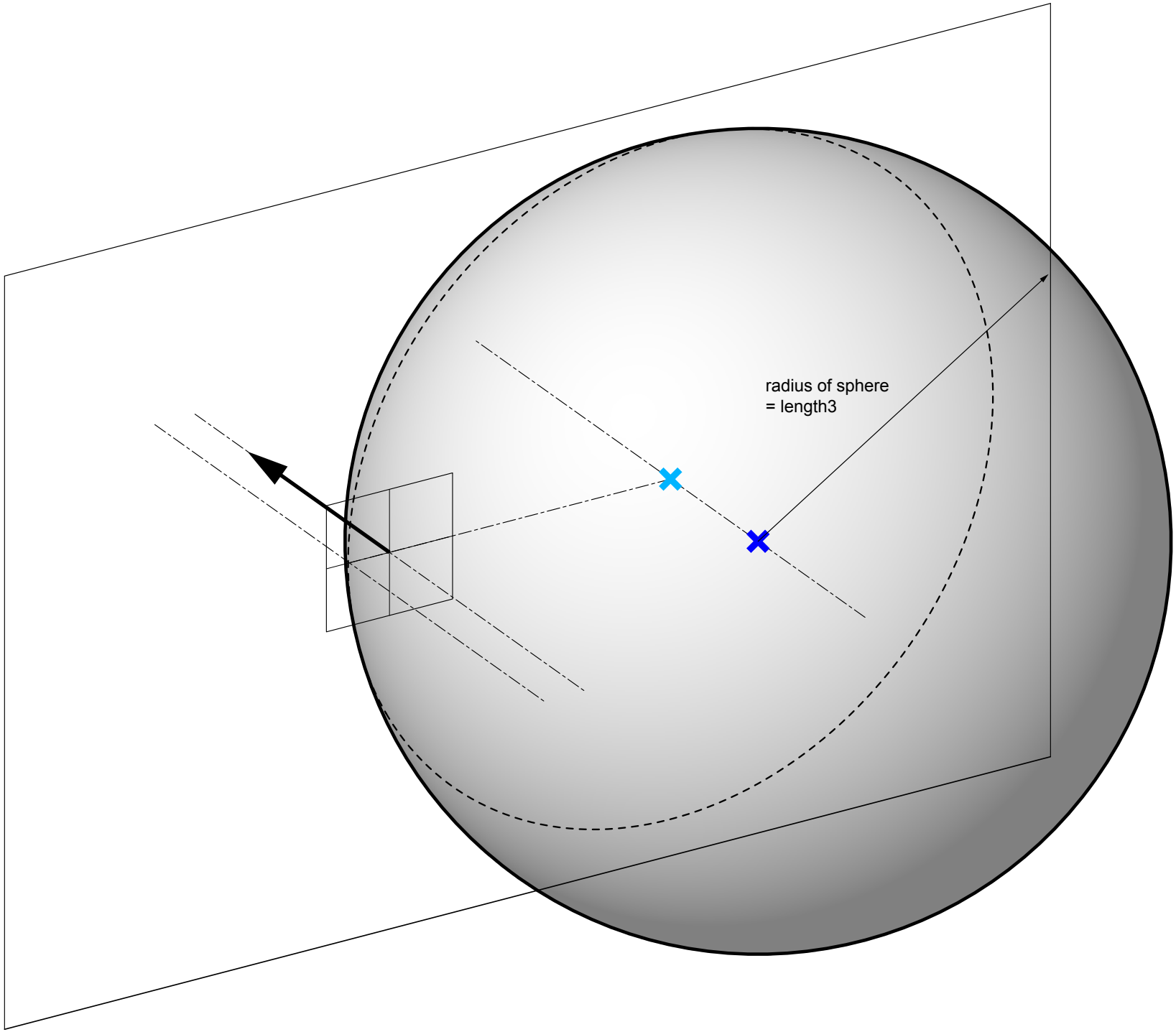
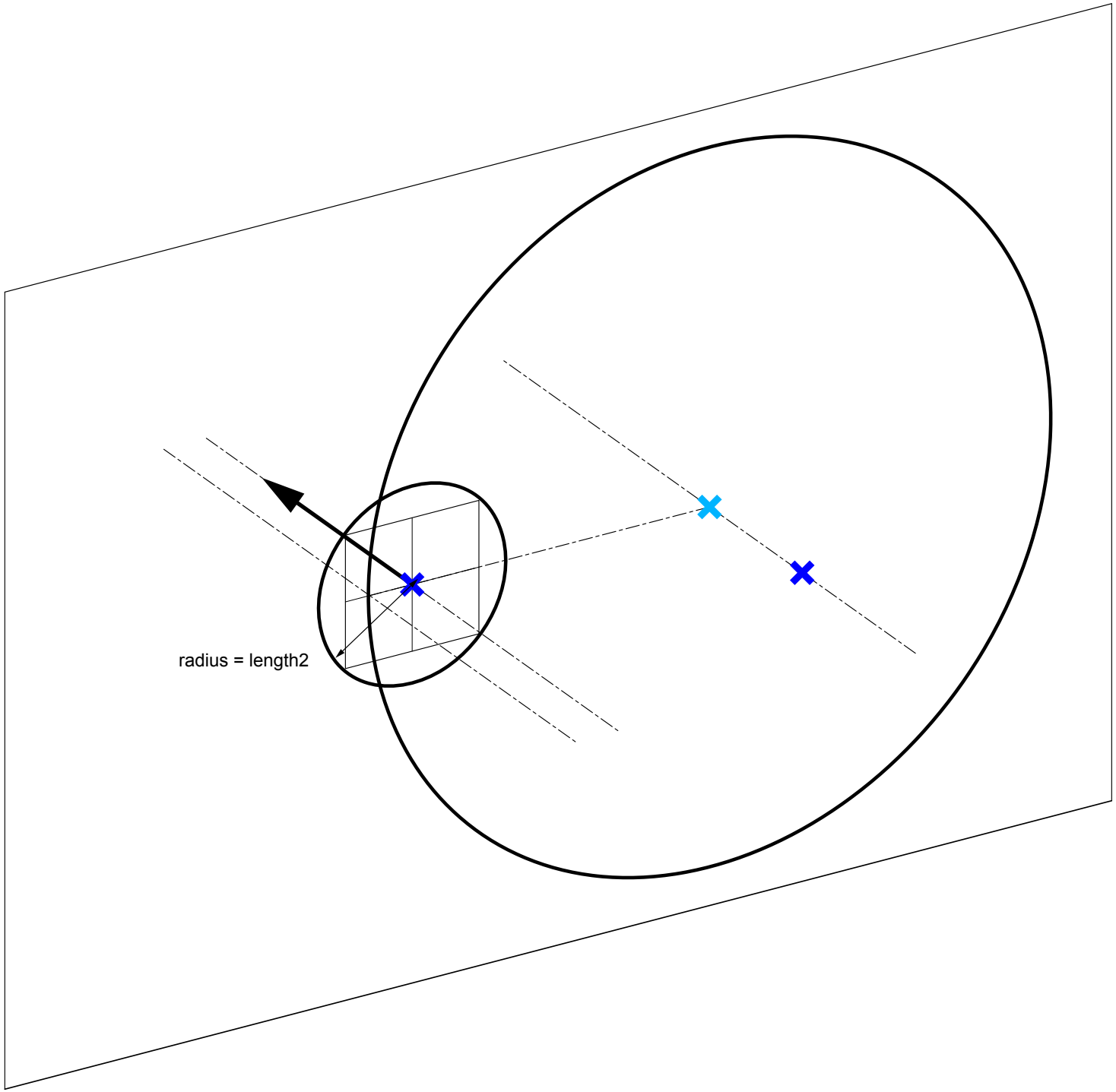
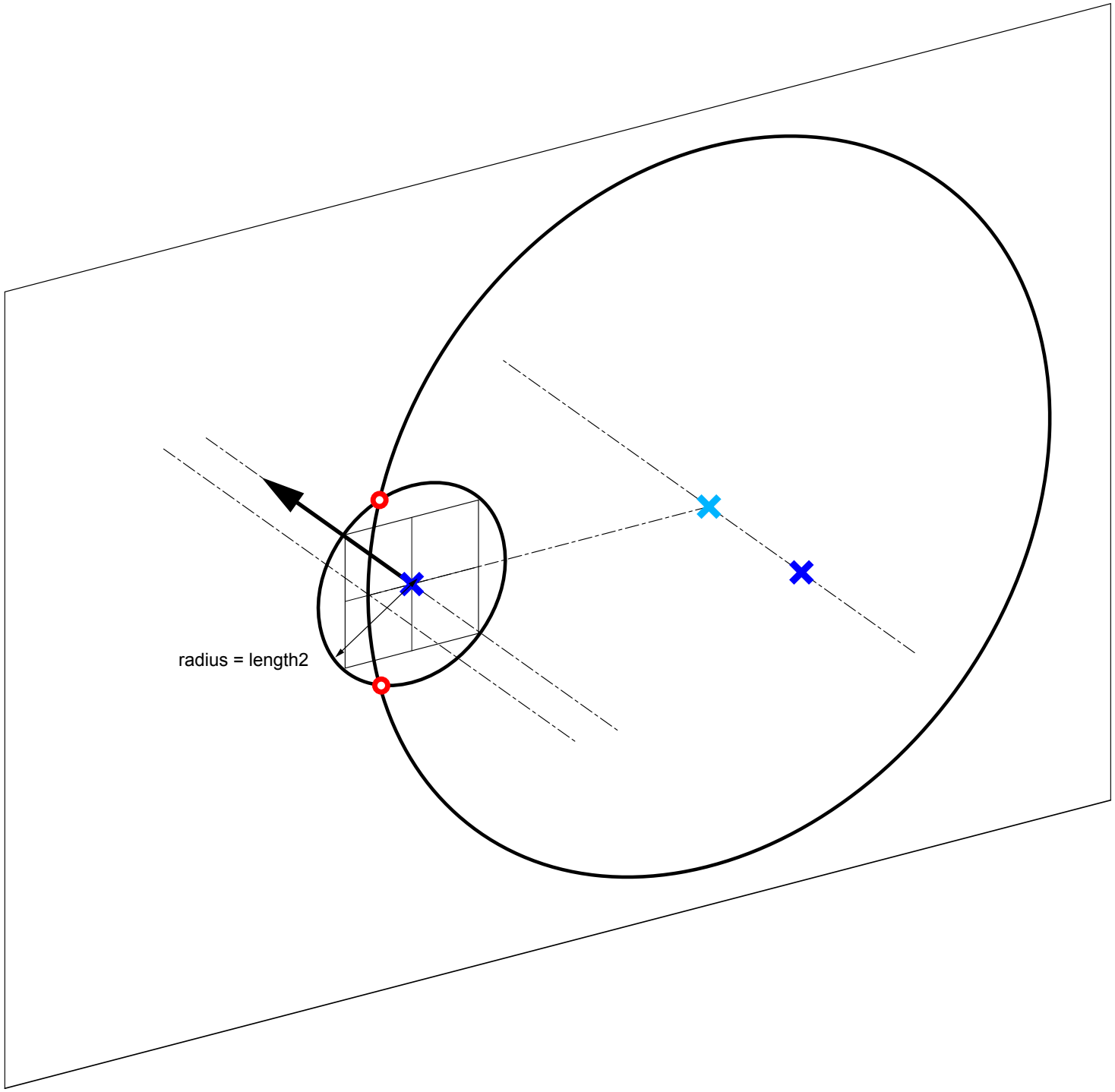


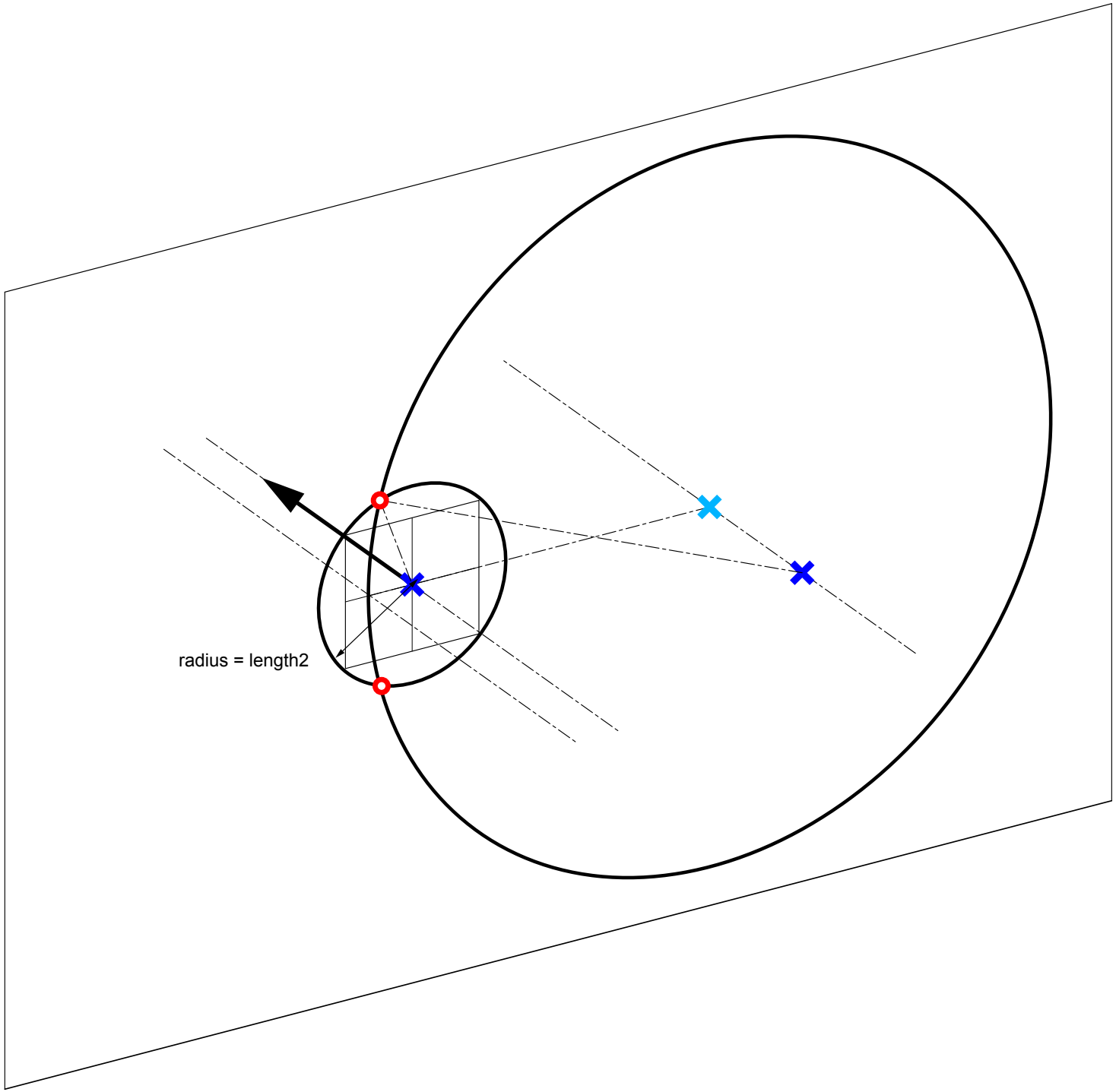
PLANE EXAMPLE : EMERSON COLLEGE SCRIM3



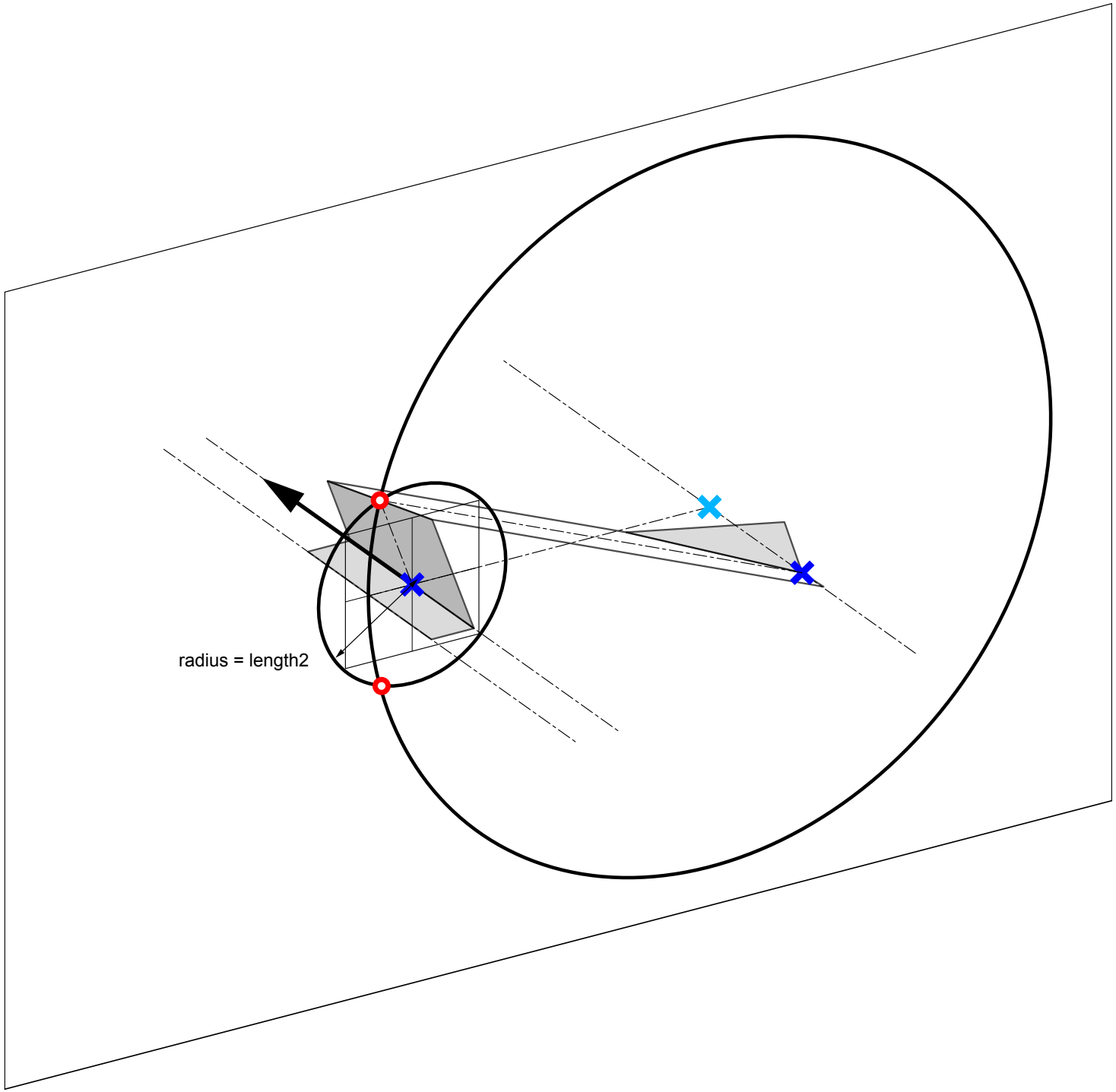
PLANE EXAMPLE : EMERSON COLLEGE SCRIM3



PLANE EXAMPLE : EMERSON COLLEGE SCRIM3



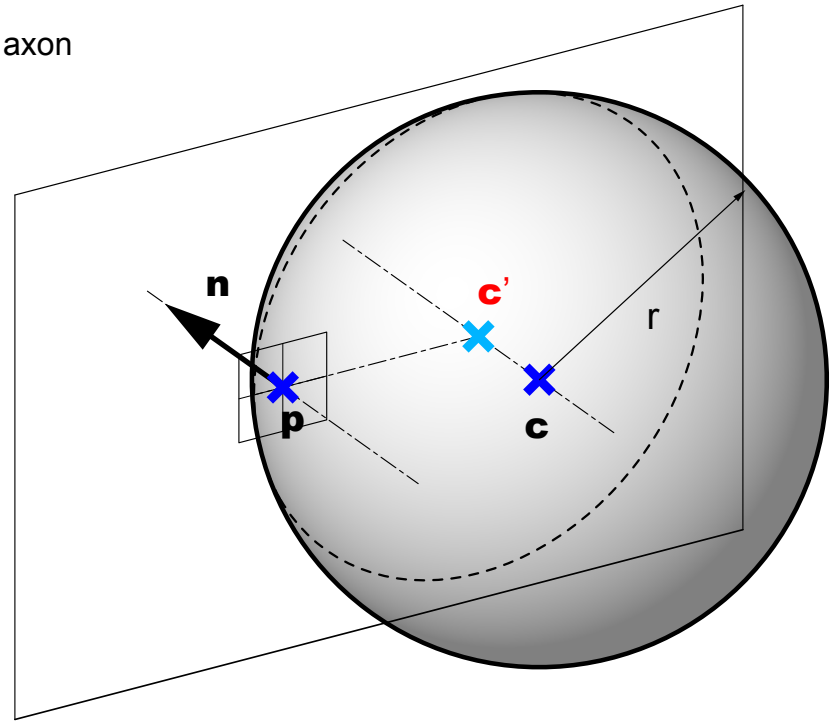
PLANE EXAMPLE : EMERSON COLLEGE SCRIM3

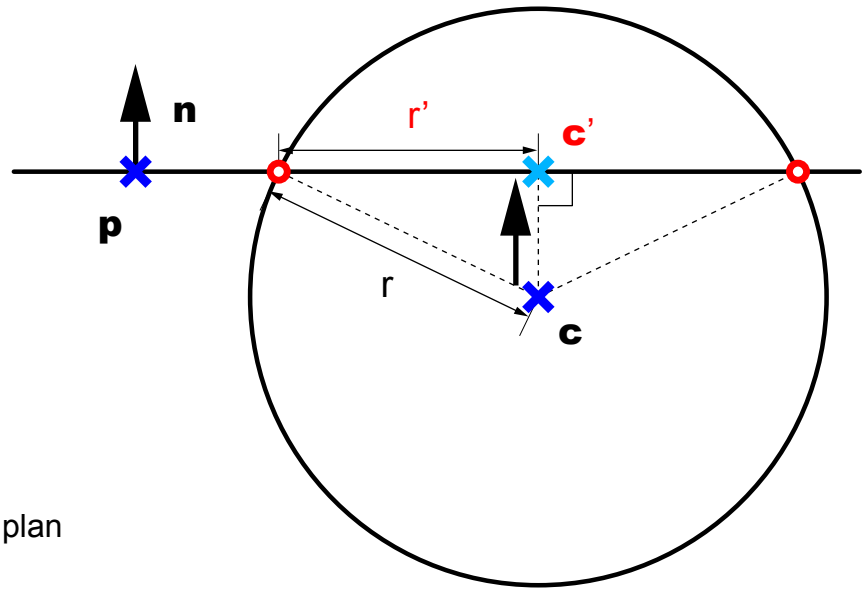


PLANE EXAMPLE : EMERSON COLLEGE SCRIM3

INTERSECTION OF PLANE AND SPHERE

axon





plan

PLANE $\mathbf{n} \cdot (\mathbf{x} - \mathbf{p}) = 0$

$|\mathbf{n}| = 1$

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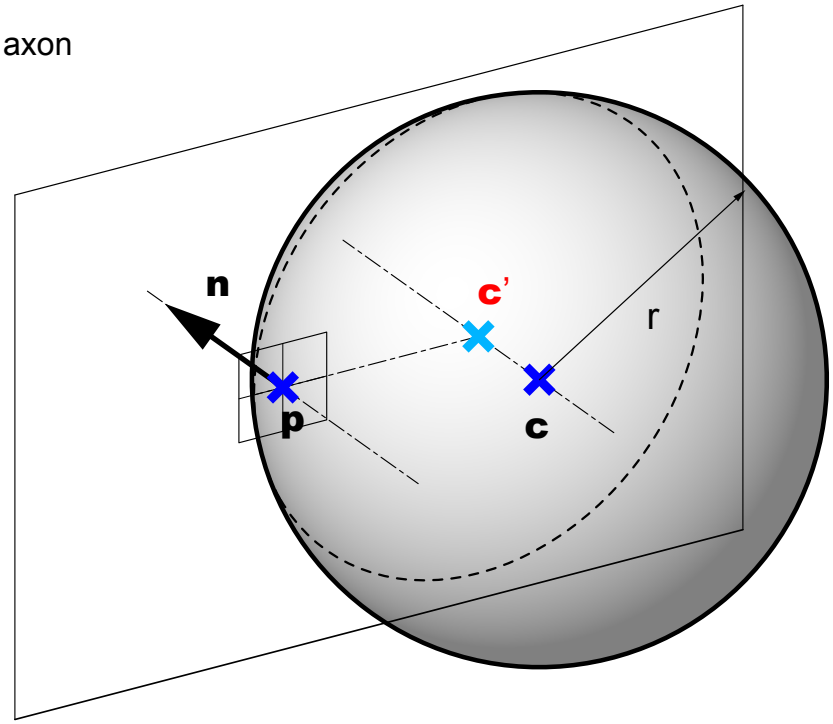
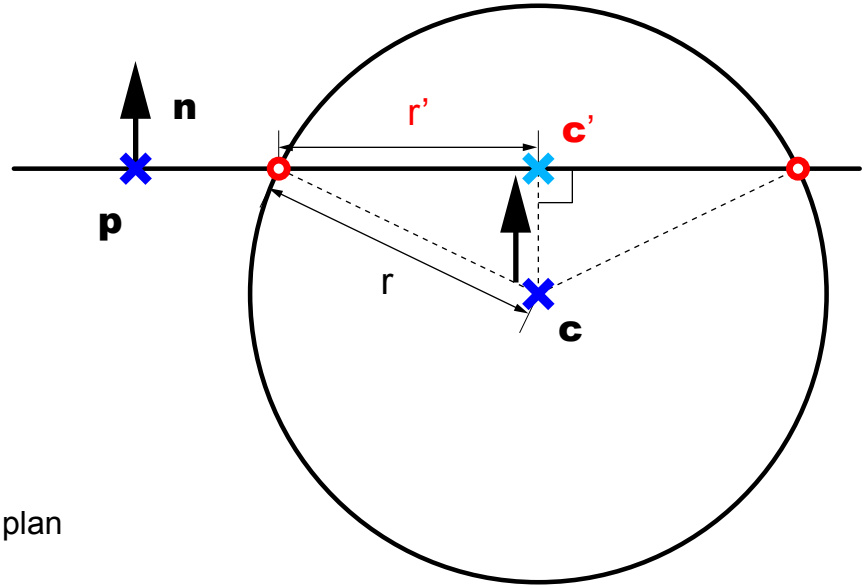
INTERSECTION OF PLANE AND SPHERE

axon





plan

PLANE $\mathbf{n} \cdot (\mathbf{x} - \mathbf{p}) = 0$

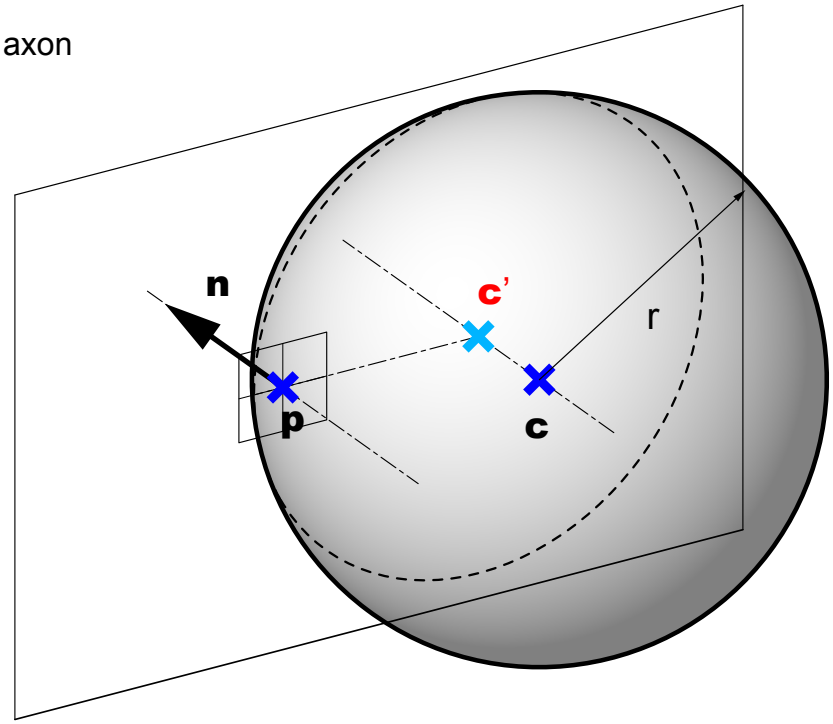
$|\mathbf{n}| = 1$

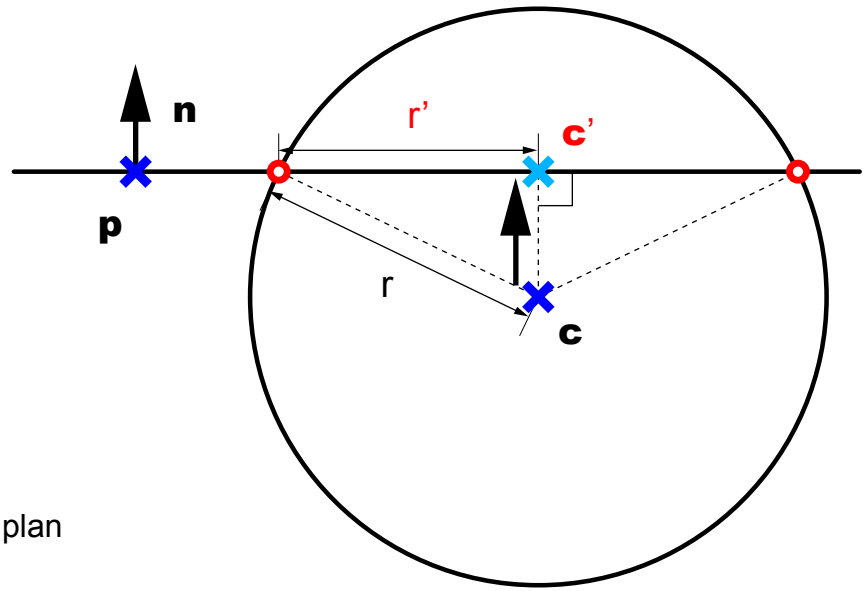
$\mathbf{c}' = k\,\mathbf{n} + \mathbf{c}$

PLANE EXAMPLE : EMERSON COLLEGE SCRIM3

INTERSECTION OF PLANE AND SPHERE

axon





plan

PLANE $\mathbf{n} \cdot (\mathbf{x} - \mathbf{p}) = 0$

$|\mathbf{n}| = 1$

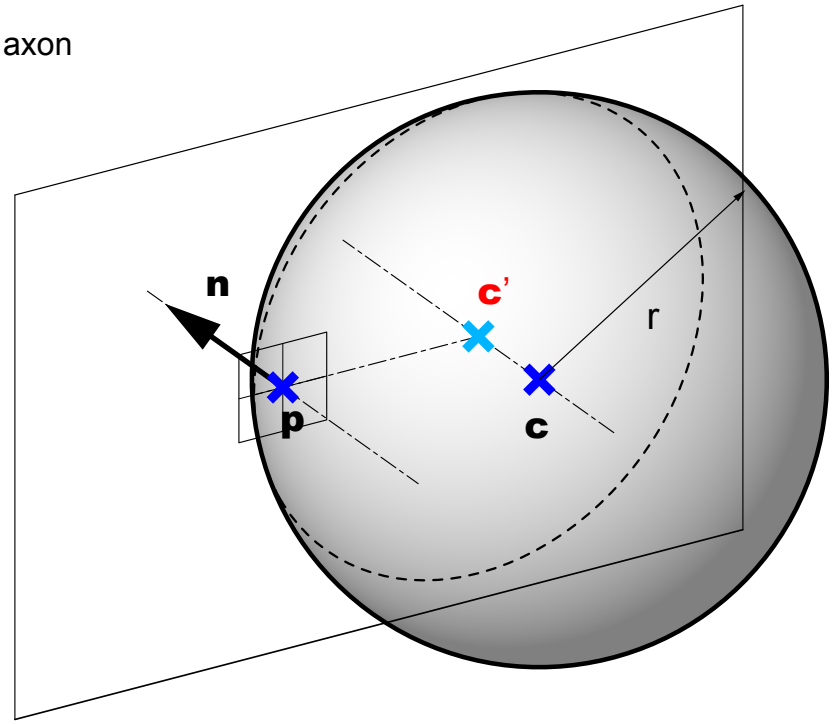
$\mathbf{c}' = k\mathbf{n} + \mathbf{c}$

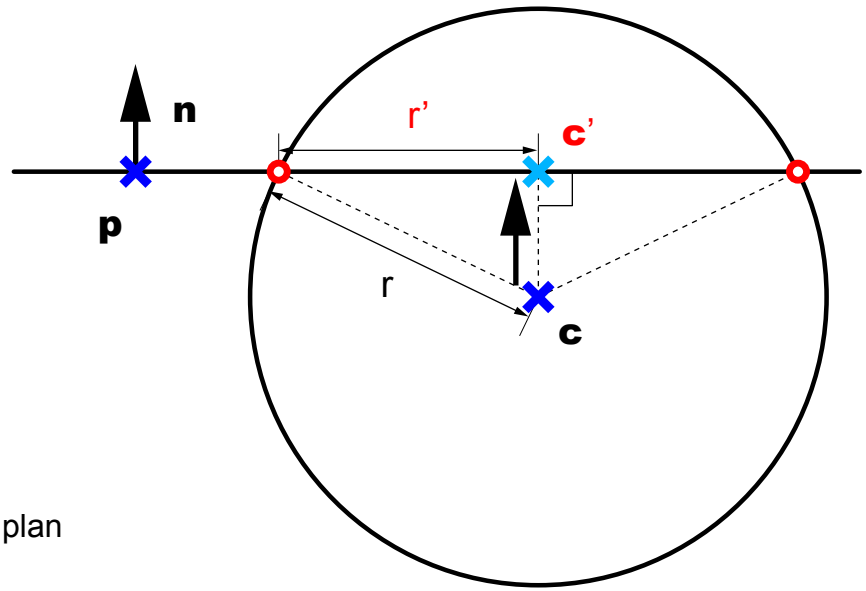
$\mathbf{n} \cdot (k\mathbf{n} + \mathbf{c} - \mathbf{p}) = 0$

PLANE EXAMPLE : EMERSON COLLEGE SCRIM3

INTERSECTION OF PLANE AND SPHERE

axon





plan

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PLANE EXAMPLE : EMERSON COLLEGE SCRIM3

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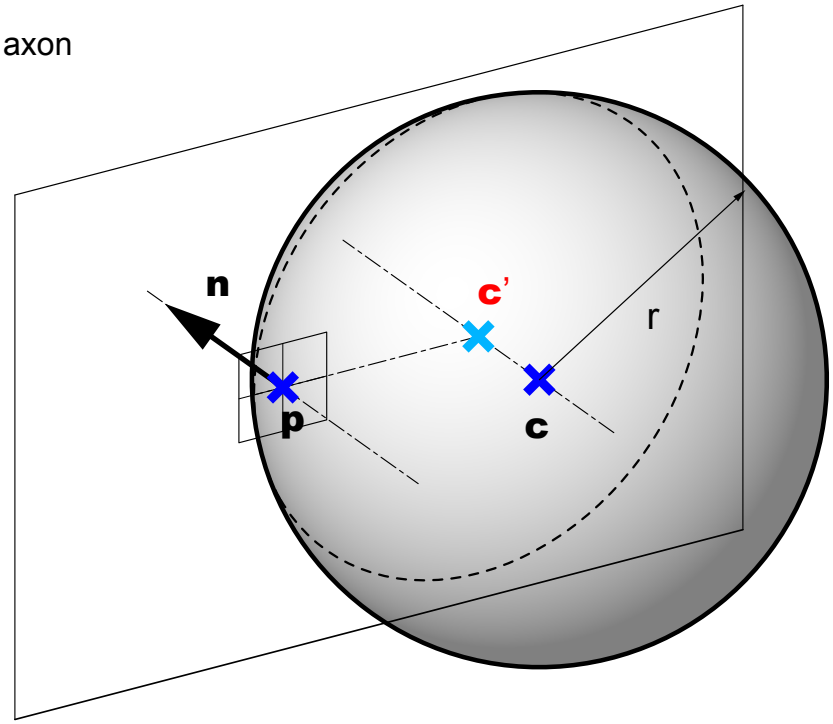

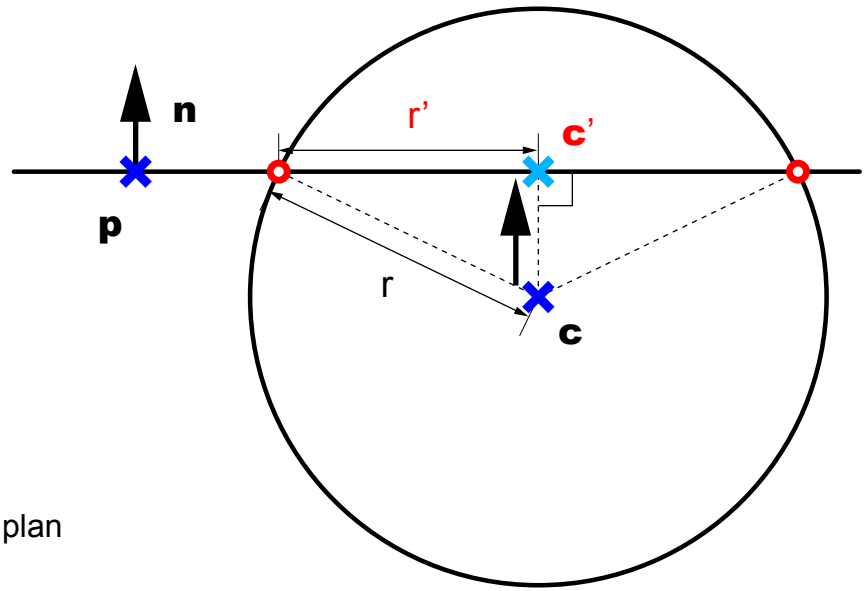

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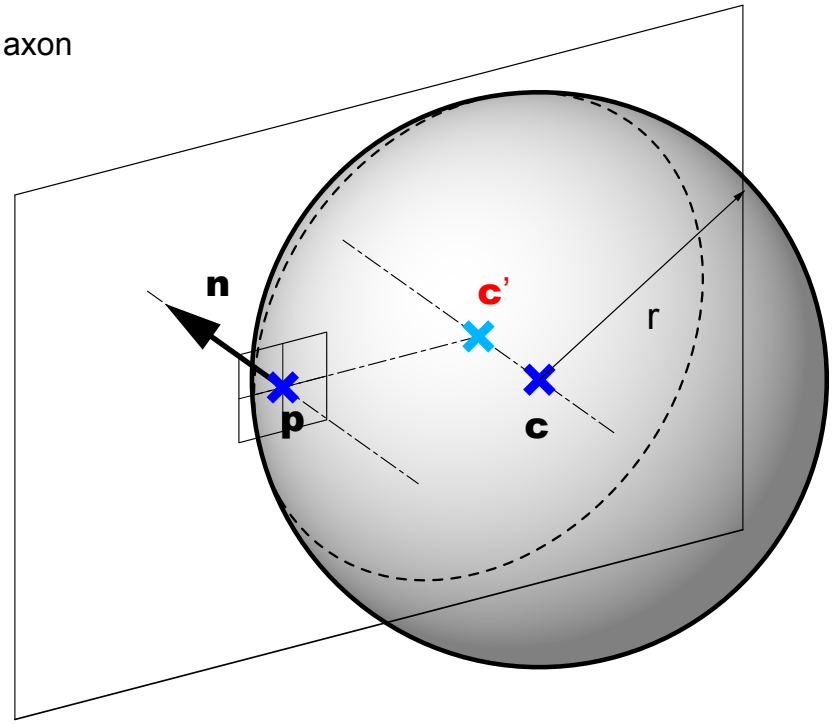

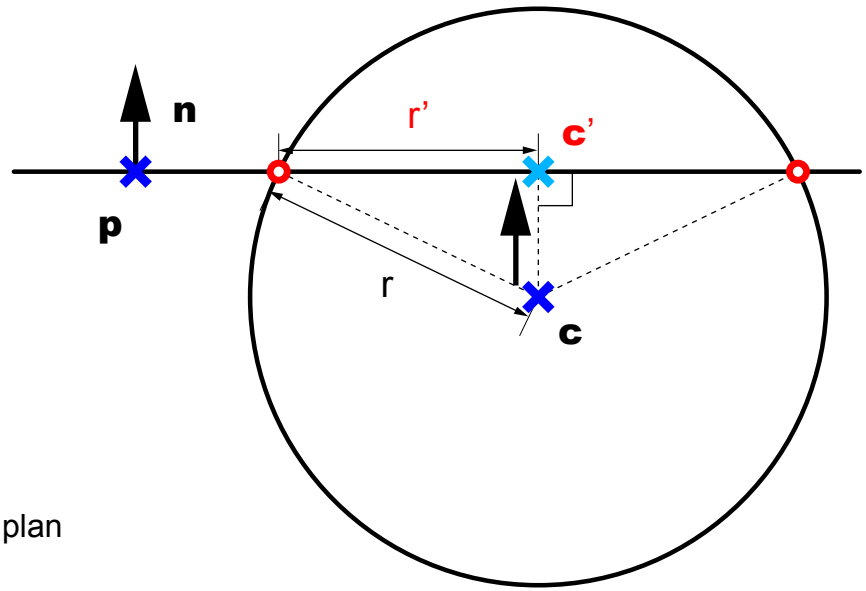

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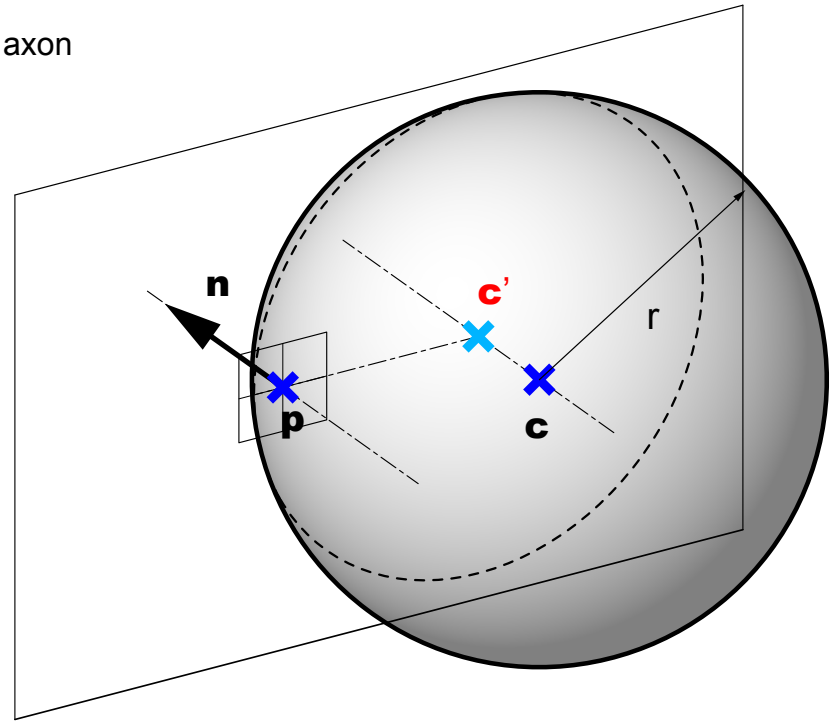

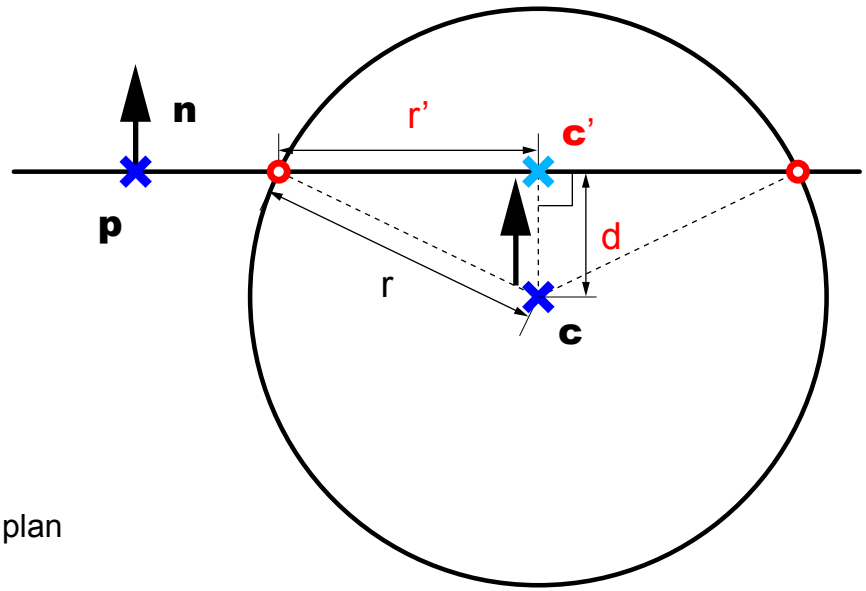

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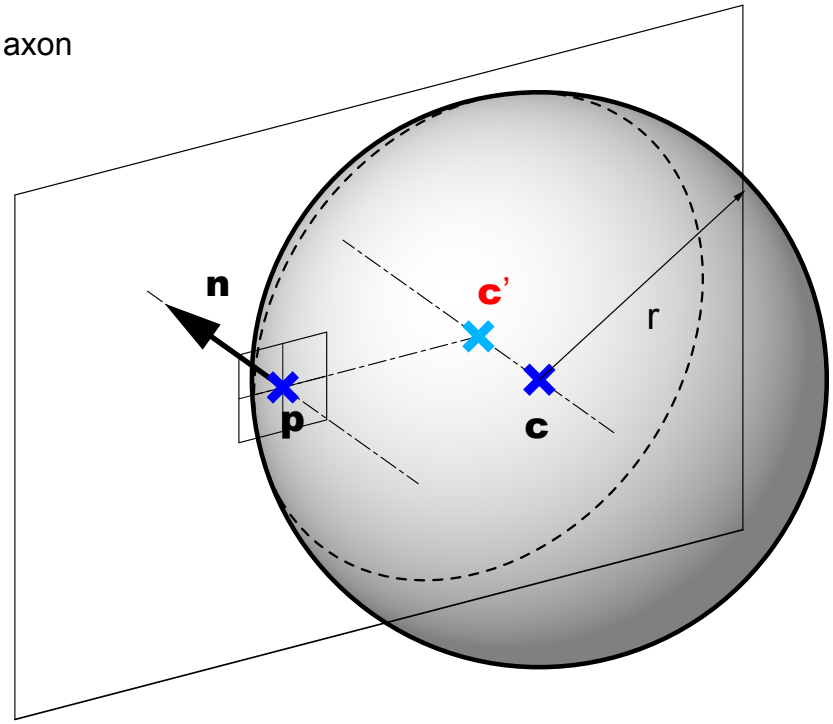

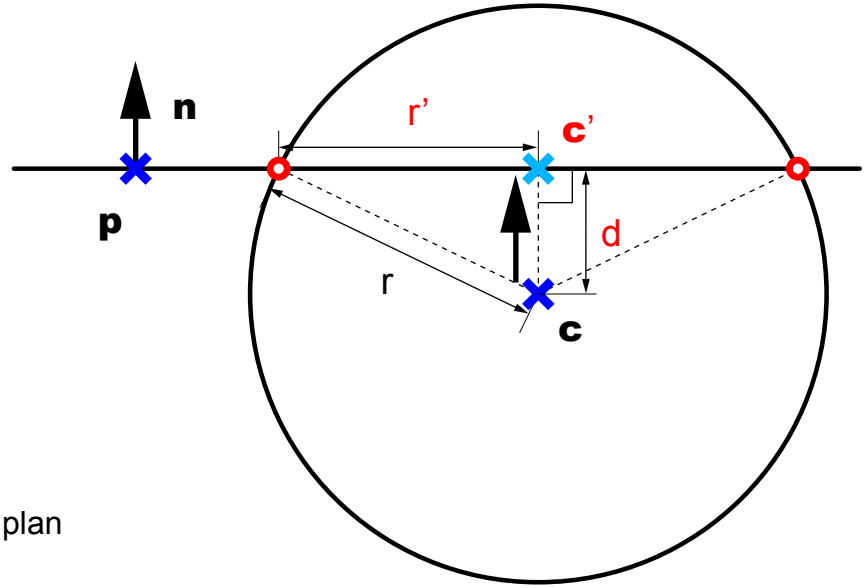

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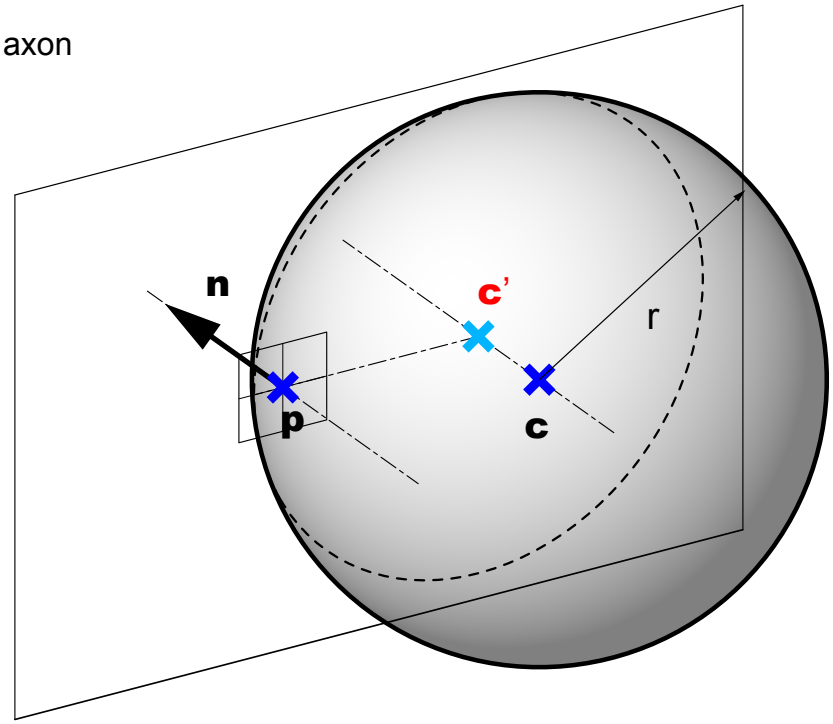

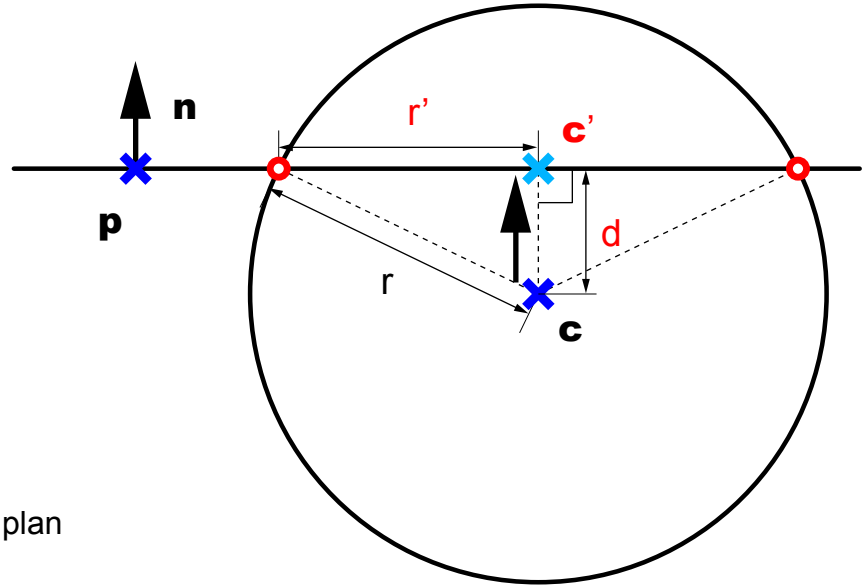

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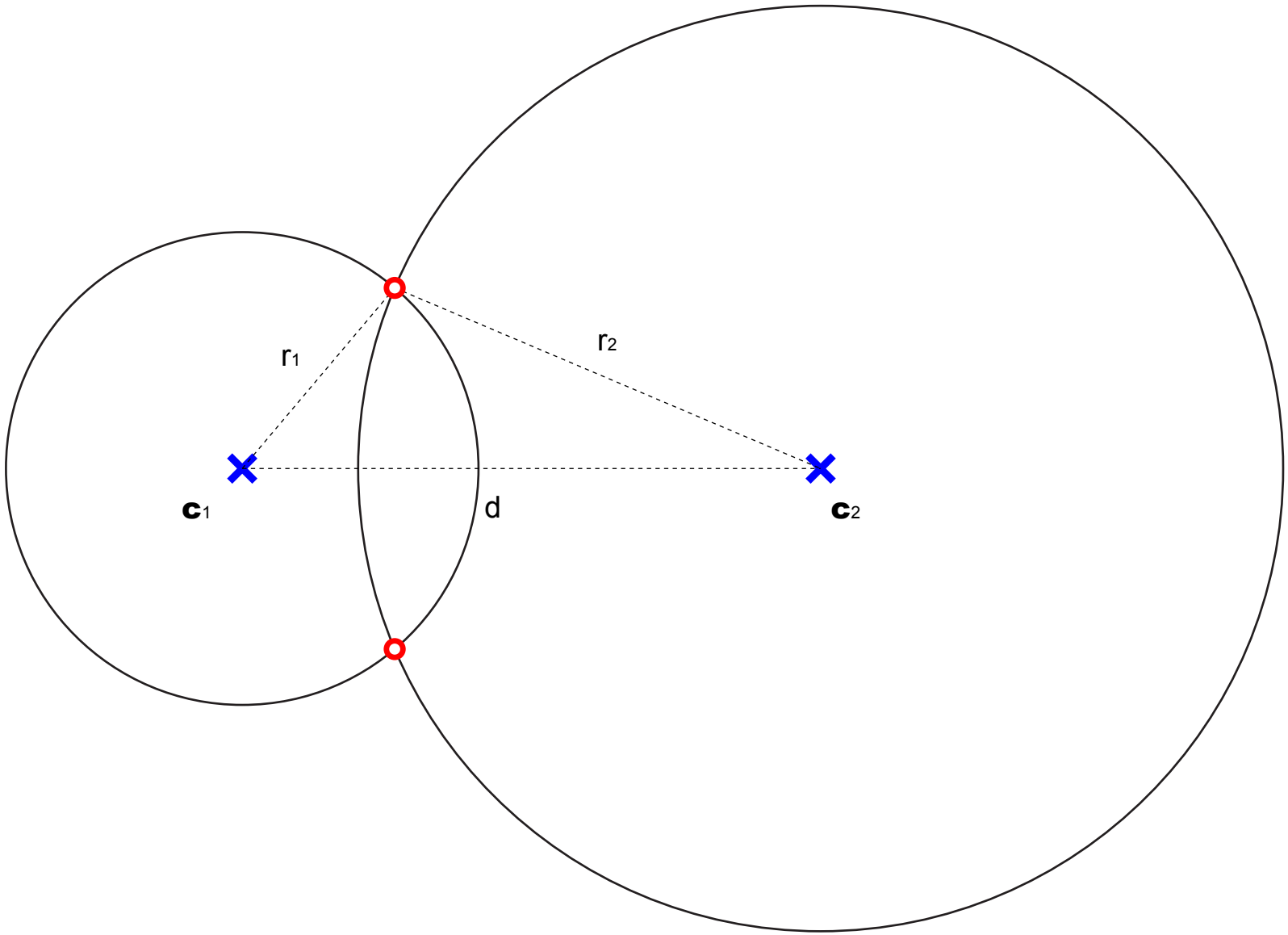
PLANE EXAMPLE : EMERSON COLLEGE SCRIM3

INTERSECTION OF TWO CIRCLES



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INTERSECTION OF TWO CIRCLES

$d = |\mathbf{c}_1 - \mathbf{c}_2|$

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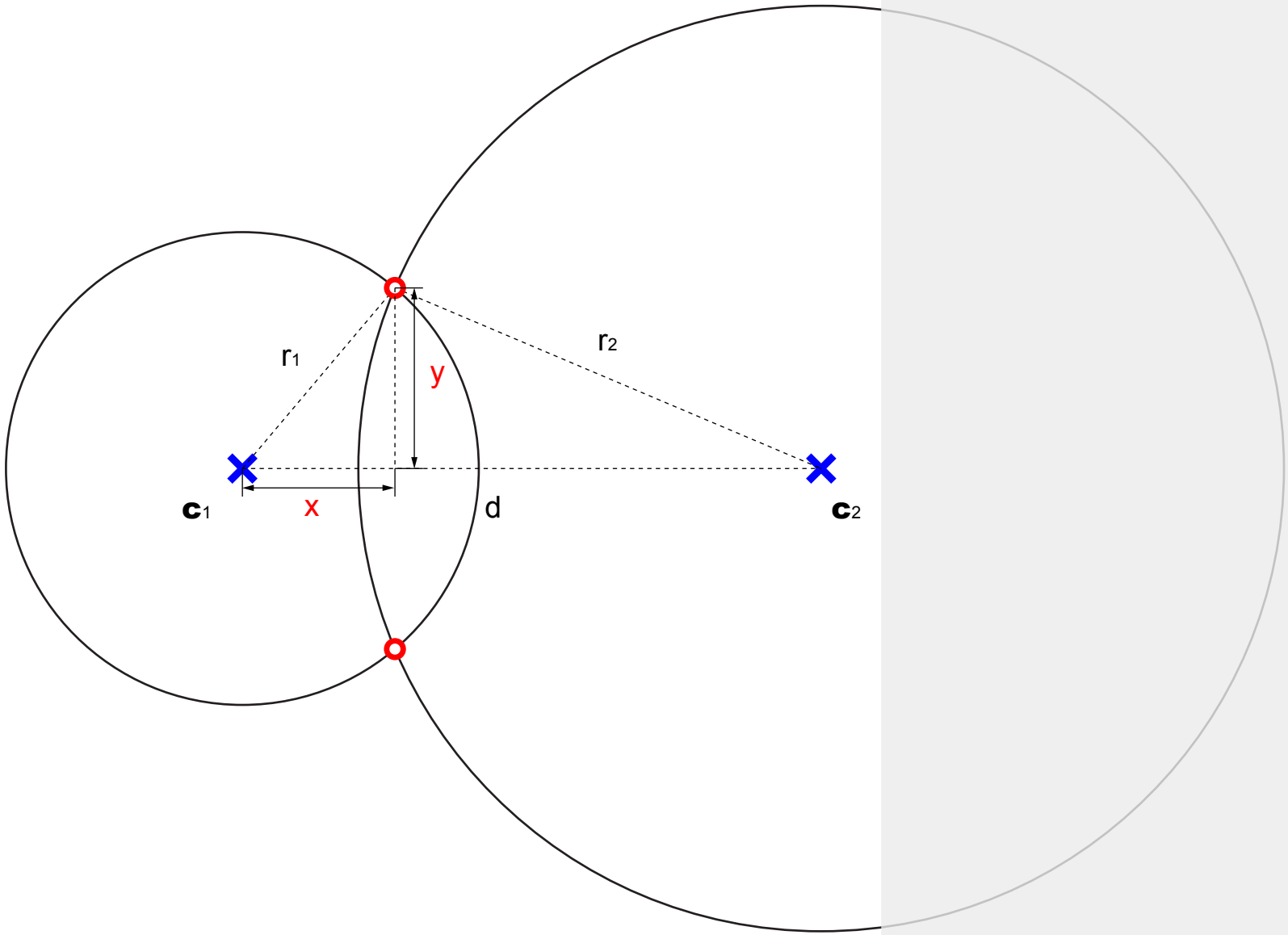

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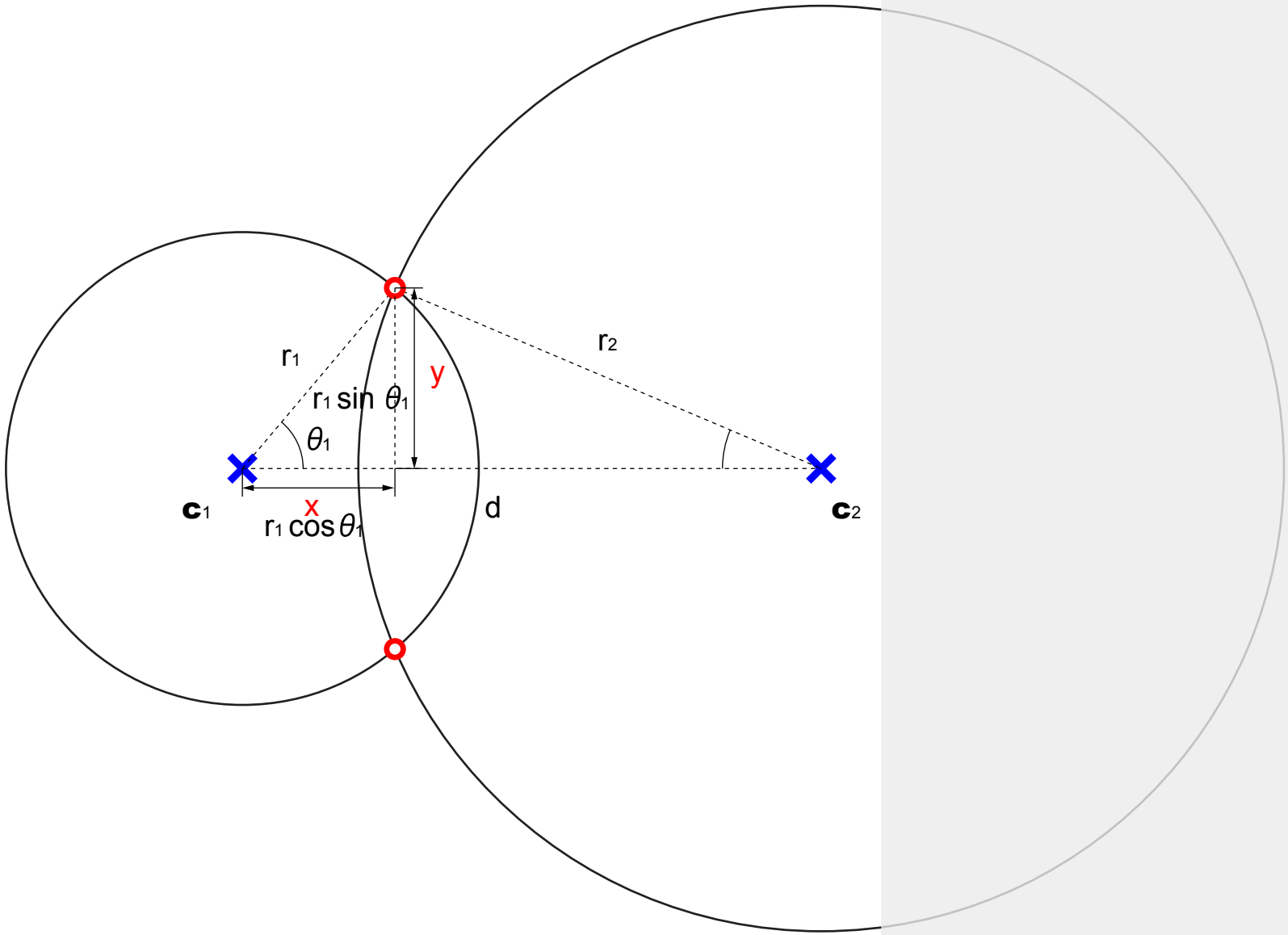

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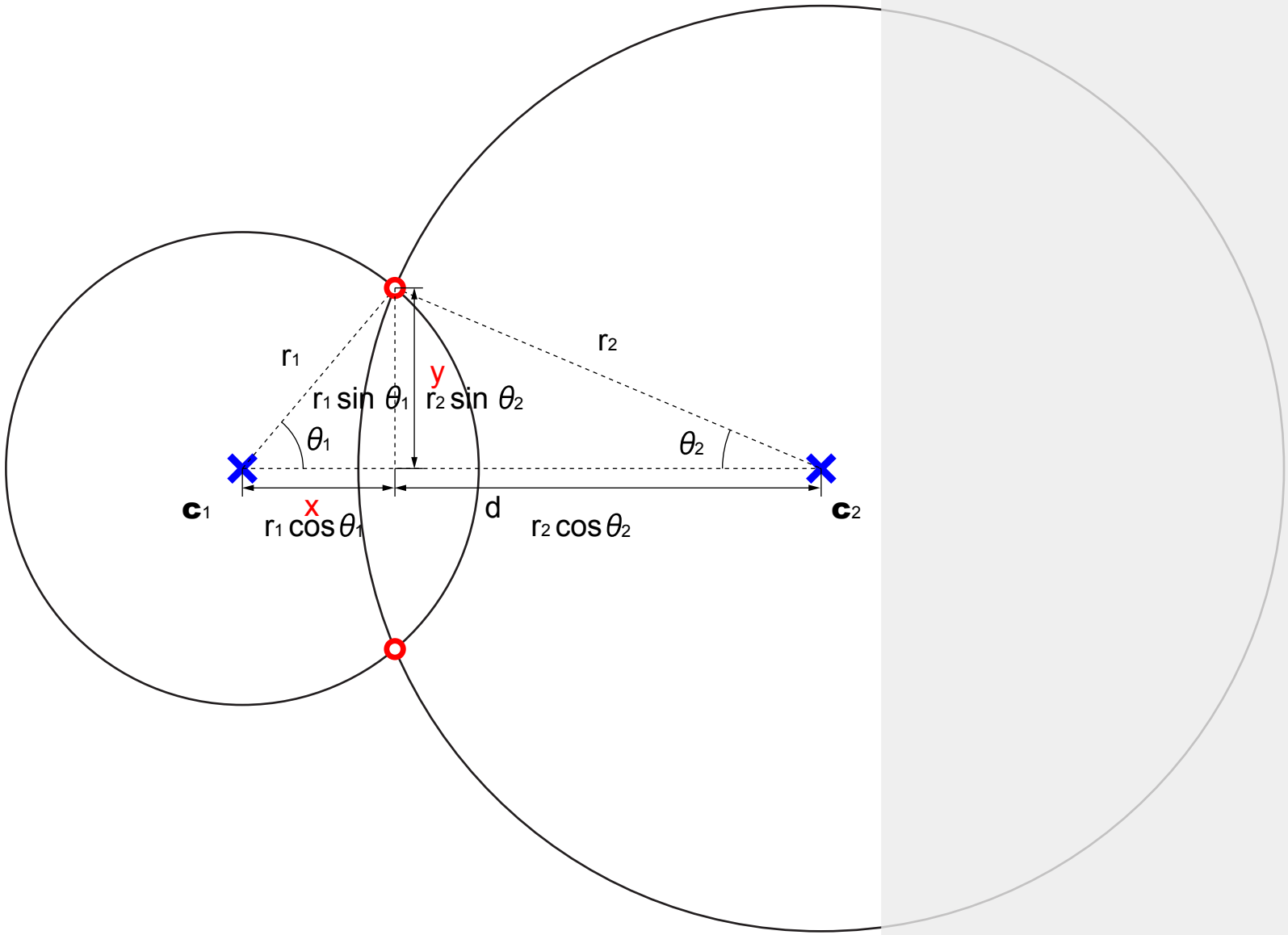

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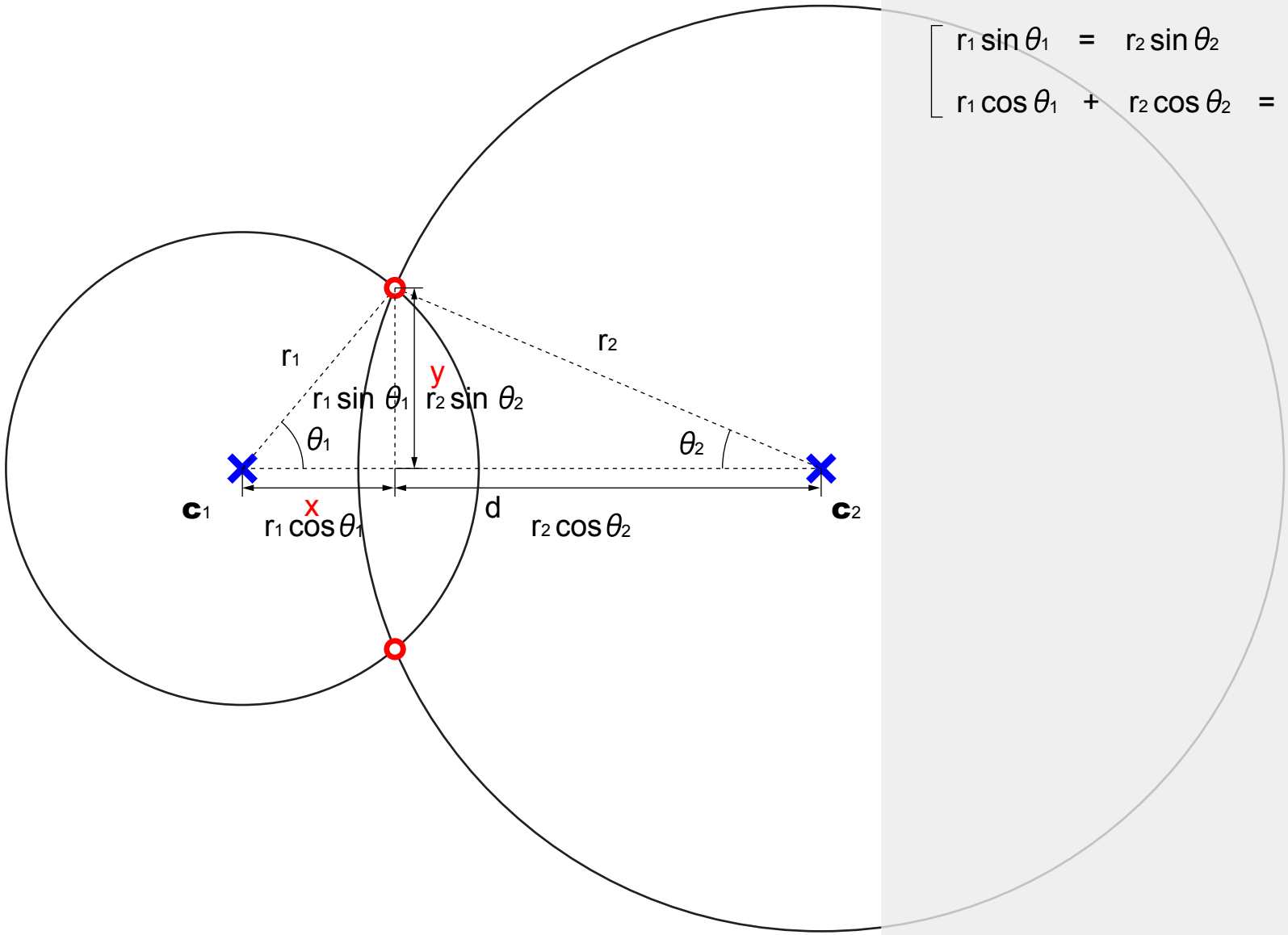

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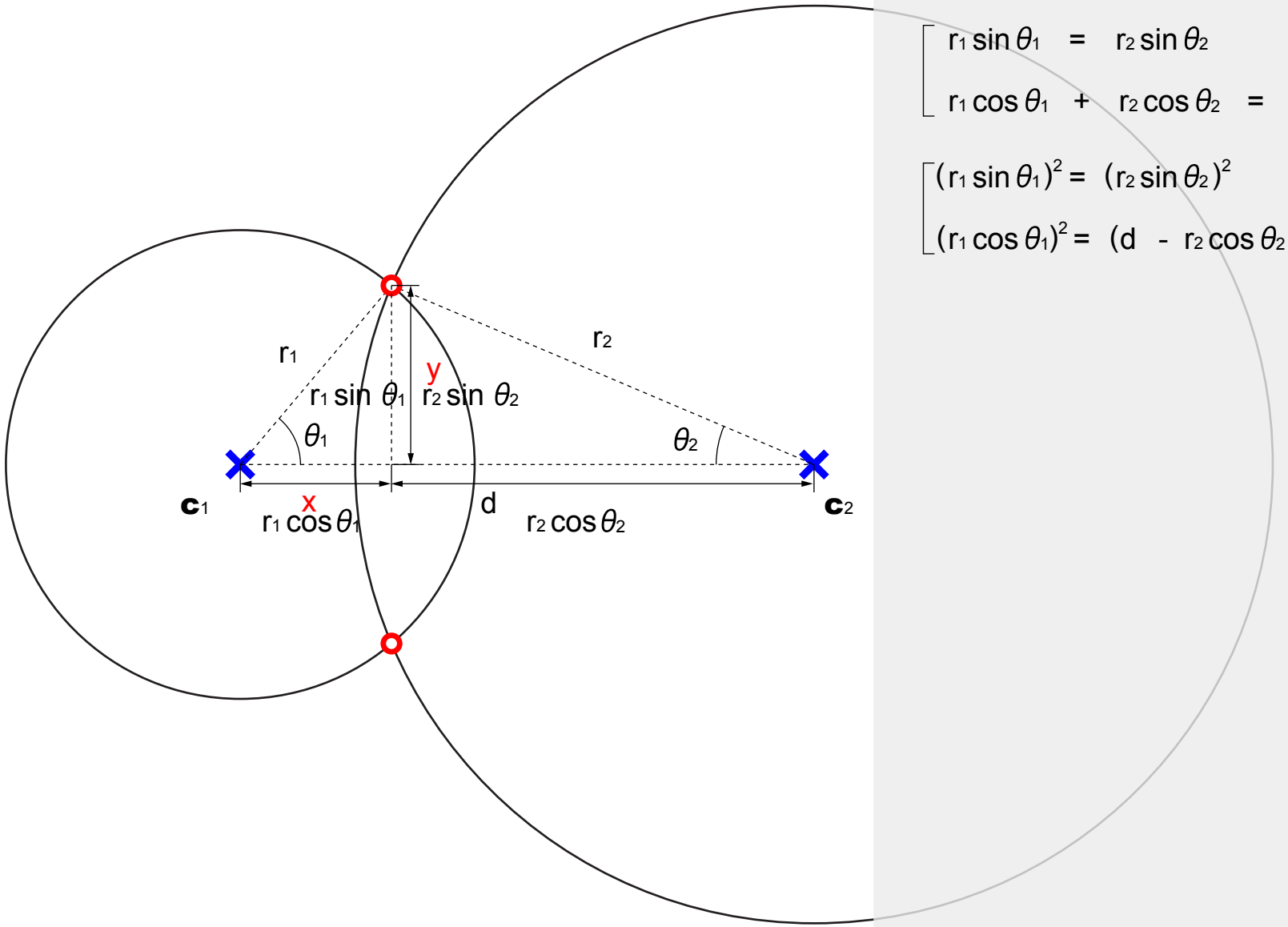

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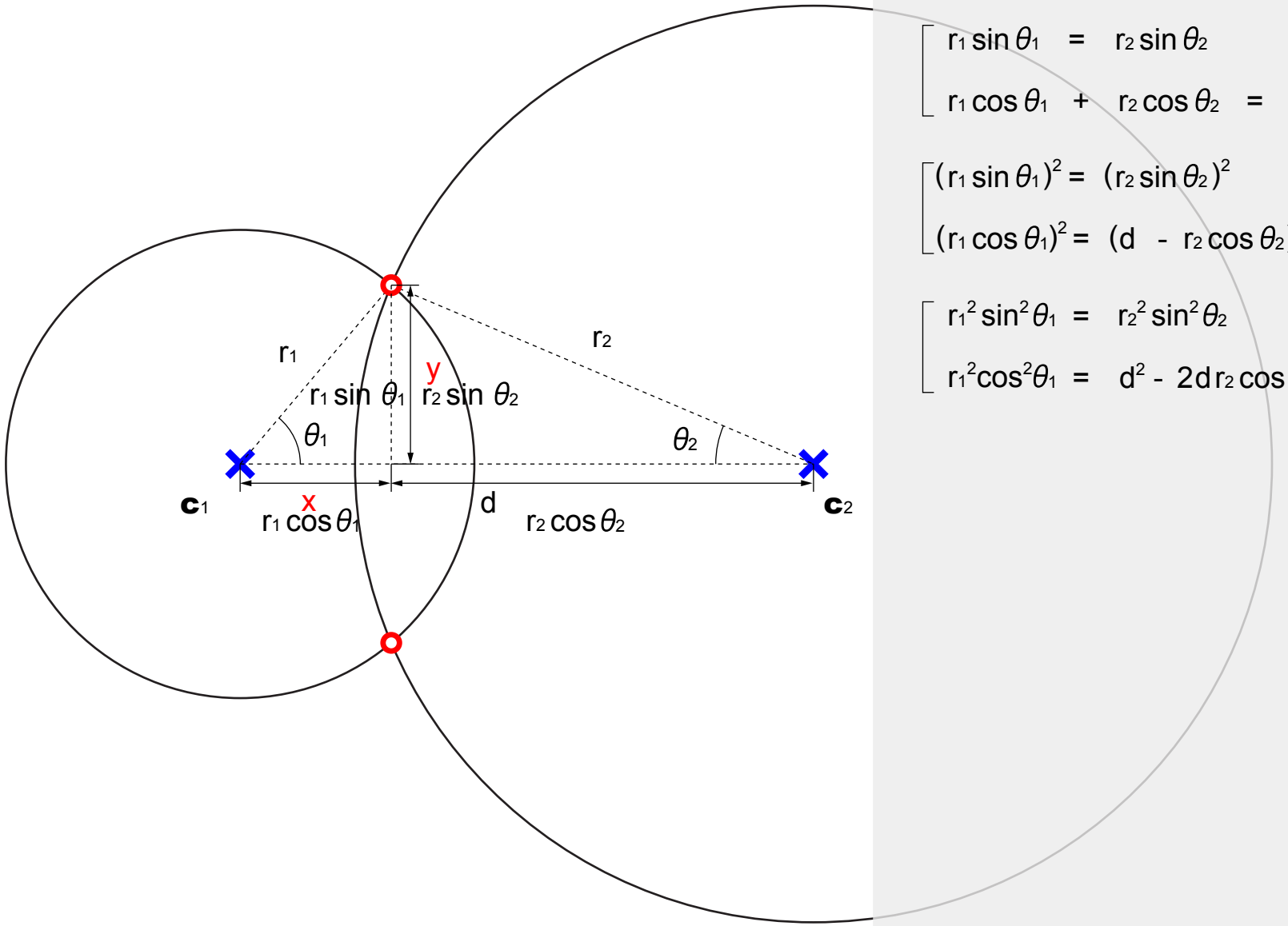

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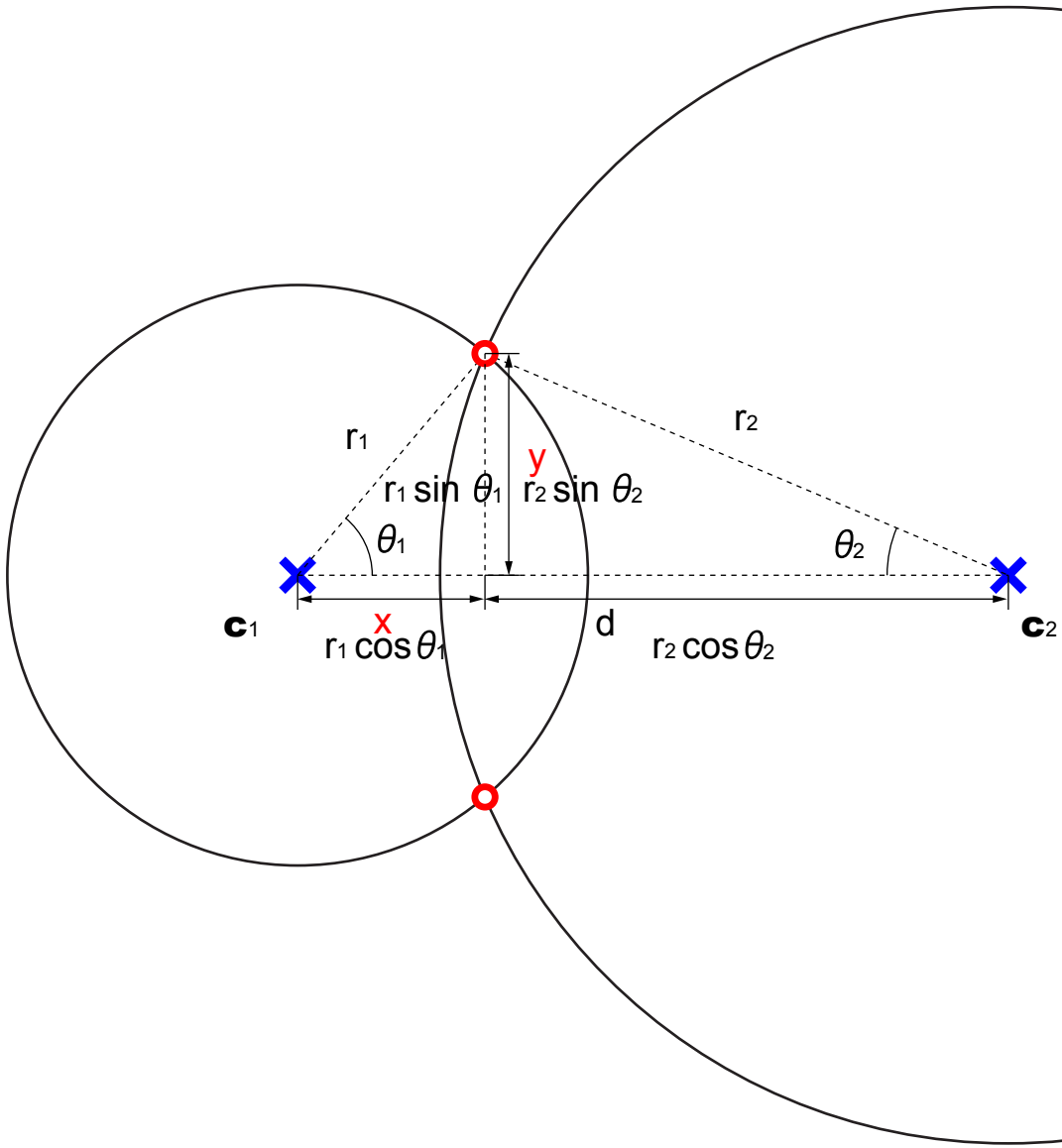

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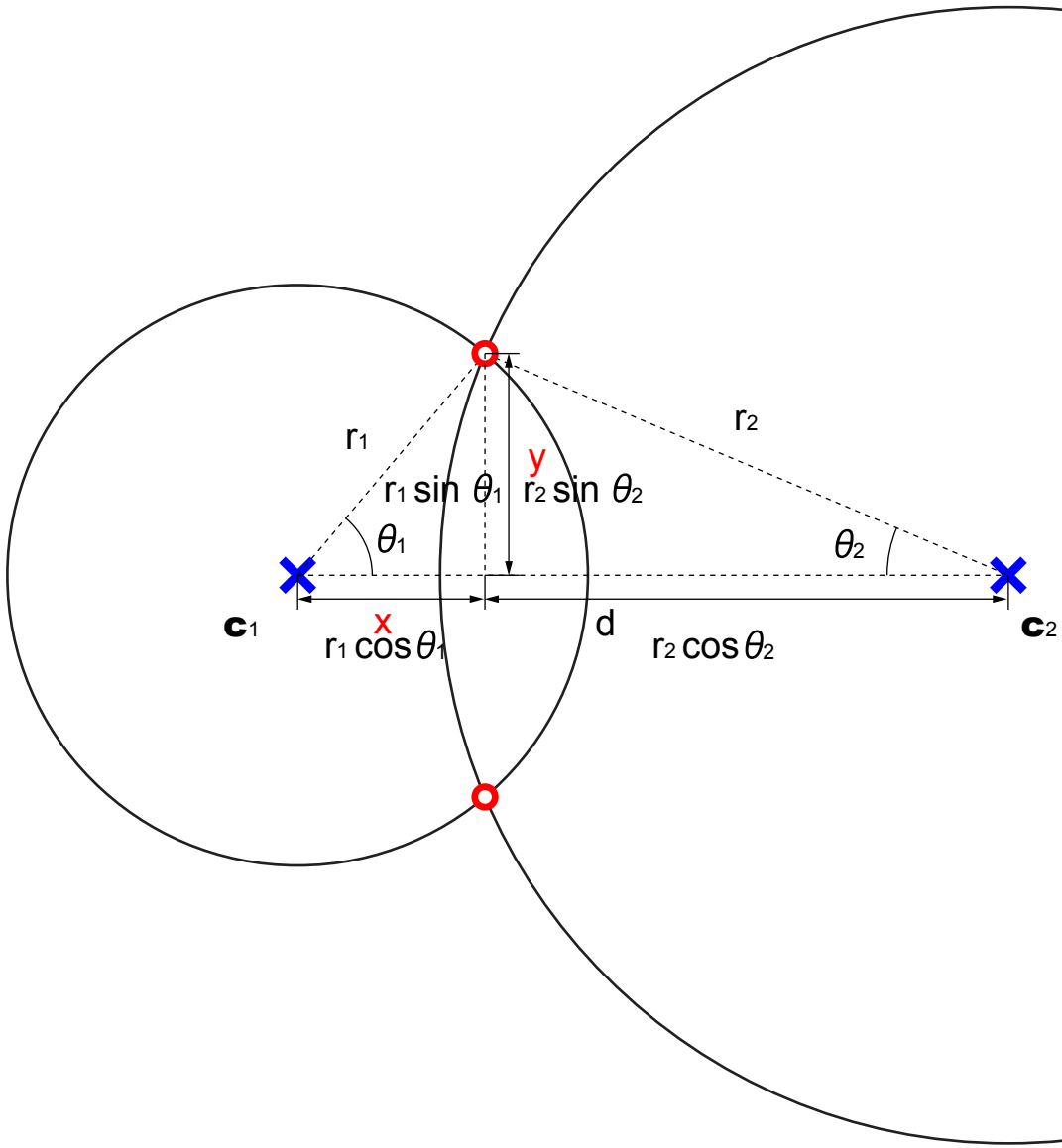

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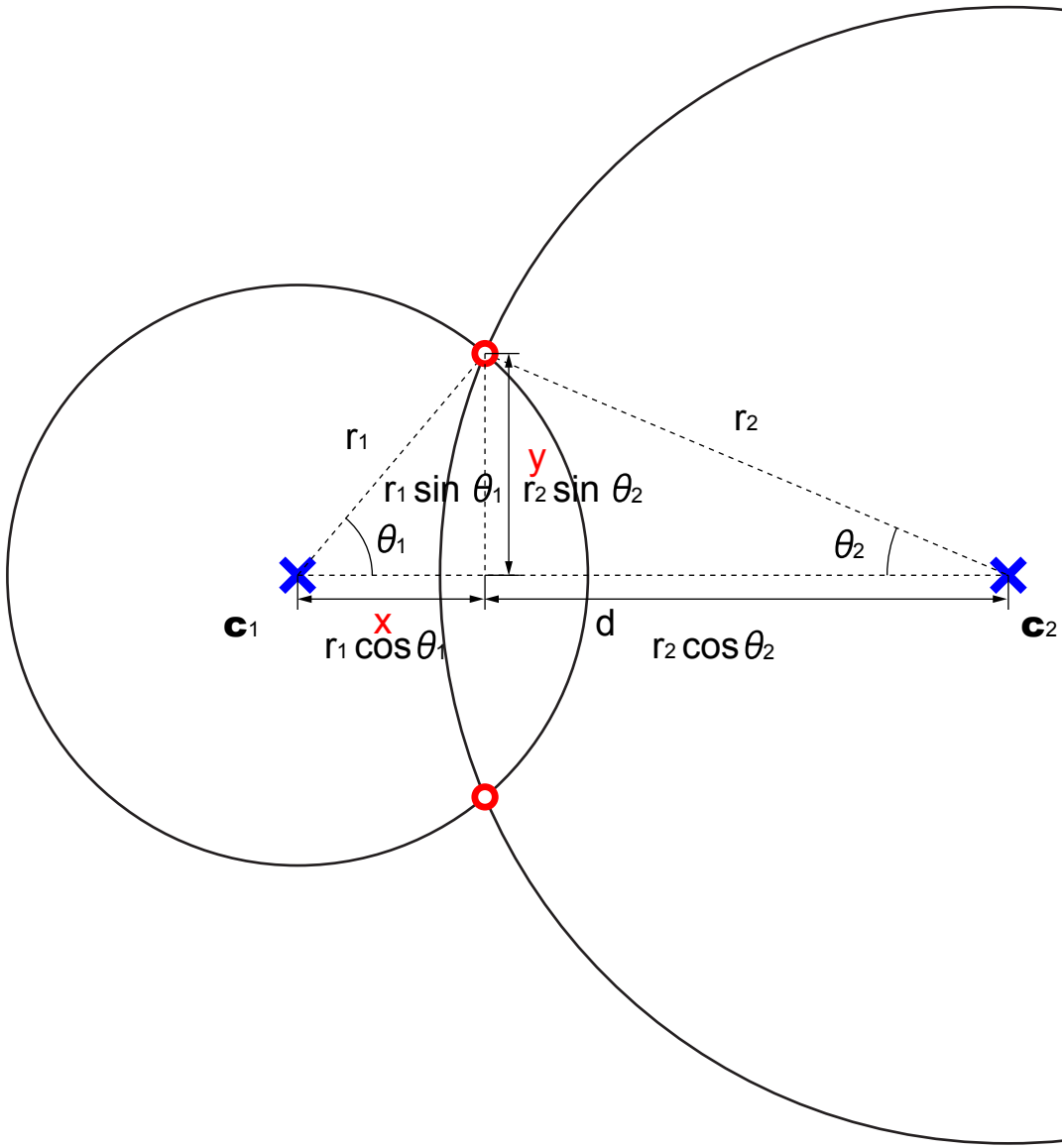

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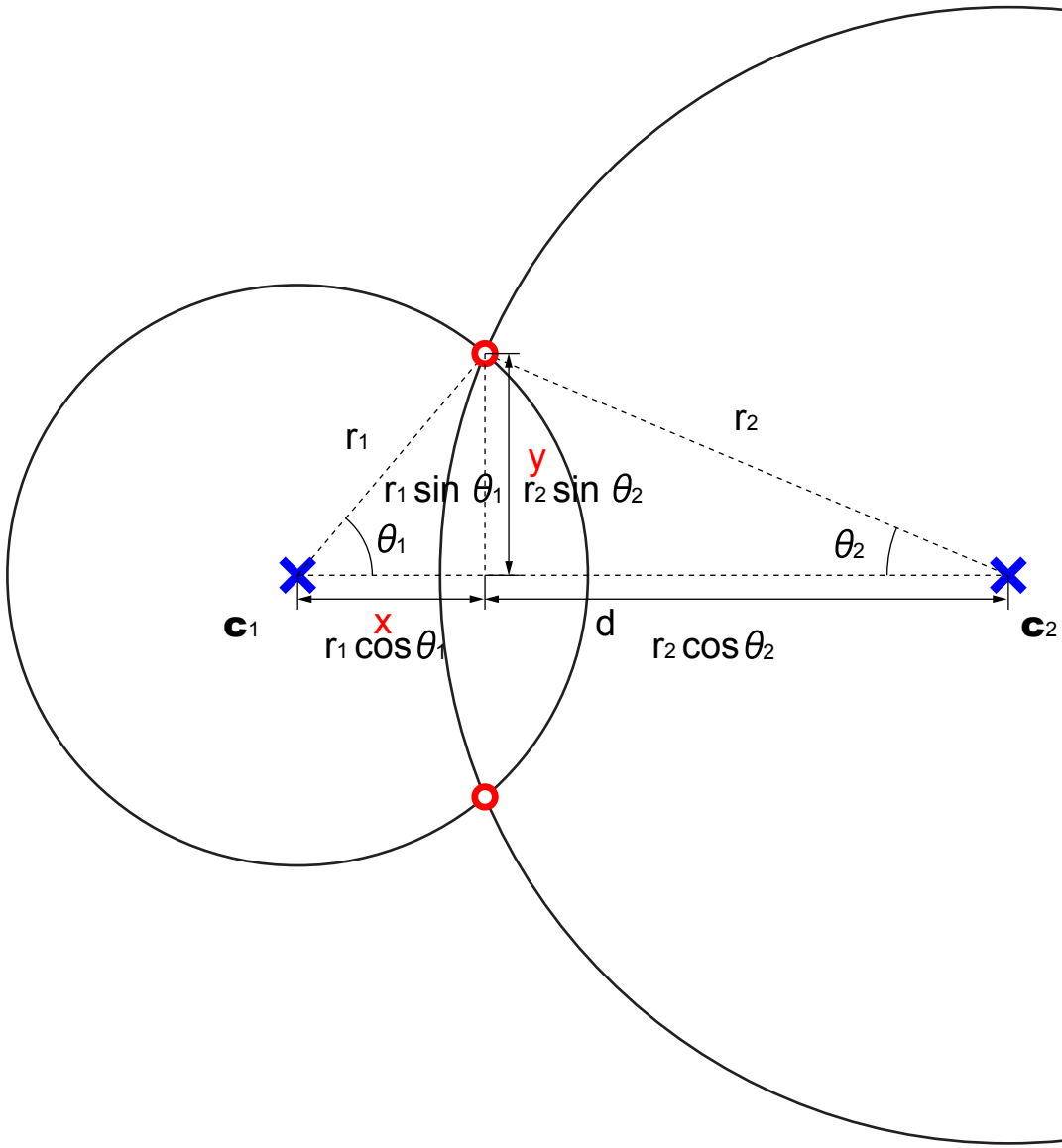

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$$\begin{aligned} x &= d - r_2 \cos\theta_2 \\ &= d - \frac{d^2 - r_1^2 + r_2^2}{2d} \\ &= \frac{d^2 + r_1^2 - r_2^2}{2d} \end{aligned}$$

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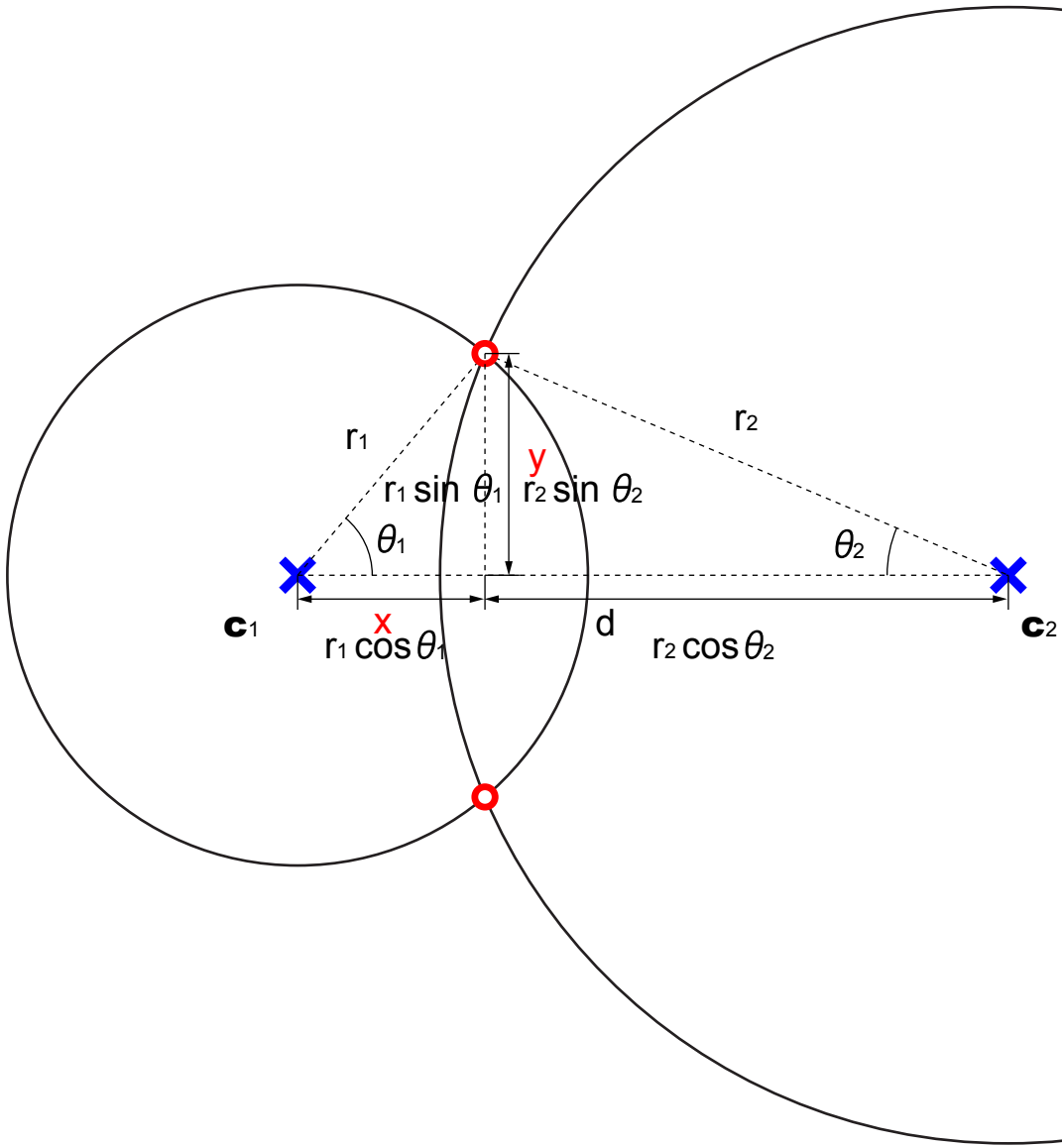

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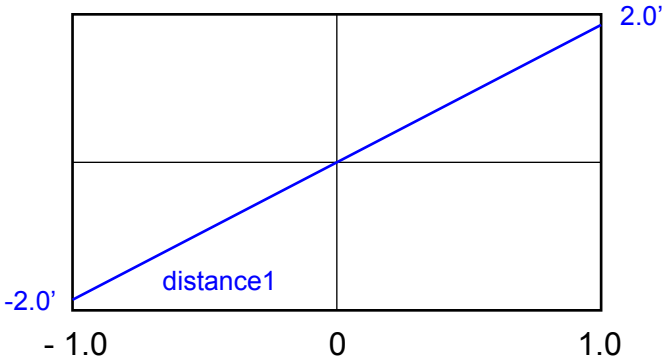
$$r_1^2 = r_2^2 + d^2 - 2d\, r_2 \cos\theta_2$$

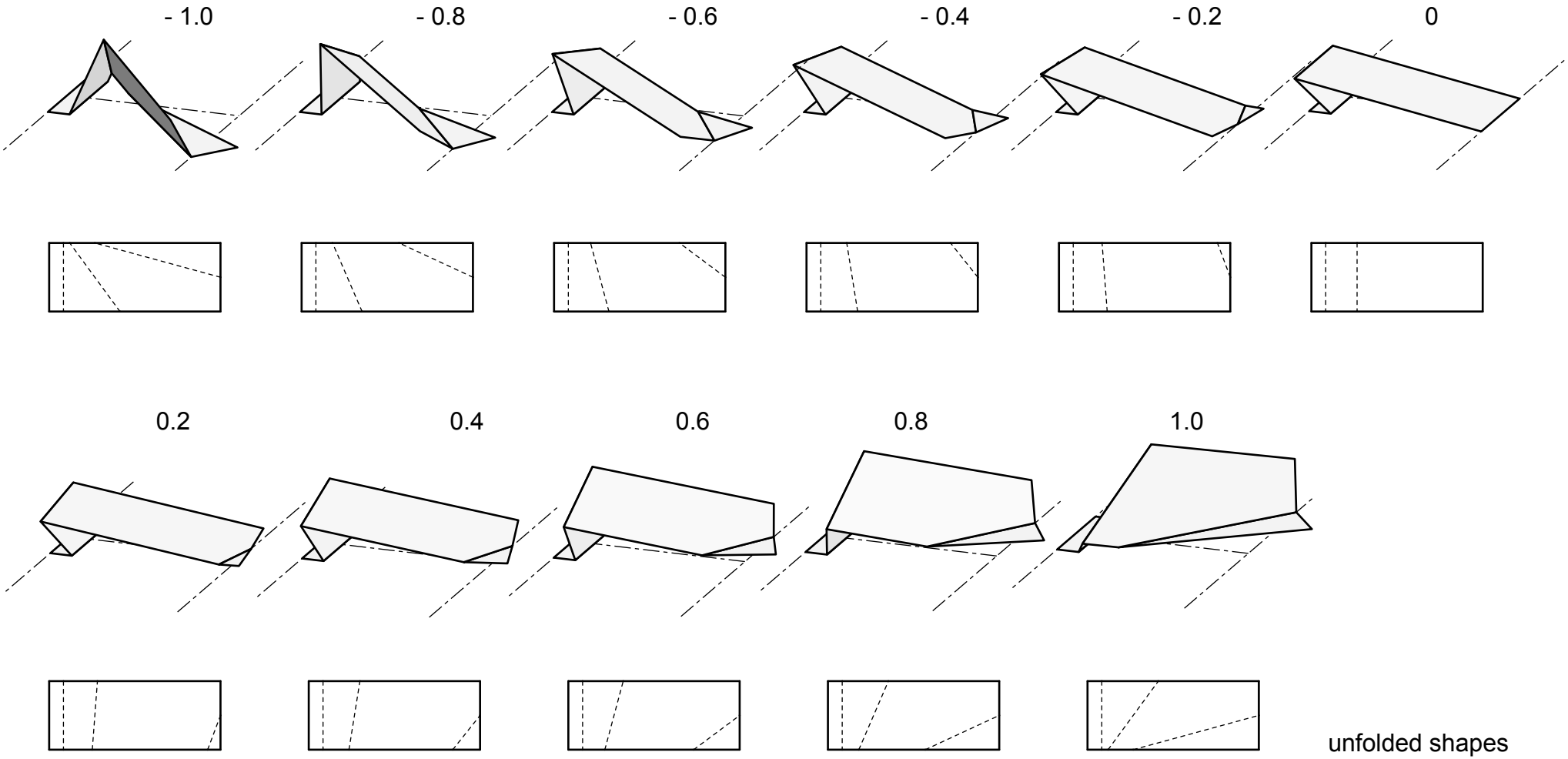
$$r_2 \cos\theta_2 = \frac{d^2 - r_1^2 + r_2^2}{2d}$$

$$\begin{aligned} x &= d - r_2 \cos\theta_2 \\ &= d - \frac{d^2 - r_1^2 + r_2^2}{2d} \\ &= \frac{d^2 + r_1^2 - r_2^2}{2d} \end{aligned}$$

$$y = \sqrt{r_1^2 - x^2}$$

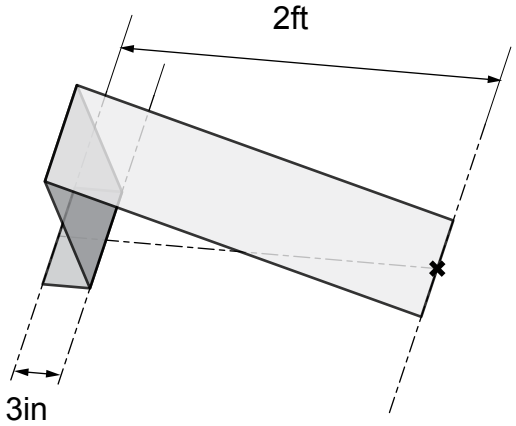
PLANE EXAMPLE : EMERSON COLLEGE SCRIM3

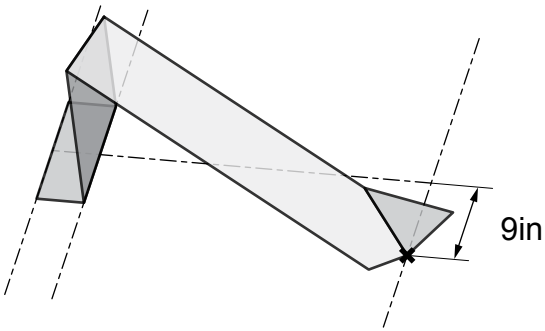


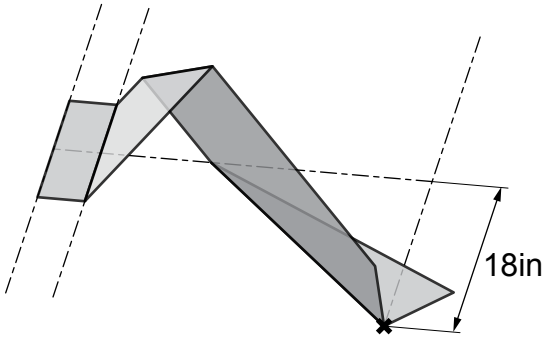
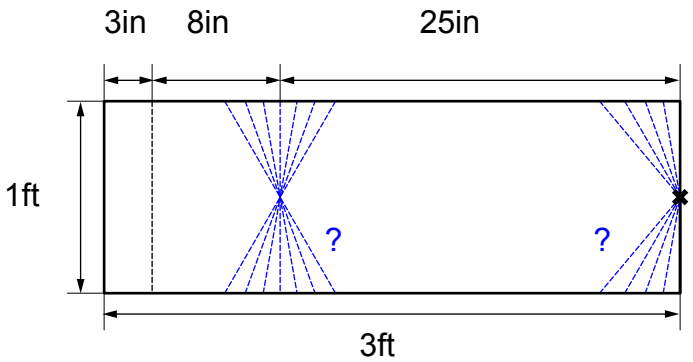


unfolded shapes

PLANE EXERCISE







FOLD 3’ x 1’ SHEET TO HAVE THE MID POINT OF RIGHT EDGE ON THE SPECIFIED LOCATION LIKE THE LEFT DIAGRAM.